

COMPOSITE FAILURE THEORIES

①

① (a) MAX STRESS FAILURE THEORY

$\theta = 60^\circ$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0.75 & 0.866 \\ 0.75 & 0.25 & -0.866 \\ -0.433 & 0.433 & -0.5 \end{bmatrix} \begin{bmatrix} 25 \\ -35 \\ 45 \end{bmatrix}$$

$$= \begin{bmatrix} 1.714 \\ -2.714 \\ -4.165 \end{bmatrix} 5$$

• MAX STRESS FAILURE THEORY

$$-(\sigma_i^c)_{ULT} < \sigma_i < (\sigma_i^t)_{ULT}, \text{ OR}$$

$$-(\sigma_z^c)_{ULT} < \sigma_z < (\sigma_z^t)_{ULT}, \text{ OR}$$

$$-(\tau_{12})_{ULT} < \tau_{12} < (\tau_{12})_{ULT}$$

∴

~~44500 < 17145 < 44500~~

FROM TABLE:

(2)

$$-1500 < 1.714 S < 1500, \text{ OR}$$

$$-246 < -2.714 S < 40, \text{ OR}$$

$$-68 < -4.165 S < 68$$

$$\therefore -875.1 < S < 875.1, \text{ OR}$$

$$-14.73 < S < 90.64$$

$$-16.33 < S < 16.33$$

$$\therefore \text{IF } S > 0 \text{ THEN } \underline{S < 16.33 \text{ MPa}}$$

ALSO THE PART WILL FAIL IN SHEAR.

(b) MAX STRAIN FAILURE THEORY

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}$$

$$\begin{aligned} S_{11} &= \frac{1}{E_1} & \cancel{S_{22}} &= \frac{1}{E_2} & S_{12} &= -\frac{\nu_{12}}{E_1} & S_{66} &= \frac{1}{G_{12}} & S_{22} &= \frac{1}{E_2} \\ &= \frac{1}{181 \times 10^9} & & & = -\frac{0.28}{181 \times 10^9} & & = \frac{1}{7.17 \times 10^9} & & = \frac{1}{10.3 \times 10^9} \\ &= \underline{5.525 \times 10^{-12}} & & & = \underline{-1.547 \times 10^{-12}} & & = \underline{1.395 \times 10^{-10}} & & = \underline{9.709 \times 10^{-11}} \end{aligned}$$

$$\therefore \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 5.525 \times 10^{-12} & -1.547 \times 10^{-12} & 0 \\ -1.547 \times 10^{-12} & 9.709 \times 10^{-11} & 0 \\ 0 & 0 & 1.395 \times 10^{-10} \end{bmatrix} \begin{bmatrix} 1.714 \\ -2.714 \\ -4.165 \end{bmatrix} S$$

$$= \begin{bmatrix} 1.367 \times 10^{-11} \\ -2.662 \times 10^{-10} \\ -5.809 \times 10^{-10} \end{bmatrix} S$$

ASSUMING MATERIAL IS LINEAR BEFORE FAILURE. THEN

(3)

$$E = \frac{\sigma}{\epsilon} \quad G = \frac{\tau}{\gamma}$$

$$\therefore \epsilon = \frac{\sigma}{E} \quad \therefore \gamma = \frac{\tau}{G}$$

$$\therefore (\epsilon_1^T)_{ULT} = \frac{(\sigma_1^T)_{ULT}}{E_1} = \frac{1500 \times 10^6}{181 \times 10^9} = \underline{\underline{8.287 \times 10^{-3}}}$$

$$(\epsilon_1^c)_{ULT} = \frac{(\sigma_1^c)_{ULT}}{E_1} = \frac{1500 \times 10^6}{181 \times 10^9} = \underline{\underline{8.287 \times 10^{-3}}}$$

$$(\epsilon_2^T)_{ULT} = \frac{(\sigma_2^T)_{ULT}}{E_2} = \frac{40 \times 10^6}{10.3 \times 10^9} = \underline{\underline{3.883 \times 10^{-3}}}$$

$$(\epsilon_2^c)_{ULT} = \frac{(\sigma_2^c)_{ULT}}{E_2} = \frac{246 \times 10^6}{10.3 \times 10^9} = \underline{\underline{2.388 \times 10^{-2}}}$$

$$(\gamma_{12})_{ULT} = \frac{(\tau_{12})_{ULT}}{G_{12}} = \frac{68 \times 10^6}{7.17 \times 10^6} = \underline{\underline{9.483 \times 10^{-3}}}$$

