

RLC Resonance

by Fergus Russell

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RLC RESONANCE

ACDA4006-Electrical & Electronic Principles
Dr Steve Mosey

Abstract

An analysis of an A.C. Series circuit with resistive, capacitive and inductive components was undertaken through theoretical, experimental and simulative methods. The findings of the experiments were such that all three techniques are valid for the analysis of an RLC circuit however highlighted some errors in data gathering of results and potential inaccurate modelling in simulation. The report goes on to suggest further areas of improvement for further work to be undertaken.

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Introduction

Resonance is a concept which is encountered in a variety of fields. Within mechanical engineering it is encountered in a whole host of applications such as spring-mass-damper systems, aero-acoustic analysis and vibration analysis. In mechanical terms resonance occurs when the stimulation frequency approaches that of the natural frequency of the object under analysis [1]. Within electrical and electronic engineering it is defined as [2]:-

"When $X_L = X_C$ the applied voltage V and current I are in phase. This effect is called series resonance"

X_L and X_C are the reactance across the Inductor and Capacitor.

It is utilised in a variety of electronic and electrical applications including bandpass and frequency filtering circuits, tuning circuits in analog radios, oscillators [3] and wah wah guitar pedals [4].

This report will outline an experiment carried out in real life and then replicated in an electronics design package called Proteus and will analyse and compare results gathered in both methods alongside those obtained through theoretical calculation.

Experimental Setup

Figure 1[5] shows the circuit diagram of the circuit under analysis:-

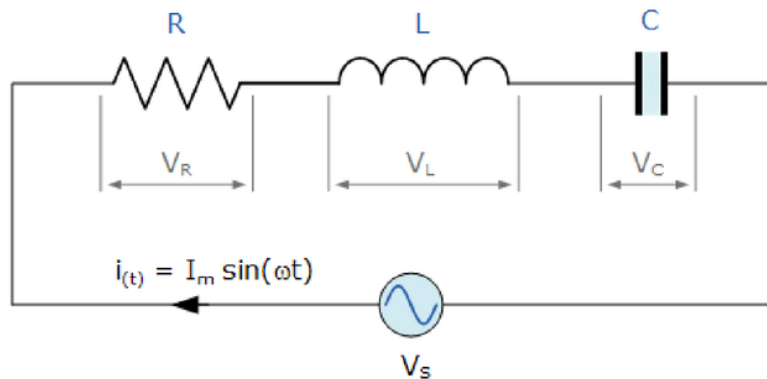


Figure 1-RLC Circuit Diagram

The circuit from left to right contains a resistor, inductor and a capacitor. V_s is an A.C. Voltage Source. For the experiment table one shows the component values selected:-

Table 1-Circuit Component Parameters

Component	Value
Voltage Source	12V @ 100Hz ranging to 12V @ 20kHz
Resistor	3.3 Ω
Inductor	1mH
Capacitor	1 μ F

The first experiment will be carried out in the digital environment using a piece of software called Proteus. Proteus is an electrical and electronic computer aided design software. It allows engineers to design and test circuits prior to constructing them for real, it also assists in the design of Printed Circuit Boards allowing rapid prototyping of circuits for quick and easy evaluations and for eventual

production. Once carried out in Proteus the experiment will be repeated under laboratory conditions in real life allowing for a comparison of results.

Proteus Experiment and Proteus Data

Figure 2 shows an image of the circuit as built in Proteus:-

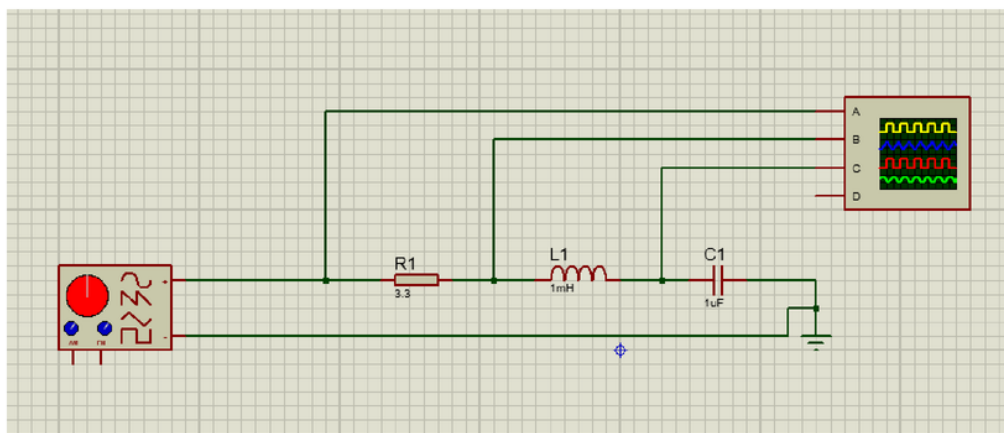


Figure 2-Circuit Built in Proteus

From this circuit the voltage across the resistor, inductor and capacitor for a series of frequencies are shown below:-

