

ONE



Prifysgol Cymru
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Complete, fold and seal
(see note 2 below)

SURNAME: BIGGS

FORENAME(S): EMMA ALICE

Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
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1	Student Number	2308431
2	Title of Examination Paper	MATHEMATICS PAPER 1
3	Module Code	ACDA4008
4	Date and Time of Examination	09/01/2024 09:00
5	Examination Venue	1Q310
6	Seat Number	017
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
li	↑
lii	
liii	
liv	25
lv	↓
2i	20
2ii	↑
3	10
4	14
5	10
6	20
Total	99

99%
Excellent!

$$\text{ii) } 6x + 2y - 5z - 3x - 9y + 3z + 2x - 5y + 8z \\ = 5x - 12y + 6z$$

$$\text{iii) } x(x+3) - x(x-4) \\ = x^2 + 3x - (x^2 - 4x) \\ = x^2 + 3x - x^2 + 4x \\ = 7x$$

$$\text{liii) } 2p^2qr \times 4pq^3 \times 6qr^2 \\ = 48p^3q^5r^3$$

$$\text{liv) } \frac{12p^5qr}{4p^3q^3r^{-2}}$$

$$= 3p^2q^{-2}r^3 \\ = \frac{3p^2r^3}{q^2}$$

$$\text{lv) } \frac{fg}{16h} \times \frac{8gh^2}{4fh} \times \frac{32fh^3}{12f^3}$$

$$= \frac{256f^2g^2h^5}{768f^4h^2}$$

$$= \frac{1}{3}(f^{-2}g^2h^3) \\ = \frac{g^2h^3}{3f^2}$$

$$\text{2i) } 5(x+3) = 2(x-8) \\ \Rightarrow 5x + 15 = 2x - 16 \\ \Rightarrow 3x + 15 = -16 \\ \Rightarrow 3x = -31 \\ \Rightarrow x = -31/3$$

$$2ii) \frac{(x-3)}{4} = x + \frac{(3x+4)}{8}$$

X 8

$$\Rightarrow 2(x-3) = 8x + 3x + 4$$

$$\Rightarrow 2x - 6 = 11x + 4$$

$$\Rightarrow 2x - 10 = 11x$$

$$\Rightarrow 9x = -10$$

$$\Rightarrow x = -10/9$$

$$3) \quad ① \quad 2x + 4y = 6$$

$$② \quad 3x - 5y = -2$$

$$5 \times ① \Rightarrow 10x + 20y = 30$$

$$4 \times ② \Rightarrow 12x - 20y = -8$$

$$\begin{array}{r} (5 \times ①) + (4 \times ②) \Rightarrow 10x + 20y = 30 \\ + \quad 12x - 20y = -8 \\ \hline 22x = 22 \\ x = 1 \end{array}$$

$$\text{sub } x=1 \text{ into } ①$$

$$2(1) + 4y = 6$$

$$\Rightarrow 2 + 4y = 6$$

$$\Rightarrow 4y = 4$$

$$\Rightarrow y = 1$$

12

10

$$4) A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

Shawals +1 $\begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix}$ etc

$$\begin{aligned} \det(A) &= 1[(-1 \times 1) - (2 \times 2)] - 2[(2 \times 1) - (3 \times 2)] + 3[(2 \times 2) - (3 \times 1)] \\ &= 1[-1 - 4] - 2[2 - 6] + 3[4 - 3] \\ &= -5 - 2(-4) + 3(1) \\ &= -5 + 8 + 3 \\ &= 6 \end{aligned}$$

matrix of minors

$$\begin{bmatrix} [(-1 \times 1) - (2 \times 2)] & [(2 \times 1) - (3 \times 2)] & [(2 \times 2) - (3 \times 1)] \\ [(2 \times 1) - (3 \times 2)] & [(1 \times 1) - (3 \times 3)] & [(1 \times 2) - (3 \times 2)] \\ [(2 \times 2) - (3 \times 1)] & [(1 \times 2) - (3 \times 2)] & [(1 \times 1) - (2 \times 2)] \end{bmatrix}$$

$$= \begin{bmatrix} (-1-4) & (2-6) & (4-3) \\ (2-6) & (1-9) & (2-6) \\ (4-3) & (2-6) & (1-4) \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -4 & 1 \\ -4 & -8 & -4 \\ 1 & -4 & -3 \end{bmatrix}$$

