



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
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1	Student Number	DAV22868331
2	Title of Examination Paper	Mathematics
3	Module Code	ISOEN4001
4	Date and Time of Examination	19/05/25 13:30
5	Examination Venue	Tycho Main Hall
6	Seat Number	H15
7	Number of Answer Books handed in	1

Complete, fold and seal
(see note 2 below)

SURNAME: Jairc Fabricic

FORENAME(S): David Sanchez

List the numbers of the questions you have attempted	Office Use
Q1	30
Q2	30
Q3	32
Total	92/100

(Gweler yr ochr arall am y fersiwn Cymraeg)

RK

92%

Question 3

Q332/32

$$i) \int (25x^4 + 12x^2 + 6x - 10) dx$$

$$= \int (25x^4) dx + \int 12x^2 dx + \int 6x dx - 10 \int dx$$

$$= \frac{25x^5}{5} + \frac{12x^3}{3} + \frac{6x^2}{2} - 10x + C$$

$$= 5x^5 + 4x^3 + 3x^2 - 10x + C$$

Q3a18/18

$$ii) \int (\sin 7x + 6 \cos(3x) + 5e^{5x}) dx$$

$$= \int \sin 7x dx + 6 \int \cos(3x) dx + 5 \int e^{5x} dx$$

$$= -\frac{\cos 7x}{7} + \frac{6 \sin(3x)}{3} + \frac{5 e^{5x}}{5} + C$$

$$= -\frac{1}{7} \cos 7x + 2 \sin(3x) + e^{5x} + C$$

$$iii) \int \frac{10\theta^4 + 2}{2\theta^5 + 2\theta + \pi} d\theta$$

$$= \ln(2\theta^5 + 2\theta + \pi) + C$$

$$iv) \int \left(\frac{16}{f} + \ln(e^2) + 3 \right) df$$

$$\therefore \ln(e) = 1$$

$$\ln(a)^2$$

$$2 \ln(a)$$

$$= 16 \int \frac{1}{f} df + 2 \int df + 3 \int df$$

$$= 16 \ln(f) + 2f + 3f + C$$

$$= 5f + 16 \ln(f) + C$$

$$v) \int_1^5 (2x^3 + 5x^2 + 3x - 9) dx$$

$$\Rightarrow \int (2x^3 + 5x^2 + 3x - 9) dx$$

$$= \frac{2x^4}{4} + \frac{5x^3}{3} + \frac{3x^2}{2} - 9x$$

$$\int_1^5 (2x^3 + 5x^2 + 3x - 9) dx = \frac{x^4}{2} + \frac{5x^3}{3} + \frac{3x^2}{2} - 9x \Big|_1^5$$

$$\Rightarrow \frac{(5)^4}{2} + \frac{5(5)^3}{3} + \frac{3(5)^2}{2} - 9(5) - \left[\frac{(1)^4}{2} + \frac{5(1)^3}{3} + \frac{3(1)^2}{2} - 9(1) \right]$$

~~$$= \frac{625}{2} + \frac{625}{3} + \frac{75}{2} - 45 - \frac{1}{2} - \frac{5}{3} - \frac{3}{2} + 9$$~~

$$\Rightarrow \frac{625}{2} + \frac{625}{3} + \frac{75}{2} - 45 - \frac{1}{2} - \frac{5}{3} - \frac{3}{2} + 9$$

$$= 312.5 + 208.333 + 37.5 - 45 - 0.5 - 1.666 - 1.5 + 9$$

$$= 518.667$$

$$vi) \int_0^4 (2e^{2x} + \sin \frac{x}{2}) dx$$

$$\Rightarrow \int 2e^{2x} dx + \int \sin(\frac{x}{2}) dx$$

$$= e^{2x} + (-2\cos \frac{x}{2})$$

3

 $e^0 = 1$

$$\int_0^4 (2e^{2x} + \sin \frac{x}{2}) dx = e^{2x} - 2\cos \frac{x}{2} \Big|_0^4$$

$$e^{2(4)} - 2\cos(4/2) - e^{2(0)} + 2\cos(0/2)$$

$$= 2980.95 - 1.998 - 1 + 2$$

$$= 2979.952$$

$$b) \int x^2 (4x^3 + 9)^5 dx$$

$$u = 4x^3 + 9$$

Q3b

$$\int \frac{(u)^5 du}{12} = \frac{1}{12} \int u^5 du$$

$$\frac{du}{dx} = 12x^2$$

$$\frac{du}{12} = x^2 dx$$

6/6

$$\frac{1}{12} \frac{u^6}{6} + C$$

$$= \frac{u^6}{72} \Rightarrow \frac{(4x^3 + 9)^6}{72} + C$$

~~$$c) \int 5x e^{5x} dx$$~~

~~$$\int u dv dx = uv - \int v \frac{du}{dx} dx$$~~

~~$$\int 5x e^{5x} dx = \frac{5x e^{5x}}{5} - \int \frac{e^{5x}}{5} \frac{du}{5}$$~~

