

ONE

Complete, fold and seal
(see note 2 below)

SURNAME: SIMPSON

FORENAME(S): George

Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes provided to the right of the question paper and seal down (using the label provided) to hide your answers.
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8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	231955 ~~56~~
2	Title of Examination Paper	mathematics ~~ACDA4008~~
3	Module Code	ACDA4008
4	Date and Time of Examination	09/01/24
5	Examination Venue	1Q310
6	Seat Number	052
7	Number of Answer Books handed in	1

[illegible]

54%

①

Simplify

Q1

$$i) \cancel{6x} + \cancel{2y} - \cancel{5z} - \cancel{3x} - \cancel{9y} + \cancel{3z} + \cancel{2x} - \cancel{8y} + \cancel{8z}$$

$$= 5x - 12y + 6z$$

$$ii) x(x+3) - x(x-4)$$

$$= x^2 + 3x - x^2 + 4x$$

$$= 2x^2 + 7x$$

$$iii) \cancel{2p^2} \cancel{qr} \times \cancel{4p} \cancel{q^3} \times \cancel{6} \cancel{qr^2}$$

$$= 48p^3 q^5 r^3$$

$$iv) \frac{\cancel{12} \cancel{p^5} \cancel{q} \cancel{r}}{\cancel{4} \cancel{p^3} \cancel{q^3} \cancel{r^{-2}}} = 3p^2 q^{-2} r^{-1} \xleftarrow{1-(-2)} r^3$$

~~FR 2402~~

$$v) \frac{fg}{16h} \times \frac{8gh^2}{4fh} \times \frac{32fh^3}{12f^3}$$

$$\frac{\cancel{f} \cancel{g}}{\cancel{2} \cancel{h}} \times \frac{\cancel{g} \cancel{h^2}}{\cancel{f} \cancel{h}} \times \frac{\cancel{2} \cancel{8} \cancel{f} \cancel{h^3}}{\cancel{3} \cancel{12} \cancel{f^3}}$$

$$= \frac{2f^2 g^2 h^5}{6h^2 f^4} = \frac{1f^{-2} h^3 g^2}{3} = \frac{1}{3} f^{-2} h^3 g^2$$

(2)

Question No.

Examiner

Q2)

$$i) 5(x+3) = 2(x-8)$$

$$5x + 15 = 2x - 16$$

$$~~5x + 31 = 2x~~$$

$$3x + 15 = -16$$

$$3x = -16 - 15 = -31$$

$$x = -31/3 \quad x = -10\frac{1}{3}$$

$$ii) \frac{(x-3)}{4} = x + \frac{(3x+4)}{8}$$

x4 here?

$$\frac{(x-3)}{1} = 4x + \frac{(3x+4)}{2}$$

$$x-3 = 4x + (1.5x+2)$$
$$x-3 = 1.5x^2 + 2x$$

$$4x - 12 = x + 24x + 32$$

$$4x = x + 24x + 44$$

$$\begin{aligned} \text{Q3)} \quad & 2x + 4y = 6 \\ & \cancel{2x} + \cancel{4y} \\ & 3x - 5y = -2 \end{aligned}$$

$$6x + 12y = 18 \rightarrow (6 + 12) = 18$$

$$- \quad 6x - 10y = -4$$

$$22y = 22$$

$$\underline{\underline{y = i}}$$

$$6x + 12 = 18$$

$$6x = (18 - 12) \div 6$$

$$\underline{x = 7}$$

10

④

Question No.

Examiner

Q4)
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

$$A^{-1} = 1 \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix} - 2 \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} + 3 \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$$

$$\begin{aligned} & \downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow \\ & + 1 \cdot ((-1) - (4)) = \underline{\underline{-5}} \quad -2(2 - 6) = \underline{\underline{8}} \quad 3(4 - 3) = \underline{\underline{21}} \end{aligned}$$

~~AA^{-1} = I~~
~~BA^{-1} = I~~
~~Det A = 24~~

C =
$$\begin{bmatrix} \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix} - \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \\ + \begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \\ + \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} \end{bmatrix}$$

→ Continued on page ⑧!