matrix of cofactors

$$\begin{bmatrix} -5 & 4 & 1 \\ 4 & -8 & 4 \\ 1 & 4 & -3 \end{bmatrix}$$

adjoint / transpose

$$\begin{bmatrix} -5 & 4 & 1 \\ 4 & -8 & 4 \\ 1 & 4 & -3 \end{bmatrix}$$

inverse $A^{-1} =$

$$A^{-1} = \frac{1}{\det(A)} \times \begin{bmatrix} -5 & 4 & 1 \\ 4 & -8 & 4 \\ 1 & 4 & -3 \end{bmatrix} = \begin{bmatrix} -5/6 & 1/3 & 1/6 \\ 1/3 & -2/3 & 1/3 \\ 1/6 & 1/3 & -1/2 \end{bmatrix}$$

$$5) \text{ let } A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = 1/24 \times \begin{bmatrix} -5 & 4 & 7 \\ 4 & -8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$

$$A \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

write as $A\underline{x} = \underline{b}$

$$\Rightarrow A^{-1} A \underline{x} = A^{-1} \underline{b}$$

$$A^{-1} \times A \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$\text{since } I \underline{x} = \underline{x}$$

$$\Rightarrow \underline{x} = A^{-1} \underline{b}$$

$$I \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \times \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \times \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1/24 \times \begin{bmatrix} -5 & 4 & 7 \\ 4 & -8 & 4 \\ 7 & 4 & -5 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$$= 1/24 \times \begin{bmatrix} (-5 \times 2) + (4 \times 5) + (7 \times 2) \\ (4 \times 2) + (-8 \times 5) + (4 \times 2) \\ (7 \times 2) + (4 \times 5) + (-5 \times 2) \end{bmatrix}$$

$$= 1/24 \times \begin{bmatrix} -10 + 20 + 14 \\ 8 - 40 + 8 \\ 14 + 20 - 10 \end{bmatrix}$$

$$= 1/24 \times \begin{bmatrix} 24 \\ -24 \\ 24 \end{bmatrix}$$

continues on next page

5)

~~$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$~~

~~$$x = 1$$~~

~~$$y = -1$$~~

~~$$z = 1$$~~

Emma, This is an
Excellent effort!

Very, well Done!

(10)

6)

~~$$\frac{x+2}{x^2+2x-3} = f(x)$$~~

Factorise Denominator and solve

~~$$x^2+2x-3=0$$~~

~~$$(x-1)(x+3)=0$$~~

~~$$x=1 \text{ or } x=-3$$~~

No need to solve?

~~$$\frac{x+2}{x^2+2x-3} = \frac{x+2}{(x-1)(x+3)}$$~~

~~$$\frac{x+2}{(x-1)(x+3)} = \frac{A}{(x-1)} + \frac{B}{(x+3)}$$~~

~~multiply both sides $\Rightarrow x+2 = \frac{A(x-1)(x+3)}{(x-1)} + \frac{B(x-1)(x+3)}{(x+3)}$
by $(x-1)(x+3)$~~

~~$$\Rightarrow x+2 = A(x+3) + B(x-1)$$~~

Finding A

~~$$\text{let } x = 1$$~~

~~$$1+2 = A(1+3) + B(1-1)$$~~

~~$$3 = 4A + 0B$$~~

~~$$A = 3/4$$~~

Finding B

~~$$\text{let } x = -3$$~~

~~$$-3+2 = A(-3+3) + B(-3-1)$$~~

~~$$-1 = 0A - 4B$$~~

~~$$B = 1/4$$~~

~~$$\frac{x+2}{x^2+2x-3} = \frac{3}{4(x-1)} + \frac{1}{4(x+3)}$$~~

(20)



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OME

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1	Student Number	2308431
2	Title of Examination Paper	MATHEMATICS PAPER 2
3	Module Code	ACDA4008
4	Date and Time of Examination	11/01/2024 3pm
5	Examination Venue	1Q 310
6	Seat Number	049
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
1a	
1b	
1c	
2a	
2b	
2c	
3a	
3b	
3c	
3d	
3e	
4a	
4b	
Total	

98%

Excellent!

1ai) $y = 8x^3 + 3$
 $\frac{dy}{dx} = 24x^2$ ✓

(2)

1aii) $y = 4x^3 + 10x^{1/2} + 5/6$
 $\frac{dy}{dx} = 12x^2 + 5x^{-1/2}$

(3)

1aiii) $y = 2x^{-5} + x^3$
 $\frac{dy}{dx} = -10x^{-6} + 3x^2$

(3)

1aiv) $y = \sin(5x)$ $\begin{matrix} -c & s \\ s & -c \end{matrix}$
 $\frac{dy}{dx} = 5\cos(5x)$

(2)

1av) $y = x^2 \cos(x)$
 $\frac{dy}{dx} = 2x \cos(x) - x^2 \sin(x)$

$u = x^2$ $v = \cos(x)$
 $\frac{du}{dx} = 2x$ $\frac{dv}{dx} = -\sin(x)$

(4)