~~$$= \frac{e^{5x}}{25} - \frac{1}{25} \int e^u du$$~~

Q3e

$$\int 5x e^{5x} dx$$

$$= 5x \frac{e^{5x}}{5} - \int \frac{e^{5x}}{5} 5 dx$$

$$= x e^{5x} - \frac{e^{5x}}{5} + C$$

$$u = 5x$$

$$du = 5 dx$$

$$dv = e^{5x} dx$$

$$v = \frac{e^{5x}}{5}$$

Q3c8/8

Question 2

a) $z_1 = 3 + 4j$, $z_2 = 5 + 3j$, $z_3 = 2 + 6j$

Q2

30/34

i) $z_1 + z_2$

$$(3 + 4j) + (5 + 3j) \checkmark \underline{\underline{1}}$$

$$= 8 + 7j \checkmark$$

Q2a

15/18

ii) $2z_2 + 3z_3$

$$2(5 + 3j) + 3(2 + 6j)$$

$$= 10 + 6j + 6 + 18j \checkmark \underline{\underline{2}}$$

$$16 + 24j \checkmark$$

iii) $z_1^2 + z_3$

$$(3 + 4j)^2 + 2 + 6j$$

$$9 + 24j - 16 + 2 + 6j \checkmark \underline{\underline{3}}$$

$$-5 + 30j \checkmark$$

iv) $z_1 \cdot z_2 \cdot z_3$

$$(3 + 4j)(5 + 3j)(2 + 6j) \checkmark$$

$$(3 + 29j)(2 + 6j) \checkmark$$

$$-168 + 76j \checkmark \underline{\underline{4}}$$

$$j^2 = -1 \checkmark$$

$$(15 + 9j + 20j - 12)$$

$$3 + 29j \checkmark$$

v) $|z_1 \cdot z_2 \cdot z_3|$

$$= |-168 + 76j| (= 168 + 76j) \times 0 =$$

$$6 + 18j + 58j - 174$$

vi) $\frac{z_1}{z_2}$

$$\Rightarrow \frac{3 + 4j}{5 + 3j} \Rightarrow \underline{\underline{5}} \frac{3 + 4j}{5 + 3j} \cdot \frac{5 - 3j}{5 - 3j} \checkmark$$

$$\checkmark$$

$$\frac{15 - 9j + 20j + 12}{25 - 15j + 15j + 9}$$

$$27 + 11j$$

$$34$$

$$b) \quad 5x^2 - 8x + 5 = 0$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(5)}}{2(5)}$$

$$x = \frac{8 \pm \sqrt{-36}}{10}$$

$$x = \frac{8 \pm 6j}{10} \quad x_1 = \frac{8}{10} + \frac{6}{10}j$$

$$x_2 = \frac{8}{10} - \frac{6}{10}j$$

$$r = \sqrt{a^2 + b^2}$$

$$r = \sqrt{\left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2}$$

$$r = \sqrt{(0.64) + 0.36}$$

$$r = 1$$

$$\tan^{-1} \theta =$$

$$\theta = \tan^{-1} \left(\frac{0.6}{0.8} \right) = 36.869^\circ$$

Polar form

$$x_1 = 1 \angle 36.869^\circ$$

$$r_2 = \sqrt{(0.8)^2 + (-0.6)^2}$$

$$r_2 = 1 \angle 323.130^\circ$$

$$\theta_2 = \tan^{-1}(-0.75)$$

$$\theta_2 = -36.869^\circ$$

$$\theta_2 + 360^\circ$$

$$\theta_1 = 323.130^\circ$$

Q2b

9/9

$$c) (3+4j)^4$$

De Moivre's Theorem

$$r = \sqrt{(3)^2 + (4)^2}$$

$$r = 5$$

$$[r \angle \theta]^n = r^n \angle n\theta = r^n (\cos n\theta + j \sin n\theta)$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\theta = 53.130^\circ$$

$$= 5^4 (\cos(4(53.130)) + j \sin(4(53.130)))$$

$$212.52$$

$$= 625(-0.84320 + (-0.53759)j)$$

$$-0.84320$$

Polar form

$$-525 \angle -335.993^\circ$$

Q206/7

Question 1

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 4 \\ -3 & 8 \end{bmatrix}$$