MAX STRAIN FAILURE THEORY

(4)

$$-(\epsilon_1^c)_{ULT} < \epsilon_1 < (\epsilon_1^T)_{ULT}, \text{ OR}$$

$$-(\epsilon_2^c)_{ULT} < \epsilon_2 < (\epsilon_2^T)_{ULT}, \text{ OR}$$

$$-(\gamma_{12})_{ULT} < \gamma_{12} < (\gamma_{12})_{ULT}$$

$$\therefore -8.287 \times 10^{-3} < 1.367 \times 10^{-10} S < 8.287 \times 10^{-3}, \text{ OR}$$

$$-2.388 \times 10^{-2} < -2.662 \times 10^{-10} S < 3.883 \times 10^{-3}, \text{ OR}$$

$$-9.483 \times 10^{-3} < -5.809 \times 10^{-10} S < 9.483 \times 10^{-3}$$

$$-606.2 \times 10^6 < S < 606.2 \times 10^6, \text{ OR}$$

$$-14.58 \times 10^6 < S < 89.71 \times 10^6, \text{ OR}$$

$$-16.33 \times 10^6 < S < 16.33 \times 10^6$$

IF $S > 0$ THEN $S < 16.33 \text{ MPa}$

NOTE THIS IS THE SAME AS FOR MAX STRESS FAILURE THEORY AS FAILURE IS IN SHEAR.

IF IT WAS DUE TO DIRECT LOADING IT WOULD BE DIFFERENT.

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$$\left[\frac{\sigma_1}{(\sigma_1^T)_{ULT}} \right]^2 - \left[\frac{\sigma_1 \sigma_2}{(\sigma_1^T)_{ULT}^2} \right] + \left[\frac{\sigma_2}{(\sigma_2^T)_{ULT}} \right]^2 + \left[\frac{\tau_{12}}{(\tau_{12})_{ULT}} \right]^2 < 1$$

NOW $\sigma_1 = 1.7145$

$\sigma_2 = -2.7145$

$\tau_{12} = -4.1655$

$$\therefore \left[\frac{1.7145}{1500 \times 10^6} \right]^2 - \left[\frac{1.7145 \times -2.7145}{(1500 \times 10^6)^2} \right] + \left[\frac{-2.7145}{40 \times 10^6} \right]^2 + \left[\frac{-4.1655}{68 \times 10^6} \right]^2 < 1$$

$$1.306 \times 10^{-18} s^2 + 2.068 \times 10^{-18} s^2 + 4.604 \times 10^{-15} s^2 + 3.752 \times 10^{-15} s^2 < 1$$

$$8.359 \times 10^{-15} s^2 < 1$$

$$s^2 < 1.196 \times 10^{14}$$

$$\underline{s < 10.94 \text{ MPa}}$$

NOTE THAT THIS THEORY DOES NOT DISTINGUISH BETWEEN TENSILE & COMPRESSIVE FAILURES .:

IT GENERALLY UNDERESTIMATES THE FAILURE STRESS BECAUSE THE TRANSVERSE TENSILE STRENGTH OF A

UNIDIRECTIONAL LAMINA IS GENERALLY MUCH LESS THAN ITS TRANSVERSE COMPRESSIVE STRENGTH.