Table 2-Data Collected From Proteus Experiment

Frequency (Hz)	V (R)	V (L)	V (C)
100	12	12	12
200	12	12	12
500	12	12	12
1000	12	12	12
2000	12	12	13
4500	12	10	60
4600	12	10	70
4700	12	8	80
4800	12	6	90
4900	12	3	100
5000	12	1.75	110
5100	12	5.5	100
5200	12	7	80
5300	12	10	66
5400	12	10	54
5500	12	10	45
10000	12	12	4
20000	12	12	0.75

A graphical form of this data is shown in figure3:-

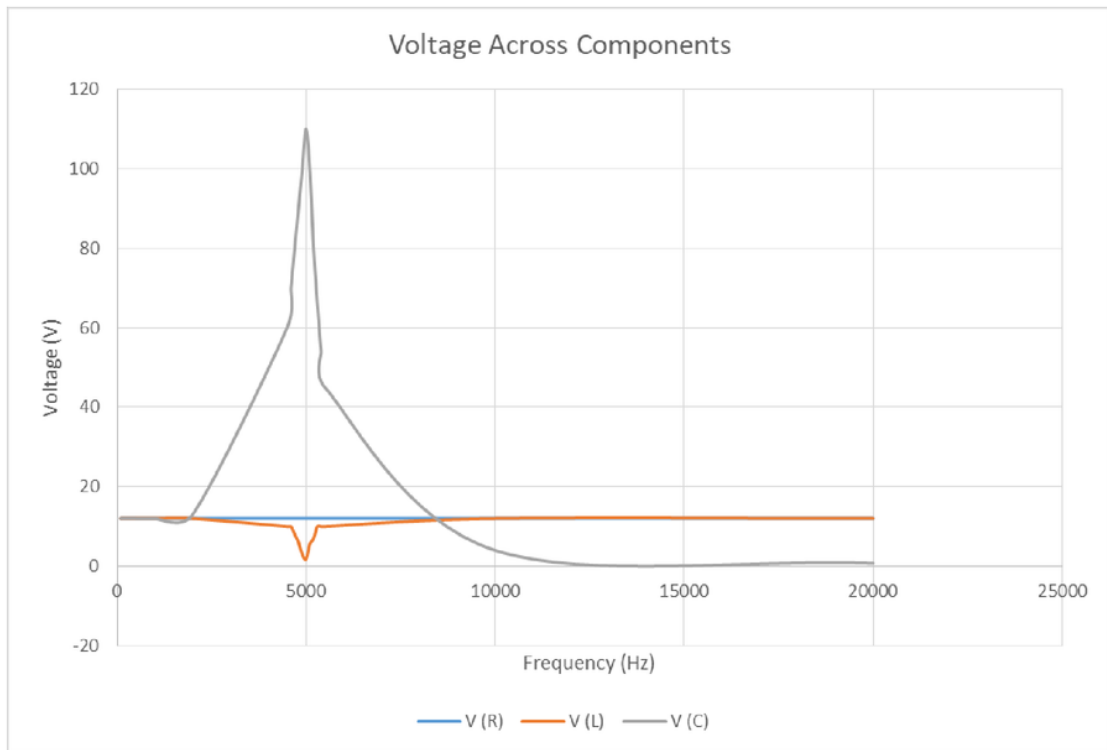


Figure 3-Voltage across Components against Frequency

Real Life Experiment and Real Life Data

For the real life experiment the same circuit in figure 1 was created using real resistors, inductors and capacitors. An A.C. Signal Generator was used to provide a sine wave frequency that could be changed along with an amplitude voltage. An oscilloscope was used to analyse the frequency response of the inductor throughout the experiment and to obtain the results. Figure 4 is a photograph of the circuit as built on a breadboard:-