i) $|A|$

$$2(0 \cdot 6) - 0(+3(2)) \\ = -12 + 6$$

$$|A| = -6$$

Q130/34Q1a15/16ii) $|C|$ is not allowed 'cause is not a square matrixiii) $|D|$

$$16 + 12 = 28$$

$$|D| = 28$$

iv

A.B

$$a_{11} = 6 + 0 + 3 = 9$$

$$a_{21} = 3 + 16 + 3 = 25$$

$$a_{31} = 0 + 4 + 0 = 4$$

$$a_{12} = 0 + 6 + 9 = 9$$

$$a_{22} = 6 + 40 + 9 = 49$$

$$a_{32} = 0 + 10 + 0 = 10$$

$$a_{13} = 4 + 0 + 18 = 22$$

$$a_{23} = 2 + 0 + 18 = 20$$

$$a_{33} = 0 + 0 + 0 = 0$$

$$A.B = \begin{bmatrix} 9 & 9 & 22 \\ 25 & 49 & 20 \\ 4 & 10 & 0 \end{bmatrix}$$

3

v) B.C

$$a_{11} = 3 + 0 + 6 = 9$$

$$a_{21} = 2 + 10 + 0 = 12$$

$$a_{12} = 12 + 0 + 2 = 14$$

$$a_{22} = 8 - 25 + 0 = -17$$

$$B.C = \begin{bmatrix} 9 & 14 \\ 12 & -17 \\ 25 & -5 \end{bmatrix}$$

3

$$a_{31} = 1 + 6 + 18 = 25$$

$$a_{32} = 4 - 15 + 6 = -5$$

vi) D.C

D.C are not allowed the matrices has different size

vii) A+D

A+D are not allowed because the matrices has different size.

b) $A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 4 & 7 \\ 0 & 5 & 2 \end{bmatrix}$

$|A| = 2(14-35) - 4(2-0) + 1(5-0)$

$|A| = -42 - 8 + 5$

$|A| = -45$

$A^{-1} = \frac{\text{Adj } A}{|A|}$

$\text{Adj } A = \begin{bmatrix} -27 & 2 & 5 \\ 3 & 4 & 10 \\ 24 & 13 & 8 \end{bmatrix}$

$a_{11} = (4-35) = -27$

$a_{21} = 8-5 = 3$

$a_{12} = (2-0) = 2$

$a_{22} = 4-0 = 4$

$a_{13} = (5-0) = 5$

$a_{23} = 10-0 = 10$

$a_{31} = 28-4 = 24$

$a_{32} = 14-1 = 13$

$a_{33} = 8-4 = 8$

$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} -27 & 2 & 5 \\ 3 & 4 & 10 \\ 24 & 13 & 8 \end{bmatrix}$

$\text{Adj } A^T = \begin{bmatrix} -27 & -3 & 24 \\ -2 & 4 & -13 \\ 5 & -10 & 8 \end{bmatrix}$

$A^{-1} = \frac{\text{Adj } A^T}{|A|} = \frac{1}{-45} \begin{bmatrix} -27 & -3 & 24 \\ -2 & 4 & -13 \\ 5 & -10 & 8 \end{bmatrix} = \begin{bmatrix} 25/45 & 3/45 & -24/45 \\ 2/45 & -4/45 & 13/45 \\ -5/45 & 10/45 & -8/45 \end{bmatrix}$

E.C.F

Q11b

8/10

$$\begin{aligned} c) \quad & 3x - 2y + z = 2 \\ & x - 3y + 2z = 1 \\ & 2x + y - 3z = -5 \end{aligned}$$

$$\begin{bmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix}$$

Determinant of "F"

$$3(9-2) - (2)(-3-4) + 1(1-(-6))$$

$$|F| = 21 - 21 + 7$$

$$|F| = 7$$

$$x = \frac{\Delta x}{\Delta F} = \frac{14}{7} = 2$$

E.C.F

$$\Delta x = \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ -5 & 1 & -3 \end{bmatrix}$$

$$\Delta x = 2(9-2) + 2(-3+10) + 1(1-15)$$

$$\Delta x = 14 + 14 - 14$$

$$\Delta x = 14$$

$$y = \frac{\Delta y}{\Delta F} = \frac{28}{7} = 4$$

E.C.F

$$\Delta y = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & -5 & -3 \end{bmatrix}$$

$$\Delta y = 3(-3+10) - 2(-3-4) + 1(-5-2)$$

$$\Delta y = 21 + 14 - 7$$

$$\Delta y = 28$$

$$z = \frac{\Delta z}{\Delta F} = \frac{42}{7} = 6$$

E.C.F

$$\Delta z = \begin{bmatrix} 3 & -2 & 2 \\ 1 & -3 & 1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$\Delta z = 3(15-1) + 2(-5-2) + 2(1+6)$$

$$\Delta z = 42 - 14 + 14$$

$$\Delta z = 42$$

Q1C

7/8