(2) $\sigma_x = 2R$
 $\sigma_y = -3R$
 $\tau_{xy} = 4R$

(6)

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$= \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\theta = 60^\circ$$

$$= \begin{bmatrix} 0.25 & 0.75 & 0.866 \\ 0.75 & 0.25 & -0.866 \\ -0.433 & 0.433 & -0.5 \end{bmatrix} \begin{bmatrix} 2R \\ -3R \\ 4R \end{bmatrix}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 1.714 \\ -2.714 \\ -4.165 \end{bmatrix} R$$

MAX STRESS FAILURE :

$$-(\sigma_1^c)_{ULT} < \sigma_1 < (\sigma_1^T)_{ULT}, \text{ OR}$$

$$-(\sigma_2^c)_{ULT} < \sigma_2 < (\sigma_2^T)_{ULT}, \text{ OR}$$

$$-(\tau_{12})_{ULT} < \tau_{12} < (\tau_{12})_{ULT}$$

(7)

$$-1500 < 1.714 R < 1500, \text{ OR}$$

$$-246 < -2.714 R < 40, \text{ OR}$$

$$-68 < -4.165 R < 68$$

$$-875.15 < R < 875.15, \text{ OR}$$

$$-14.74 < R < 90.64, \text{ OR}$$

$$-16.33 < R < 16.33$$

$$\therefore R = 16.33$$

THUS THE LOAD THAT CAN BE APPLIED JUST BEFORE FAILURE IS

$$\sigma_x = 16.33 \times 2 = \underline{\underline{32.66 \text{ MPa}}}$$

$$\sigma_y = 16.33 \times -3 = \underline{\underline{-48.99 \text{ MPa}}}$$

$$\tau_{xy} = 16.33 \times 4 = \underline{\underline{65.32 \text{ MPa}}}$$

$$(3) \tau_{xy} = 24 \text{ MPa}$$

(8)

$$\text{NOW } \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = [T] \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$= \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$= \begin{bmatrix} 0.25 & 0.75 & 0.866 \\ 0.75 & 0.25 & -0.866 \\ -0.433 & 0.433 & -0.5 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ 24 \end{bmatrix}$$

$$= \begin{bmatrix} 0.25\sigma_x + 0.75\sigma_y + 20.784 \\ 0.75\sigma_x + 0.25\sigma_y - 20.784 \\ -0.433\sigma_x + 0.433\sigma_y - 12 \end{bmatrix}$$

MAX STRESS FAILURE THEORY :

$$-(\sigma_1^c)_{ULT} < \sigma_1 < (\sigma_1^T)_{ULT} , \text{ OR}$$

$$-(\sigma_2^c)_{ULT} < \sigma_2 < (\sigma_2^T)_{ULT} , \text{ OR}$$

$$-(\tau_{12})_{ULT} < \tau_{12} < (\tau_{12})_{ULT}$$

$$\therefore -1500 < 0.25\sigma_x + 0.75\sigma_y + 20.784 < 1500 , \text{ OR}$$

$$-246 < 0.75\sigma_x + 0.25\sigma_y - 20.784 < 40 , \text{ OR}$$

$$-68 < -0.433\sigma_x + 0.433\sigma_y - 12 < 68$$

NOW FIND POINTS ON ENVELOPE:

(9)

