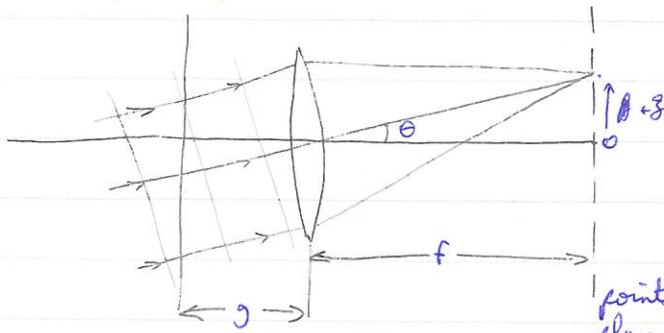


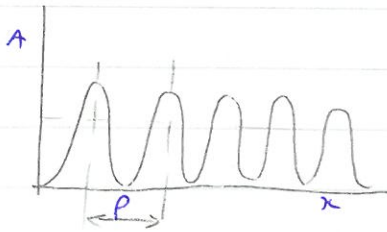


Plane wave focused to point in back plane, angle of incidence controls distance of point from 0.

points in this plane corresponds to different spatial frequencies.

$$A(x, y) \equiv \underset{\text{Ampl. dist.}}{g}(\underset{\text{F.T.}}{\omega_x, \omega_y})$$

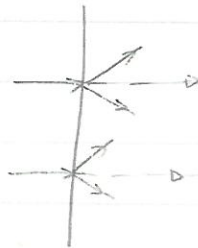
By interpreting the spatial phase distribution of some arbitrary wave is capable of being resolved into a set of fourier distⁿ amplitude components of differing angle of out of lens, spatial angle dependent on frequency



$$\text{spatial } f = \frac{1}{p}$$

$$\text{spatial } \omega = \frac{2\pi}{p}$$

Illuminate a uniaxial diffraction grating with a plane wave



Then put a lens in the way, find they focus to 3 different points. The distances between the points are prop. to the spatial frequencies in diffraction gratings

Relative phases of spatial frequencies only correct if $f = g$. ~~the~~

Now can modify frequencies at back plane by some means.
 Can so reform the plane wave using another lens (of the same type) and so get a (in some way) filtered or altered wave. eg just block out one component. If take out first component, can double the frequency.

FT.

$$f.T. \quad [fT(A) \times fT(B)] = A \otimes B$$

$$fT \quad (A \otimes B) = fT(A) \times fT(B)$$



Convolution important in imaging. Convolution gives the degradation of image.

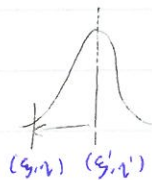
To work out intensity of point in image plane, sum the point spread functions (P) about it.

~~Intensity~~

4

$$I(\xi, \eta) = \iint I(\xi', \eta') \cdot P(\xi - \xi', \eta - \eta') d\xi' d\eta'$$

$$= I \otimes P$$



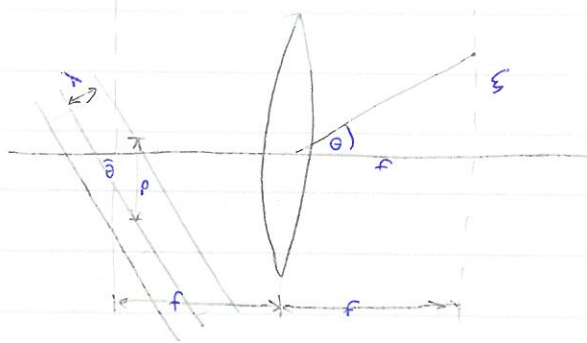
If put in front of a lens & illuminate at the front focal length of a lens. In back focal plane get

$$fT(I \otimes P) = \{ fT(I) \times fT(P) \}$$

If create a filter ~~with~~ $\frac{1}{fT(P)}$, get back a perfect image!

Though in principle possible, cannot actually fully achieve,
but can get close.

Reiteration
on last
lecture



$$\theta = \frac{w}{L}$$

$$\frac{w}{p} = \frac{2\pi}{\lambda} = \frac{2\pi \sin \theta}{\lambda} \quad \text{for small } \theta \quad \approx \frac{2\pi}{\lambda} \cdot \theta \quad \approx \frac{2\pi}{\lambda} \frac{f}{F}$$

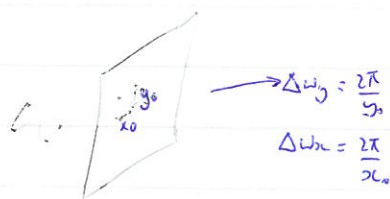
ie spatial frequency does affect position.

for the picture improvement, ~~the~~ getting the phase relationship right may be as (or more) important

Sampling

Use delta functions in 2-D ie $\delta(mx_0 - x_0, ny_0 - y)$

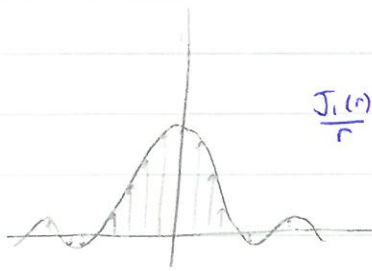
A regular array of delta functions transforms to another regular array of delta functions