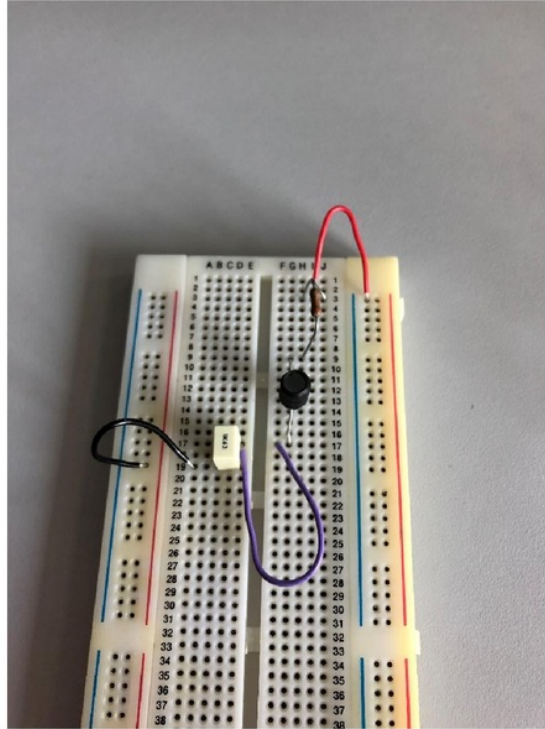


Figure 4-Circuit Built on Breadboard

The off white colour box is a capacitor, the black cylinder is the inductor and the brown cylinder is the resistor. The red and black wires are connected to the V+ and Ground bus rails for connection to the signal generator. Figure 5 shows the circuit with a probe fitted that is connected to the oscilloscope that will monitor the frequency response of the inductor:-

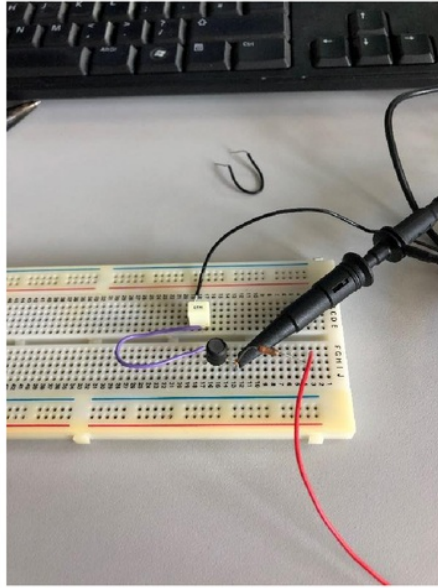


Figure 5-Circuit with voltage probe attached

Once connected the signal generator was powered up. The same frequencies that were used in the Proteus experiment were used however a voltage of 3V was used. Figure 6 shows a typical oscilloscope response for the circuit.

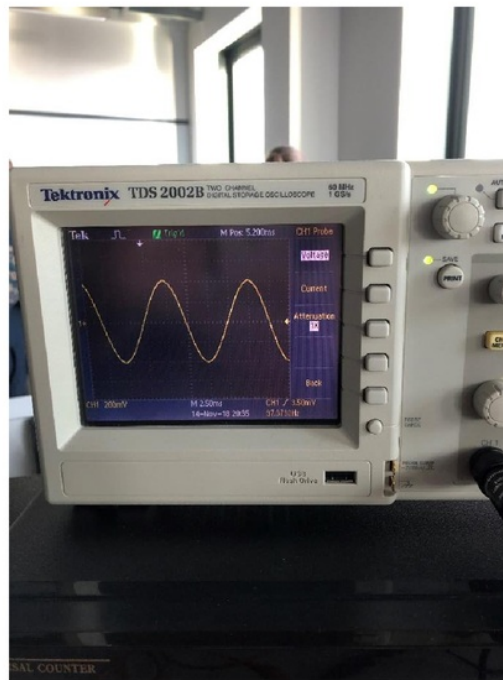


Figure 6-Voltage response for Inductor

Table 3 shows the collected data from the experiment:-

Table 3-Real Life Experimental Data

Freq	Vs	VL
100	3	2.94
200	3	2.94
500	3	2.94
1000	3	2.82
2000	3.06	2.34
4500	3.06	0.66
4600	3.06	0.54
4700	3.06	0.42
4800	3.06	0.36
4900	3.06	0.316
5000	3.06	0.26
5100	3.06	0.248
5200	3.06	0.288
5300	3.06	0.3
5400	3.06	0.34
5500	3.06	0.348
10000	3.06	1.88
20000	3.06	2.7