• SAY $\sigma_x = -25$

$$-1500 < -6.25 + 0.75\sigma_y + 20.784 < 1500, \text{ OR}$$

$$-246 < -18.75 + 0.25\sigma_y - 20.784 < 40, \text{ OR}$$

$$-68 < 10.825 + 0.433\sigma_y - 12 < 68$$

$$\therefore -1500 < 0.75\sigma_y + 14.534 < 1500, \text{ OR}$$

$$-246 < 0.25\sigma_y - 39.534 < 40, \text{ OR}$$

$$-68 < 0.433\sigma_y - 1.175 < 68$$

$$\therefore -1514.53 < 0.75\sigma_y < 1485.47, \text{ OR}$$

$$-206.47 < 0.25\sigma_y < 79.53, \text{ OR}$$

$$-66.83 < 0.433\sigma_y < 69.18$$

$$\therefore -2019.37 < \sigma_y < 1980.63, \text{ OR}$$

$$-825.88 < \sigma_y < 318.12, \text{ OR}$$

$$-154.34 < \sigma_y < 159.77$$

• WHEN $\sigma_x = -25 \text{ MPa}$

$$\sigma_y = -154.34 \text{ MPa} \text{ \& } 159.77 \text{ MPa}$$

IF THIS IS DONE FOR VARIOUS VALUES OF σ_x

WE WOULD SEE THE FOLLOWING

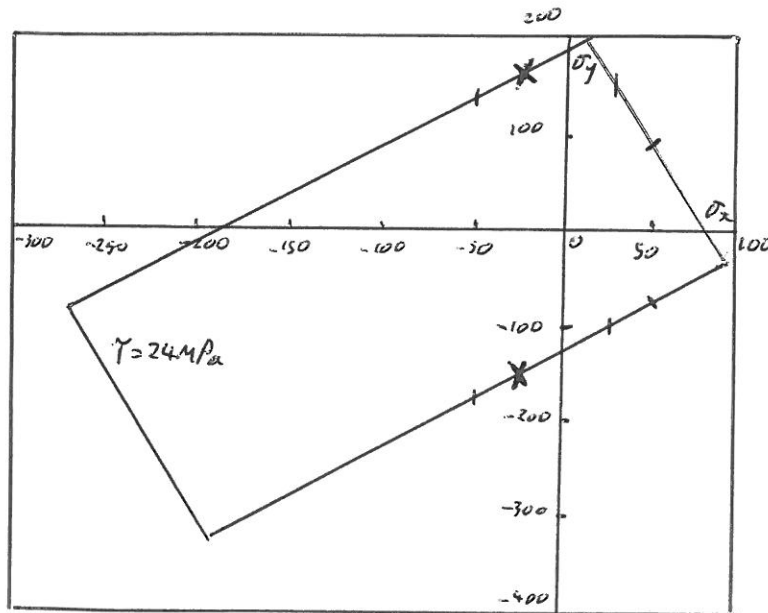
$$\sigma_x = -25 \text{ MPa}, \sigma_y = -154 \text{ MPa} \text{ \& } 160 \text{ MPa}$$

$$\sigma_x = -50 \text{ MPa}, \sigma_y = -135 \text{ MPa} \text{ \& } 179 \text{ MPa}$$

$$\sigma_x = 25 \text{ MPa}, \sigma_y = -104 \text{ MPa} \text{ \& } 168 \text{ MPa}$$

$$\sigma_x = 50 \text{ MPa}, \sigma_y = -79.3 \text{ MPa} \text{ \& } 93.1 \text{ MPa}$$

$$\sigma_y = -79.3 \text{ MPa} \text{ \& } 93.1 \text{ MPa}$$



NOTE IF $\sigma_x = 100$

$$-1500 < 25 + 0.75\sigma_y + 20.784 < 1500, \text{ OR}$$

$$-246 < 75 + 0.25\sigma_y - 20.784 < 40, \text{ OR}$$

$$-68 < -43.3 + 0.433\sigma_y - 12 < 68$$

$$\therefore -1500 < 0.75\sigma_y + 45.784 < 1500, \text{ OR}$$

$$-246 < 0.25\sigma_y + 54.216 < 40, \text{ OR}$$

$$-68 < -55.3 + 0.433\sigma_y < 68$$

$$\therefore -2061 < \sigma_y < 1938.95, \text{ OR}$$

$$-1200 < \sigma_y < -56.86, \text{ OR}$$

$$-29.33 < \sigma_y < 284.76$$

THIS IS FOR MAX STRESS FAILURE, i.e. σ_y CAN NOT BE LESS THAN -56.86 & GREATER THAN -29.33 i.e. THERE IS NO POSSIBLE SOLUTION i.e. IT ~~DOES NOT~~ FAILS - IT FALLS OUTSIDE OF THE ENVELOPE! IF IT IS INSIDE THE ENVELOPE IT DOES NOT FAIL: