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Analytical Methods Test 2

Question 2

a) $z_1 = 3 + 7j$ $z_2 = 2 - 10j$ $z_3 = -4 - j$

i) $z_1 + z_3$
 $(3 + 7j) + (-4 - j)$
 $3 + 7j - 4 - j$
 $-1 + 6j$

ii) $z_1 - z_3$
 $(3 + 7j) - (-4 - j)$
 $3 + 7j + 4 + j$
 $7 + 8j$

iii) $\frac{z_1}{z_2}$
 $\frac{z_1 = (3 + 7j)}{z_2 = (2 - 10j)}$
 $\frac{3 + 7j}{2 - 10j} \times \frac{2 + 10j}{2 + 10j}$
 $\frac{(3 + 7j)(2 + 10j)}{(2 - 10j)(2 + 10j)}$

FOIL

$$\frac{6 + 30j + 14j + 70j^2}{4 + 20j - 20j - 100j^2}$$

$$\frac{6 + 44j - 70}{4 + 100}$$

+70x-1
-100x-1

$$\frac{-64 + 44j}{104}$$

$$\frac{-64}{104} + \frac{44}{104}j$$

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iv) $Z_1 - Z_2$ $Z_1 = 3 + 7j$ $Z_2 = 2 - 10j$
 $(3 + 7j) - (2 - 10j)$
 $3 + 7j - 2 + 10j$
 $1 + 17j$

v) $(Z_3)^3 - Z_1$ $Z_3 = -4 - j$ $Z_1 = 3 + 7j$
 $(-4 - j)(-4 - j)(-4 - j) - (3 + 7j)$

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b) $8x^2 = -3x - 18$

$$8x^2 + 3x + 18 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 8 \times 18}}{2 \times 8}$$

$$x = \frac{-3 \pm \sqrt{9 - 576}}{16}$$

$$x = \frac{-3 \pm \sqrt{-1 \times 567}}{16}$$

$$x = \frac{-3 \pm \sqrt{-1 \times 567}}{16}$$

$$x = \frac{-3 \pm \sqrt{1} \times \sqrt{16}}{16}$$

$$x = \frac{6 \pm j \times 4}{16}$$

$$x = \frac{6 + 4j}{16} \quad x = \frac{6 - 4j}{16}$$

$$0.375 + 0.25j$$

$$0.375 - 0.25j$$

$0.375 + 0.25j$ in p.f

$0.25j$

rounded

$$r^2 = 0.375^2 + 0.25^2$$

to

$$r = \sqrt{0.141 + 0.0625}$$

(3dp)

$$r = \sqrt{0.2035}$$

3dp)

$$r = 0.451 \text{ (3dp)}$$

$$\tan \theta = \frac{0.25}{0.375}$$

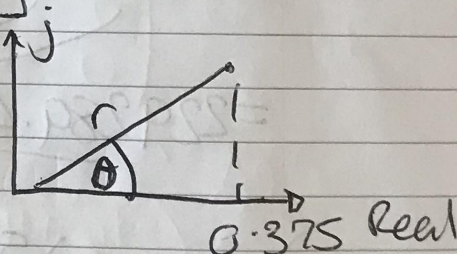
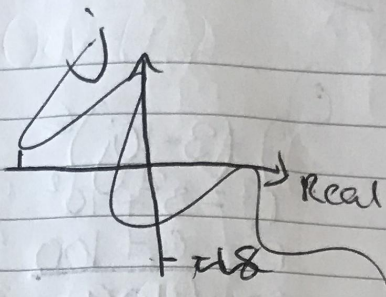
$$\theta = \tan^{-1} \left(\frac{0.25}{0.375} \right)$$

$$\theta = 33.69^\circ$$

Polar form

$$r(\cos \theta + j \sin \theta)$$

$$0.451(\cos 33.69 + j \sin 33.69)$$



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c)

i) $(3-10j)^6$

$r^2 = 3^2 + 10^2$

$r^2 = 9 + 100$

$r = \sqrt{109}$

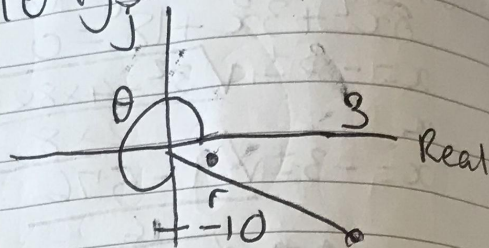
$r = 10.44$

$\tan \theta = \frac{10}{3}$

$\theta = \tan^{-1}\left(\frac{10}{3}\right)$

$\theta = 73.3^\circ$

$r^n(\cos n\theta + jsin n\theta)$



$\theta = 360 - 73.3$

$\theta = 286.7$

$10.44^6(\cos 6 \times 286.7 + jsin 6 \times 286.7)$

$1294800.898(\cos 1720.2 + jsin 1720.2)$ Polar form

$1294800.898(0.1770847403 + -0.984195608j)$

$= 229289.4808 - 1274337.357j$ Cartesian

ii) $(-3+8j)^5$

$r^2 = 3^2 + 8^2$

$r^2 = 9 + 64$

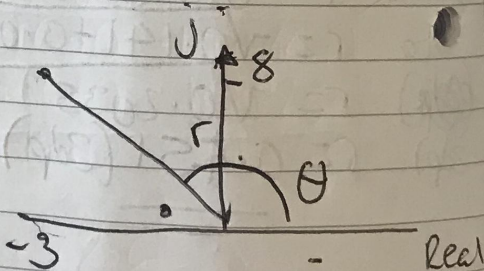
$r = \sqrt{73}$

$r = 4.179$ (3dp)

$\tan \theta = \frac{8}{-3}$

$\theta = \tan^{-1}\left(\frac{8}{-3}\right)$

$\theta = 69.443$ (3dp)



$\theta = 180 - 69.443$

$\theta = 110.557$

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$$4.1795 (\cos 5 \times 110.557 + j \sin 5 \times 110.557)$$

$$1274.564611 (\cos 552.835 + j \sin 552.835) \text{ P.F}$$

$$1274.564611 (-0.9750138361 + -0.2221441411j)$$

$$= -1242.718131 - 283.1374048j \text{ cartisian}$$

d) $j^5 \times j^3 = j^8$

a) i) $A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$ $B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix}$

3×3 3×3 can multiply

$AB = \begin{pmatrix} -16 + -2 + 0 & 64 + -5 + -6.3 & 4 + -2 + 7 \\ -10 + 4 + 0 & -40 + 10 + -4.5 & -2.5 + 4 + 5 \\ -6 + 0 + 1 & 24 + 0 + -18 & 1.5 + 0 + 2 \end{pmatrix}$

$AB = \begin{pmatrix} -18 & -4 & 9 \\ -6 & -7.5 & 6.5 \\ -5 & 6 & 3.5 \end{pmatrix}$

ii) $B = \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix}$ $A = \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$

3×3 3×3

$BA = \begin{pmatrix} -16 + 40 + 1.5 & 2 + 16 + 0 & -14 + 40 + 1 \\ 16 + -25 + 6 & -2 + 10 + 0 & 14 + 25 + 4 \\ 0 + 45 + 3 & 0 + -18 + 0 & 0 + -45 + 2 \end{pmatrix}$

$BA = \begin{pmatrix} 54.5 & 18 & 26 \\ -3 & 8 & 43 \\ 48 & -18 & -43 \end{pmatrix}$

$AB \neq BA$ matrices are non commutative (matrices)

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iii) $A \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad C \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix}$
 $3 \times 3 \quad 3 \times 2$
 cannot multiply

iv) ~~$CA \quad C \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix} \quad A \begin{pmatrix} 8 & -1 & 7 \\ 5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$~~
 ~~$2 \times 3 \quad 3 \times 3$~~
~~cannot multiply~~

$AC \begin{pmatrix} 16+6+7 & 40+0+84 \\ -10+12+5 & -25+0+60 \\ 6+0+2 & 15+0+24 \end{pmatrix}$
 $AC \begin{pmatrix} 29 & 124 \\ -17 & 45 \\ 8 & 39 \end{pmatrix}$

iv) $CA \quad C \begin{pmatrix} 2 & 5 \\ -6 & 0 \\ 1 & 12 \end{pmatrix} \quad A \begin{pmatrix} 8 & -1 & 7 \\ 5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$
 $2 \times 3 \quad 3 \times 3$
 cannot multiply

v) $A \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix}$
 $|A| = 8(2 \times 0) - (-1)(15 - 10) + 7(6 - 0)$
 $= 8(0) + 1(5) + 7(6)$
 $= 24 + 5 + 42 = \underline{\underline{71}}$

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$$vi) |B| \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix}$$

$$\begin{aligned} |B| &= -2(-18-5) - 8(0-2) + 0.5(-18-0) \\ &= -2(-23) - 8(-2) + 0.5(-18) \\ &= -46 + 16 - 9 \\ &= \underline{\underline{-49}} \end{aligned}$$

vii) A+B

$$A \begin{pmatrix} 8 & -1 & 7 \\ -5 & 2 & 5 \\ 3 & 0 & 2 \end{pmatrix} \quad B \begin{pmatrix} -2 & 8 & 0.5 \\ 2 & 5 & 2 \\ 0 & -9 & 1 \end{pmatrix}$$

$$A+B \begin{pmatrix} 6 & 7 & 7.5 \\ -3 & 7 & 7 \\ 3 & -9 & 3 \end{pmatrix}$$