This data is represented graphically in figure 7:-

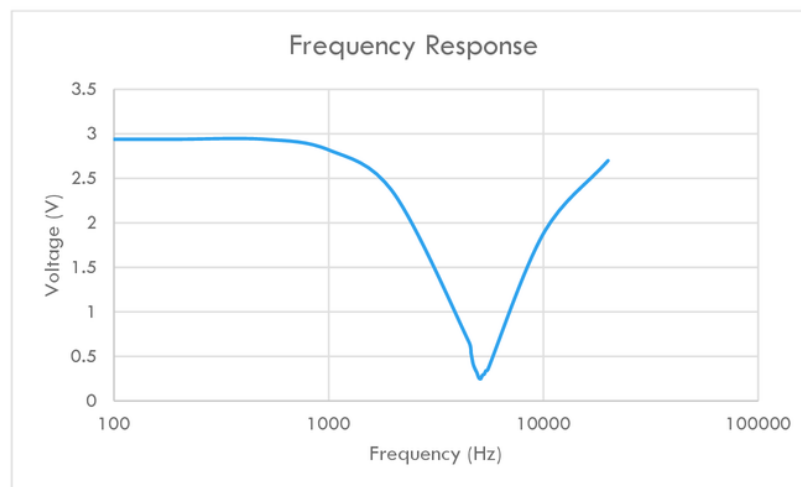


Figure 7-Real Life Frequency Response

Theoretical Calculations

There are a number of theoretical calculations that can be carried out that can determine a number of parameters for an RLC circuit. Using the parameters that were used in the Proteus experiment the resonant frequency of the circuit, circuit reactance, current flowing in the circuit, voltage across the capacitor and inductor and the quality factor of the circuit were all calculated.

The resonant frequency of the circuit was calculated using equation 1.1[6]:-

$$\omega r = \frac{1}{2 * \pi * \sqrt{L * C}} \quad (1.1)$$

Where ωr is the resonant frequency, L is the value of the inductor and C is the value of the capacitor. Using the values of 1mH and 1 μ F the following frequency was calculated:-

$$\omega r = \frac{1}{2 * \pi * \sqrt{L * C}} = \frac{1}{2 * \pi * \sqrt{(1 * 10^{-3}) * (1 * 10^{-6})}} = 5032.92121 \text{ Hz}$$

Now that the resonant frequency has been obtained the reactance across the inductor and capacitor can be calculated using equations 1.2 and 1.3 [2]:-

$$X_L = 2 * \pi * f * L \quad (1.2)$$

$$X_C = \frac{1}{2 * \pi * f * C} \quad (1.3)$$

Using these equations the following values were calculated:-

$$X_L = 2 * \pi * f * L = 2 * \pi * 5032.92121 * (1 * 10^{-3}) = 31.6227 \Omega$$

$$X_C = \frac{1}{2 * \pi * f * C} = \frac{1}{2 * \pi * 5032.92121 * (1 * 10^{-6})} = 31.6227 \Omega$$

At resonance the current in the circuit is equal to the current that is following across the resistor, this value can be calculated using Ohm's Law using equation 1.4[7]:-

$$V = IR \quad (1.4)$$

Transposing this equation to make I the subject yields the following result for current across the circuit:-

$$I = \frac{V}{R} = \frac{12}{3.3} = 3.63 \text{ A}$$

Once the reactance have been calculated the voltage across the inductor and capacitor can be calculated using equations 1.5 and 1.6 [7]

$$V_L = I * X_L \quad (1.5)$$

$$V_C = I * X_C \quad (1.6)$$

Using these equations the following values were calculated:-

$$V_L = I * X_L = 3.63 * 31.6227 = 114.790 \text{ V}$$

$$V_C = I * X_C = \frac{I}{2 * \pi * f * C} = \frac{3.63}{2 * \pi * 5032.92121 * (1 * 10^{-6})} = 114.790 \text{ V}$$

Once these were calculated the Quality factor was calculated. Q factor is a figure of how good a resonant circuit is at storing power in the reactive components of the circuit. Using Equation 1.7 [8] the Q factor of the circuit at resonance can be determined.

$$Qr = \frac{1}{R} * \sqrt{\frac{L}{C}} \quad (1.7)$$

At resonance the follow Q factor value was calculated:-

$$Qr = \frac{1}{R} * \sqrt{\frac{L}{C}} = \frac{1}{3.3} * \sqrt{\frac{1 * 10^{-3}}{1 * 10^{-6}}} = 9.5826$$

Q represents a factor of magnification of voltage across components when compared to the supply voltage [8].