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Question No

Examiner Only

$$Q5) x + 2y + 3z = 2$$

$$2x - y + 2z = 5$$

$$3x + 2y + z = 2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix}$$

$AX = b$
 $\Rightarrow X = A^{-1}b$

Determinant $\Delta = +1 \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$

Determinant $\Delta = +1(-5) - 2(-4) + 3(7)$
 $-5 + 8 + 21 = 24$
 ~~$= -5 - 8 - 21 = -34$~~

$$\text{Adj } A = \begin{bmatrix} \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -5 & 4 & 7 \\ -4 & -8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$

⑥

$$\begin{array}{ccc} + & - & + \\ - & \swarrow & + \\ + & - & + \end{array}$$

$$\begin{bmatrix} -5 & -4 & 7 \\ -4 & 8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$

$$A^{-1} = \left(\frac{1}{\det A} \right) \text{adj } A$$

$$A^{-1} = \begin{bmatrix} -5/24 & -4/24 & 7/24 \\ -4/24 & 8/24 & 4/24 \\ 7/24 & 4/24 & -5/24 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -2/3 \\ -10/3 \\ 2/3 \end{bmatrix}$$

$$\begin{array}{l} x = -2/3 \\ y = -10/3 \\ z = 2/3 \end{array}$$

THEN

Reflect!

⑥

=

~~$$\begin{bmatrix} -5 & -4 & 7 \\ -4 & 8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$~~

$$\begin{bmatrix} -5 & -4 & 7 \\ -4 & 8 & 4 \\ 7 & 4 & -5 \end{bmatrix}$$

7

$$x = -2/3$$

$$y = -10/3$$

$$z = 2/3$$

Q6 $f(x) = \frac{x+2}{x^2+2x-3}$

$$x^2 + 2x - 3 =$$

$$f(x) = \frac{x+2}{(x+3)(x-1)}$$

Let $f = \frac{x+2}{(x+3)(x-1)}$

$$x = (x+3)(x-1)$$

$$f(x) = x^3 + 2x - 6$$

$$(x^2 + 3)(x-2)$$

$$\frac{x+2}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

$$\begin{aligned} (x+3)(x-1) &= (x-1)(x+2) \\ x^2 + 2x - 3 &= x^2 + x - 2 \\ (x+3)(x-1) &= x^2 - 2x + x \\ (x+3)(x-1) &= x^2 - x - 1 + 2 \\ (x+3)(x-1) &= x^2 + 3x - x - 3 \end{aligned}$$

2

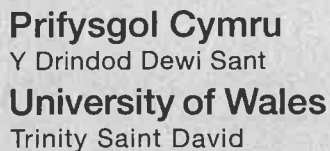
⑧

$$\begin{bmatrix} -5 & 8 & 21 \\ -4 & -8 & -4 \\ 7 & -5 & -5 \end{bmatrix}$$

Reflect!

$$\begin{bmatrix} -5 & +4 & 7 \\ 8 & -8 & -5 \\ 21 & +4 & -5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -5/24 & -4/24 & 7/24 \\ 8/24 & 8/24 & -5/24 \\ 21/24 & -4/24 & -5/24 \end{bmatrix} \begin{matrix} -1/3 \\ = 5/24 \\ = 5/4 \end{matrix}$$



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2	Title of Examination Paper	Mathematics
3	Module Code	ACDA4008
4	Date and Time of Examination	11/01/24
5	Examination Venue	1A310
6	Seat Number	015
7	Number of Answer Books handed in	01

[illegible]

32%

(Gweler yr ochr arall am y fersiwn Cymraeg)

a

$$i) y = 8x^3 + 3 \quad \frac{dy}{dx} = 24x^2$$

(2)

$$ii) y = 4x^3 + 10x^{1/2} + 5/6$$

$$\frac{dy}{dx} = 12x^2 + 5x^{-1/2}$$

(3)

$$iii) y = \frac{2}{x^5} + x^3$$

$$y = 2x^{-5} + x^3$$

$$\frac{dy}{dx} = -\frac{10}{x^6} + 3x^2$$

$$\frac{dy}{dx} = -10x^{-6} + 3x^2$$

Sin

Cos

-Sin

-Cos

(2)

$$iv) y = \sin(5x)$$

$$\frac{dy}{dx} = 5 \cos(5x)$$

(2)

~~sin~~

$$v) y = x^2 \cos(x)$$

$$u = x^2 \quad v = \cos(x)$$

$$u' = 2x \quad v' = -\sin(x)$$

$$\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = (x^2 - \sin(x)) + (x \cos(x))$$

factorise if you can.