1avi) $y = (9x+3)^{10}$
 $\frac{dy}{dx} = 10(9x+3)^9 (9)$
 $= 90(9x+3)^9$

$u = 9x+3$ $y = u^{10}$
 $\frac{du}{dx} = 9$ $\frac{dy}{du} = 10u^9$
 $\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

(3)

1avii) $y = x + 3\tan(x)$
 $\frac{dy}{dx} = 1 + 3\sec^2(x)$

(2)

$$1b) \quad y = x^{-1} + \cos(3x) \quad -c^3 c$$

$$\frac{dy}{dx} = -x^{-2} - 3\sin(3x) \quad -5$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 9\cos(3x)$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 27\sin(3x)$$

$$\frac{d^4y}{dx^4} = 24x^{-5} + 81\cos(3x)$$

$$1c) \quad y = \frac{(2x+1)}{(3+x^2)}$$

$$\text{let } f(x) = 2x+1 \quad \text{let } g(x) = 3+x^2$$

$$f'(x) = 2 \quad g'(x) = 2x$$

$$\frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2(3+x^2) - 2x(2x+1)}{(3+x^2)^2}$$

$$= \frac{6+2x^2-4x^2-2x}{(3+x^2)^2}$$

$$= \frac{6-2x^2-2x}{(3+x^2)^2}$$

$$= \frac{2(3-x^2-x)}{(3+x^2)^2}$$

when $x=0$

$$\frac{dy}{dx} = \frac{2(3-(0^2)-0)}{(3+0^2)^2}$$

$$= 6/9 = 2/3$$

$$2ai) \int 10x + x^3 dx$$

$$I = 5x^2 + \frac{1}{4}x^4 + c$$

(3)

$$2aii) \int 2x + 3e^x dx$$

$$I = x^2 + 3e^x + c$$

(3)

$$2aiii) \int 2x^{-1} + 4 dx$$

$$I = 2 \ln x + 4x + c$$

(3)

$$2aiv) \int (x-1)(3x+4) dx$$

$$= \int 3x^2 + x - 4 dx$$

$$I = x^3 + \frac{1}{2}x^2 - 4x + c$$

(3)

$$2av) \int_3^6 (3x^2 - 2x) dx$$

$$I = [x^3 - x^2]_3^6$$

$$= (6^3 - 6^2) - (3^3 - 3^2)$$

$$= (216 - 36) - (27 - 9)$$

$$= 180 - 18$$

$$= 162$$

(3)

2b) $\int_0^1 \frac{e^x}{\sqrt{e^x+1}} dx$

let $u = e^x + 1$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$\frac{du}{dx}$	u
1	$e+1$
0	2

$\int_2^{e+1} \frac{1}{\sqrt{u}} du$

$= \int_2^{e+1} u^{-1/2} du$

$I = \left[2u^{1/2} \right]_2^{e+1}$

$= 2(e+1)^{1/2} - 2(2^{1/2})$

$= 3.85656937 - 2.828427125$

$= 1.028 \text{ (3 dp)}$

Final

5

2c) $\int \sec(x) dx$ $-e^s$ $-s$ e

$I = \sin(x) \sec(x) - \int \sin(x) (1) dx$

$= x \sin(x) - \int \sin(x) dx$

$= x \sin(x) + \cos(x) + C$

5

2c ii)

$$\int_1^e \frac{1}{x} \ln x \, dx$$

$$I = \left[x(\ln x) - \int x \left(\frac{1}{x} \right) dx \right]_1^e$$

$$= \left[x(\ln x) - \int 1 \, dx \right]_1^e$$

$$= \left[x(\ln x) - x \right]_1^e$$

express
Better:

$$= (e(\ln e) - e) - (1(\ln 1) - 1)$$

$$= (e(1) - e) - (1(0) - 1)$$

$$= (e - e) - (0 - 1)$$

$$= 1$$

$$3ai) \quad (3+2i)(1-i) \\ = 3 - 3i + 2i + 2 \\ = 5 - i$$

2

$$3aii) \quad \frac{3}{(2-i)} \\ = \frac{3(2+i)}{(2-i)(2+i)}$$

$$= \frac{6+3i}{4+1}$$

$$= \frac{6}{5} + \frac{3}{5}i$$

2

$$3aiii) \quad \frac{1}{is}$$

$$\left. \begin{array}{l} i \times i = -1 \\ -i \times i = 1 \\ i \times i = -1 \\ -i \times i = 1 \end{array} \right\} is = i$$