Using the values of the real life experiment the calculations carried out above were repeated. As the only difference was voltage used then the values of resonance and reactance will remain the same. The value for current was calculated in the following manner:-

$$I = \frac{V}{R} = \frac{3}{3.3} = 0.9A$$

The value for V_c and V_L were calculated in the following manner:-

$$V_L = I * X_L = 0.9 * 31.6227 = 28.4604V$$

$$V_C = I * X_C = \frac{I}{2 * \pi * f * C} = \frac{0.9}{2 * \pi * 5032.92121 * (1 * 10^{-6})} = 28.4604V$$

Q factor also remains the same as before.

Bandwidth of an RLC circuit is another parameter that has a variety of uses. Bandwidth is determined by using Equation 1.8 [9]:-

$$Qr = \frac{fr}{f2-f1} \quad (1.8)$$

Where Qr is the quality factor at resonance, fr is the resonant frequency and $f2-f1$ is the intersection where I or V of a component equals 70%. For theoretical values the bandwidth of the circuit is determined in the following manner:-

$$Qr = \frac{fr}{f2-f1} = f2-f1 = \frac{fr}{Qr} = \frac{5032.92121}{9.5826} = 525.214577Hz$$

Discussion of Results

Proteus Results

From the Proteus experiment data some results can be drawn and calculated. The graph in Figure 3 highlights a resonant frequency spike at around 5000Hz. Compared with a theoretical value of 5032.92Hz this is very close and the simulation is running correctly at this stage. At this frequency value there is a voltage spike in the capacitor to a value of around 110V and a voltage drop to 1.75V across the inductor. This is due to the fact that at resonance the reactance of the capacitor are equal in magnitude but opposite in phase to each other[5] and thus cancelling each other out, the capacitor will act as a short circuit and the inductor will act as an open circuit and have a voltage drop approaching 0V. From the graph a Quality Value for the Proteus circuit can be calculated by measuring the band width in the spike, this is done at a point that is 70% of the total voltage drop of the spike and the band width frequency is taken [10]. For the Proteus circuit the graph was constructed in the manner shown in Figure 8:-

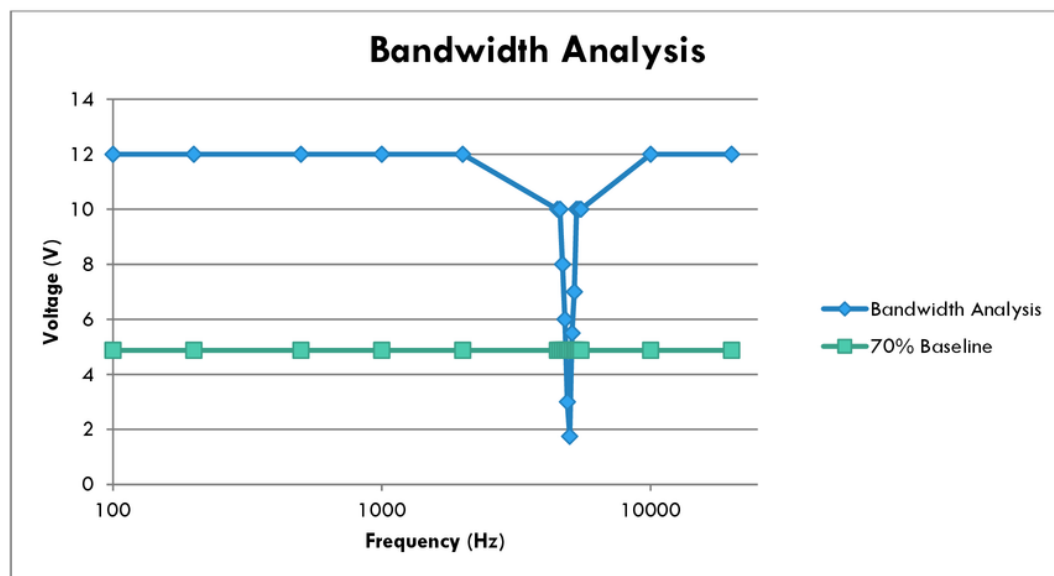


Figure 8-Bandwidth Analysis

From this graph the points on the voltage drop that intersect the 70% base line provide the bandwidth frequencies. These frequencies are approximately 4800Hz and 5200Hz. The Q factor of the circuit can now be calculated using the following formula in equation 2.1[5]:-

$$Q = \frac{f_r}{BW} \quad (2.1)$$

Where f_r is the resonant frequency and BW is the bandwidth. The Q factor for the Proteus circuit is calculated in the following manner:-

$$Q = \frac{f_r}{BW} = \frac{5000}{400} = 12.5$$

This figure is an improved quality factor over the one calculated from theory and therefore its validity is questionable. There are however a number of errors that could have crept in to the Proteus circuit that may have had an impact on results. As the data collection from Proteus requires the reading of on screen oscilloscopes then the values collected may be somewhat incorrect, incorrect data can have a compounding effect on calculations and therefore results.