(4)

$$b) \frac{dy}{dx} = -x^{-2} - 3 \sin(3x) \quad \checkmark$$

$$\frac{d^2 y}{dx^2} = -2x^{-3} - 9 \cos(3x) \quad (\checkmark) \quad \text{ecf}$$

$$\frac{d^3 y}{dx^3} = 6x^{-4} + 27 \sin(3x) \quad (\checkmark)$$

$$\frac{d^4 y}{dx^4} = 24x^{-5} + 81 \cos(3x) \quad \checkmark$$

③



vi) ~~1/10~~

$$y = (9x + 3)^{10}$$

$$u = 9x + 3 \quad v = 10$$

$$u' = 9 \quad v' = 0$$

$$\frac{dy}{dx} = 10(9x + 3) \cdot 9$$

CHAIN

$$\frac{dy}{dx} = f'[g(x)] \cdot g'(x)$$

m s ad

$$9x' \quad 9x'$$

(2)

sin

cos

-sin

-cos

vii) $y = (x + 3 \cdot \tan(x))$

$$u = x$$

$$v = 3 \tan(x)$$

$$u' = 1$$

$$v' = 3 \sec^2(x)$$

no! not
product
Rule!

$$x + \sec^2(x) + \tan(x)$$

(1)

b) given $y = \frac{1}{x} + \cos(3x)$, find $\frac{d^4 y}{dx^4}$

$$\frac{dy}{dx} = -x^{-2} - 3 \sin(3x)$$

$$\frac{d^2 y}{dx^2} = \frac{1}{x^3} + 9 \cos(3x)$$

$$\frac{d^3 y}{dx^3} = -\frac{3}{x^4} - 27 \sin(3x)$$

$$\frac{d^4 y}{dx^4} = \frac{12}{x^5} + 81 \cos(3x)$$

$$c) y = \frac{(2x+1)}{3+x^2} = \frac{u}{v}$$

$$u = 2x+1$$

$$v = 3+x^2$$

$$u' = 2$$

$$v' = 2x$$

$$\frac{dy}{dx} = \frac{(3+x^2)2 - (2x+1)(2x)}{(3+x^2)^2}$$

$$\frac{dy}{dx} = \frac{x^2 + 3}{(3+x^2)^2}$$

Mistake
Error in Simplifying

if $x = 0$ then

$$\frac{dy}{dx} = \frac{3}{3^2} = \frac{1}{3}$$

④

2

$$i) \int (10x' + x^3) dx$$

$$U \frac{dU}{dx} \cdot dx = UV - \int U \frac{dU}{dx}$$

$$\frac{dy}{dx} = \cancel{10x^2} \left(\frac{10x^2}{2} + \frac{x^4}{4} \right) + C$$

no!
not
diff

$$\frac{dy}{dx} = 5x^2 + x^{-4} + C$$

(2)

$$ii) \int (2x' + 3e^x) dx$$

$$\frac{dy}{dx} = \frac{2x^2}{2} + C + 3e^x \quad \frac{dy}{dx} = x^2 + C$$

(2)

$$iii) \int \frac{2}{x} + 4(dx) = x^{-2} + 4(dx)$$

$$\frac{dy}{dx} = x + 4(dx)$$

$$\frac{dy}{dx} = x + C$$

(1)

$$\int f[g(x)] \cdot g'(x) dx = \int f(u) du$$

iv) $\int (x-1) \cdot (3x+4) dx$ expand & integrate the normal

$$\int f[g(x)] \cdot g'(x) dx = \int f(u)$$

v) $\left(\frac{x^2}{2}\right) + \left(\frac{3x^2}{2}\right) dx \quad \frac{dy}{dx} (x^{-2}) (3x^{-2}) + C$

$$\int_3^6 (3x^2 - 2x) dx$$

$$\left[x^3 - x^2 \right]_3^6 =$$

$$\left[\frac{3x^3}{3} - \frac{2x^2}{2} \right] = [x^3 - 2x^2]$$

$$\int = (6^3 - 6^2) - (3^3 - 3^2) = 162$$

2

b $\int_0^1 \frac{e^x}{\sqrt{e^x + 1}} dx$

~~1~~ $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

$$= \int f[g(x)] \cdot g'(x) dx = \int f(u) \cdot du$$

let $u = g(x)$

$$\int_a^b f(x) dx = \left[F(x) \right]_a^b$$

α

$$= F(b) - F(a)$$

NO 3

$$i) (3+2i)(1-i)$$

$$= 3 + 3i \quad X$$

$$3 - 3i + 2i + 3i$$

$$= 2i + 3$$

C

⑥

No 4

$$i. 4^{2x} = 3^{x+1}$$

$$4 \log 2^x = 3 \log^{x+1} \quad \checkmark$$

①