$$\frac{1}{is} = \frac{1}{i}$$

$$\frac{1}{i} \times \frac{-i}{-i} = \frac{-i}{1}$$

$$\frac{-i}{1} = -i$$

2

$$3b) \quad z_1 = \frac{2-j}{2+j}$$

$$\frac{2-j}{2+j} = \frac{(2-j)(2-j)}{(2+j)(2-j)}$$

$$= \frac{4-4j-1}{4+1}$$

$$= \frac{3-4j}{5}$$

$$= \frac{3}{5} - \frac{4}{5}j$$

$$z_2 = \frac{3+j}{1-2j}$$

$$\frac{3+j}{1-2j} = \frac{(3+j)(1+2j)}{(1-2j)(1+2j)}$$

$$= \frac{3+7j-2}{1+4}$$

$$= \frac{1+7j}{5}$$

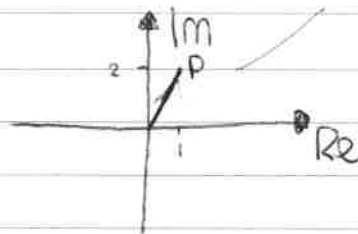
$$= \frac{1}{5} + \frac{7}{5}j$$

6

$$3ci) \quad p = z_1 + 2z_2$$

$$= \frac{3}{5} - \frac{4}{5}j + 2\left(\frac{1}{5} + \frac{7}{5}j\right)$$

$$= 1 + 2j$$

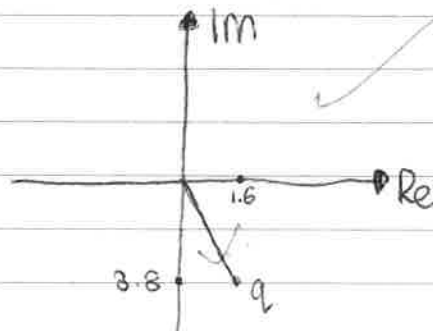


3

$$3cii) \quad q = 3z_1 - z_2$$

$$= 3\left(\frac{3}{5} - \frac{4}{5}j\right) - \left(\frac{1}{5} + \frac{7}{5}j\right)$$

$$= \frac{8}{5} - \frac{19}{5}j$$



3

$$\begin{aligned} 3a) \quad z &= (1+2i)(1+i) \\ &= 1+3i-2 \\ &= 3i-1 \end{aligned}$$

$$\begin{aligned} |z| &= \sqrt{3^2 + (-1)^2} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} \arg(z) &= \tan^{-1}(3/-1) \\ &= 71.565^\circ \text{ (3dp)} \end{aligned}$$

$$z = \sqrt{10}(\cos(71.565^\circ) + i\sin(71.565^\circ))$$

6

$$3e) \quad \frac{3+i}{3-i} = \frac{(3+i)(3+i)}{(3-i)(3+i)}$$

$$= \frac{9+6i-1}{9+1}$$

$$= \frac{8+6i}{10}$$

$$= \frac{4}{5} + \frac{3}{5}i$$

$$\begin{aligned} \text{modulus} &= \sqrt{(4/5)^2 + (3/5)^2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{argument} &= \tan^{-1}(3/4) \\ &= 36.87^\circ \text{ (2dp)} \end{aligned}$$

$$\frac{3+i}{3-i} = 1(\cos(36.87^\circ) + i\sin(36.87^\circ))$$

6

4ai) $4^{2x} = 3^{x+1}$
 $\Rightarrow \log(4^{2x}) = \log(3^{x+1})$
 $\Rightarrow 2x \log(4) = (x+1) \log(3)$
 $\Rightarrow 2x \log(4) = x \log(3) + \log(3)$
 $\Rightarrow 2x \log(4) - x \log(3) = \log(3)$
 $\Rightarrow x(2 \log(4) - \log(3)) = \log(3)$
 $\Rightarrow x = \frac{\log(3)}{2 \log(4) - \log(3)}$
 $\Rightarrow x = 0.66 \text{ (2dp)}$

4aii) $2 \ln x = \ln(4x-4)$
 $\Rightarrow \ln x^2 = \ln(4x-4)$
 $\Rightarrow x^2 = 4x-4$
 $\Rightarrow x^2 - 4x + 4 = 0$
 $\Rightarrow (x-2)(x-2) = 0$
 $\Rightarrow x = 2 \text{ (repeated)}$

4bi) $\log(x^4 y z)$
 $= \log(x^4) + \log(y) + \log(z)$
 $= 4 \log(x) + \log(y) + \log(z)$

4bii) $\ln\left(\frac{x^4 y}{z}\right)$
 $= \ln(x^4) + \ln(y) - \ln(z)$
 $= 4 \ln(x) + \ln(y) - \ln(z)$

This is excellent Emma
 Very well done!