Real Life Results

Similarly to the way the Proteus results were calculated the real life experimental data can be analysed in a similar way. From the real life experiment graph a resonant frequency of 5100 Hz was observed. This value is close to the theoretical value calculated of 5032.92Hz. This proves that the real life circuit is following theoretical predictions at this stage. No voltage spike over the capacitor was observed in the analysis however a voltage drop to 0.248V was at 5100Hz. A supply voltage of 3V was utilised in the experiment, this was due to the fact that when resonance is approached the capacitor will store voltage until it either becomes fully charged or exceeds its design voltage, "releases all its farads" and fails, this would have the potential of occurring if 12v was selected as the experimental voltage. As with the Proteus experiment a graph depicting voltage and frequency was generated and a baseline set at 70% was added to determine the bandwidth of the circuit. This graph is shown in figure 9:-

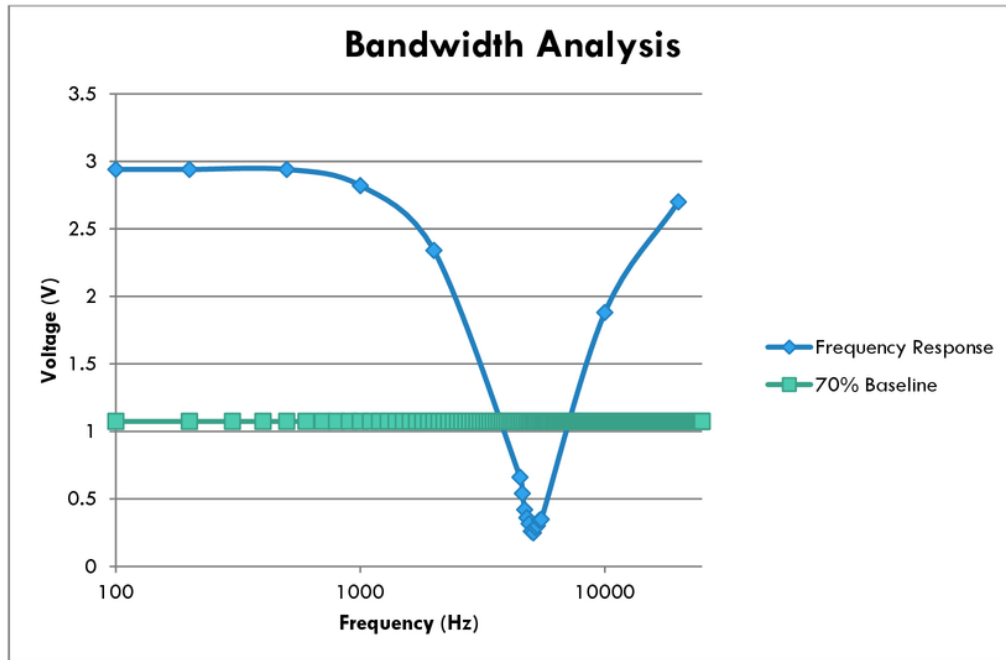


Figure 9-Real Life Bandwidth Analysis

Here the 70% baseline intersects the frequency response when the frequency is at 3800Hz and at 7200Hz. This is a total bandwidth of 3400Hz. Using the resonant frequency from the circuit analysis and equation 2.1 the Q factor of the circuit was determined in the following method:-

$$Q = \frac{fr}{BW} = \frac{5100}{3400} = 1.5$$

The real life circuit has a lower quality factor than that calculated and a lower value than calculated from the Proteus circuit.

Discussion of Results: Proteus Vs Real Vs Theory

All three experiments have produced a wide selection of results, some of which can be compared with each other. This section will discuss what areas of the experiments are coherent with each other and which areas require further analysis and investigation.

The first area where all three experiments performed well was in the determination of the resonant frequency of the circuit. From theoretical calculations a resonant frequency 5032.92121Hz was obtained, compared to the Proteus determination of 5000Hz and the real life determination of 5100Hz. The Proteus result has a deviation of 0.8% from theoretical and the real life result has a deviation of 0.8%. This difference could be accounted for as a reading error from graphical sources such as that produced by the oscilloscopes in the laboratory and in Proteus, any result that requires the use of the "Mk1 Eyeball" to determine it is likely to present a degree of inaccuracy in any result.

The next area of discussion is the voltage that was observed across the capacitor. Through theoretical calculations a value of 114.79V was determined. In Proteus a voltage of 110V was observed as acting across the capacitor in the circuit. This result is a deviation 4.172% from that calculated. This can be accounted for in a number of ways, one factor is that the result relied on being determined by the human eye. The voltage across the capacitor in the real life experiment was not obtained during the experiment so cannot be compared in this study.

Bandwidth and Quality Factor of the circuit is another point where all three experiments produced comparable results. For theory the bandwidth was calculated as 525.214577Hz, the bandwidth of the Proteus circuit was determined to be 400Hz whilst the real life experiment bandwidth was determined to be 3400Hz. As quality factor and bandwidth are properties which are related to each other then these can be compared between experiments as well. Theoretical quality factor was calculated as being 9.5826 whereas the Proteus experiment's q factor was determined to be 12.5 and the real life experiment had a q factor of 1.5. There are many reasons as to why these results have such a spread. As the real life experiment produced the lowest Q factor then it will be discussed first. The real life Q number could be as low as it is for a number of reasons. The first reason is that the way the circuit was built may be introducing stray noise into the circuit, this is because nothing is shielding them from any interference, all components are exposed and so are the connections between them. The use of the breadboard may be introducing noise to the circuit as well as the connection quality between components may be less than ideal for the circuit.

Another reason for the real life circuit having a low quality factor may be due to the components themselves. Although the components may be just a resistor, inductor and capacitor the fact is that all three components have a degree of internal resistance, inductance and capacitance. Figure 10, 11 and 12 [11] show equivalent circuits that more closely represent the internal resistance, inductance and capacitance characteristics that are present in the components.

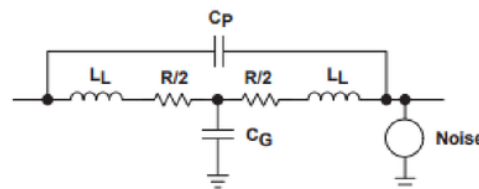


Figure 10-Ideal Resistor

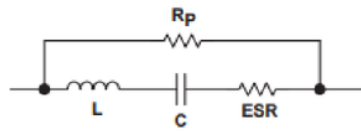


Figure 11-Capacitor Equivalent Circuit

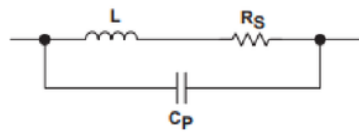


Figure 12-Inductor Equivalent Circuit

Each equivalent circuit is built upon a series of factors that are present within each component. In the resistor L_L simulates the inductance present on each lead of the resistor while C_P represents the capacitance across the resistor hence why it appears in parallel [11]. C_G in the resistor represents the capacitance that is formed between the resistor body and ground. These effects have a correlation to the physical size of the resistor, the smaller the resistor is in physical size the smaller the stray affects represented by the circuit are.

Figure 11 shows the equivalent circuit for a capacitor. L is present to model the inductance on the capacitor leads and its internal inductance, the presence of an internal inductance and the inductance from the leads enables the capacitor to become self-resonant [11]. At frequencies higher than that of the resonant frequency the capacitor will appear as an inductor [11]. ESR is the Equivalent Series Resistance of the capacitor at the operating frequency [11]. R_P is the parallel resistance of the capacitor and is dependent on operating voltage and temperature, this causes this value to fluctuate a significant amount [11].

Figure 12 shows the equivalent circuit for the inductor. L is the value of the inductor, R_s is the series resistance and C_P is the parallel capacitance. C_P is present to represent the capacitance generated by the size and shape of the inductor [11]. R_s is the series resistance of the inductor and contributes to lowering the Q factor of the inductor.

Scientific method may also have had a detrimental effect on the results obtained, particularly on the results obtained from the real life experiment and trying to compare them with theoretical and the Proteus experiment. In the Proteus experiment a voltage of 12v was used for the analysis however in real life a voltage of 3v was used. This was done in order to avoid a failure of components on the breadboard. Because of this re running the Proteus experiment at 3v or running the real life experiment at 12v would assist in providing a 100 percent comparable data set.

Conclusions and Further Work

This report has worked through a problem involving a series RLC circuit from problem statement, through procedure and results to discussing where results agree and differ for theoretical, simulated and real life experiments. In conclusion it has been shown that all three methods produce comparable results that in some aspects are close together with minimal error and in other areas deviates a significant amount.

It has been shown that Proteus has provided results which are close to reflecting those obtained through theoretical methods yet are significantly different from results obtained through real life experimentation.

The real life experiment has provided at least a single result which reflects theory and was within 5% of the theoretically calculated result. The error present in the other results including a significantly reduced Q factor may come from other sources such as how the components are actually made.

Further work to be undertaken should be able to address a number of the errors present in the results. Using a different medium to build the circuit on and the method of building the circuit such as a printed circuit board with hardwired test points should be able to reduce any stray noise from entering the circuit. Obtaining capacitor voltage data in the real life experiment would also be able to validate theoretical calculations and provide another level of comparison to Proteus results. Further Proteus experiments in particular simulating a full RLC circuit through use of the equivalent circuits and comparing the response it gives with the results obtained from the original Proteus experiment and the real life experiment.

References

- [1] "Resonance and Its Effects on Mechanical Structures", *Pumps & Systems*, 2019. [Online]. Available: <https://www.pumpsandsystems.com/topics/motors/resonance-effects-mechanical-0612>. [Accessed: 15-Feb- 2019].
- [2] J. Bird, *Electrical circuit theory and technology*. Amsterdam: Elsevier/Newnes, 2010, p. 202.
- [3] "Series LCR Circuits (Theory) : Electric Circuits Virtual Lab (Pilot) : Physical Sciences : Amrita Vishwa Vidyapeetham Virtual Lab", *Vlab.amrita.edu*, 2019. [Online]. Available: <http://vlab.amrita.edu/?sub=1&brch=75&sim=330&cnt=1>. [Accessed: 15- Feb- 2019].
- [4] *Courses.physics.illinois.edu*, 2019. [Online]. Available: https://courses.physics.illinois.edu/phys406/sp2017/Student_Projects/Spring12/Robert_Osiol_P406_Project_Report_Sp12.pdf. [Accessed: 15- Feb- 2019].
- [5] "Series RLC Circuit Analysis", *electronics-tutorials*, 2019. [Online]. Available: <https://www.electronics-tutorials.ws/accircuits/series-circuit.html>. [Accessed: 15- Feb- 2019].
- [6] D. Dailey, *Electronics for guitarists*. New York, NY: Springer, 2013.
- [7] J. Bird, *Electrical circuit theory and technology*. Amsterdam: Elsevier/Newnes, 2010, p. 206.
- [8] J. Bird, *Electrical circuit theory and technology*. Amsterdam: Elsevier/Newnes, 2010, p. 394.
- [9] J. Bird, *Electrical circuit theory and technology*. Amsterdam: Elsevier/Newnes, 2010, p. 400.
- [10] "Series Resonance", *electronics-tutorials*, 2019. [Online]. Available: <https://www.electronics-tutorials.ws/accircuits/series-resonance.html>. [Accessed: 15- Feb- 2019].
- [11] Texas Instruments, "Understanding Basic Analog-Passive Devices", 1999.

RLC Resonance

GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

Instructor

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Comment 1

When you cite several different sections from the same source, it is common to list it once in your reference list and cite it in your text as [7, p.206], and then later on you could talk about something else and reference that as [7, p.394].

FERGUS RUSSELL

65.15

ENGAGEMENT WITH LITERATURE

including reading, referencing, academic conventions and academic honesty



Inclusion of a wide range of research-informed literature, including sources retrieved independently. Selection of relevant and credible sources. Good use of referencing conventions, consistently applied, although when citing multiple sections from one source it is common to list the source once in the references section and mark the section in your text to include the reference number and page [7, p230]

3.25

KNOWLEDGE & UNDERSTANDING

Knowledge of the basic underlying concepts and principles of a subject.



Knowledge is extensive. Exhibits understanding of the breadth and depth of established views.

20.4

COGNITIVE & INTELLECTUAL

Evaluate underlying concepts and principles of a subject and interpret

qualitative and quantitative data in order to develop lines of argument and make sound judgements.

-  Sound, logical, analytical thinking; synthesis and evaluation. Ability to devise and sustain persuasive arguments, and to review the reliability, validity & significance of evidence. Ability to communicate ideas and evidence accurately and convincingly. Sound, convincing conclusions.

20.4

PRACTICAL APPLICATION SKILLS

Different approaches to solving problems in particular contexts.

-  A good and appropriate application of standard methods, tools and techniques. Good appreciation of the context of the application, with some use of examples, where relevant.

17.7

TRANSFERABLE SKILLS

Communicate the results of their study/work accurately and reliably, and with structured and coherent arguments; the qualities needed for employment requiring the exercise of some personal responsibility within a structured and managed environment.

-  Can communicate well, confidently and consistently in a suitable format. Work is coherent and fluent and is well structured and organised.

3.4