~~$$\sum_{m,n} \delta(mx_0 - x, ny_0 - y)$$~~

$$FT \left[\sum_{m,n} \delta(mx_0 - x, ny_0 - y) \right] = \sum_{m,n} \delta \left(m \frac{2\pi}{x_0} - w_x, n \frac{2\pi}{y_0} - w_y \right)$$

Consider F.T. of δ (small round hole)



sample with $\delta(mx_0 - x, ny_0 - y)$

T_r = transform of

$$T_r [T_r(\delta) \times \delta(\cdot)] = T_r(T_r(\delta)) \otimes T_r(\delta)$$

$$\boxed{\delta \otimes \delta} \longrightarrow$$



regular array of small holes

for slide, have resolution of $\delta \otimes \delta$, but also have a large aperture circle, so multiply by that

$$\text{ie } T_r(\bigcirc) \times [\delta \otimes \delta] = T_r(\bigcirc) \otimes \delta \otimes \delta \otimes T_r[\delta \otimes \delta]$$

$$\therefore = T_r(\bigcirc) \otimes [T_r(\bigcirc) \times T_r(\delta)]$$

$$T_r \bigcirc = \text{concentric circles}$$

$$T_r \bigcirc = \text{small circle}$$

Object $\times \delta(\text{array})$ = sampling

$$T_r [\text{Obj} \times \delta(\text{Array})] = T_r(\text{Obj}) \otimes \underset{\delta'}{T_r(\delta(\text{array}))}$$

$$\text{i.e. } \text{Tr} [\text{Obj} \times S(\text{Any})] = \text{Tr} (\text{Obj}) \otimes \otimes S'(\text{any})$$

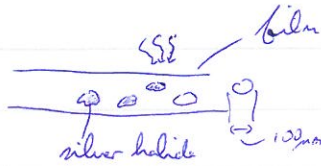


from any single pulse can reconstitute the full picture continuously by retransforming (if no overlap).

Holography

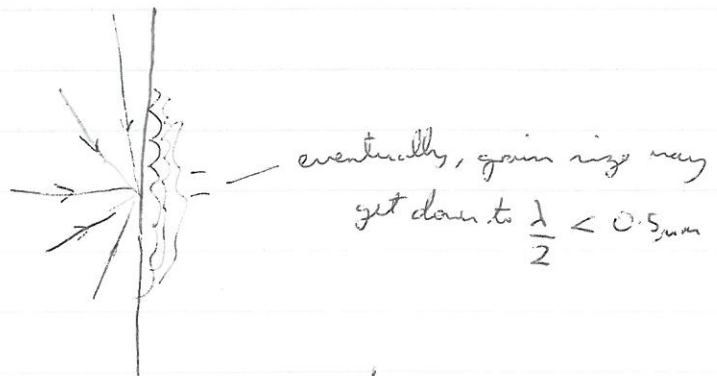
Film

Intensity $\rightarrow$ transmission

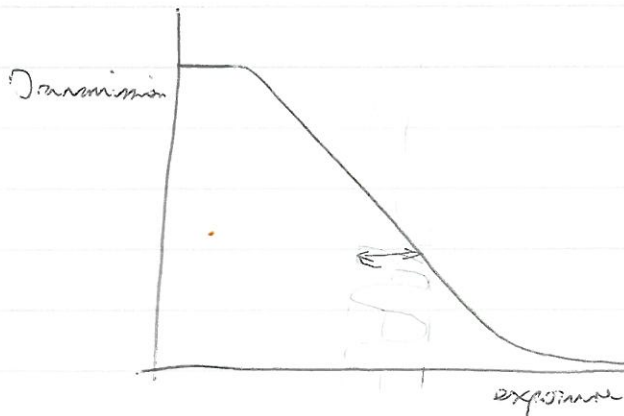


Need a small finite no. of photon to hit a grain. This catalyzes a reaction which does not cease till the whole grain has transformed to silver

To make holograms



But because grain are so small, need ^{much} larger exposure - it still takes 10 photon to start the reaction. This is ^{size} several thousand line pairs per millimeter for holography, as opposed to 40 for photographs



would like to be able to bias into linearish region

We have on $(A_1^2 + A_2^2)$ in the equations which can be used to give the bias. Thus it is usual to ~~ex~~ form the reference wave stronger

than the object wave (approx 3:1). Now can illuminate with the reference wave to reform the object image exactly, including parallax.

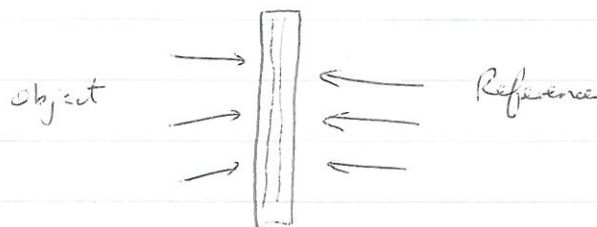
Described Holograming is not really most common.

May use a chemical etching technique to remove the silver. This leaves the film altered - where the silver was, the refractive index of the film is changed. Then can get a sort of "phase modulated" hologram, as opposed to the normal light - dark amplitude modulation.

Several third order terms terms may occur because of non-linearity of film response. 2nd order term doesn't matter - 3rd order does. not terribly dependent on linearity.

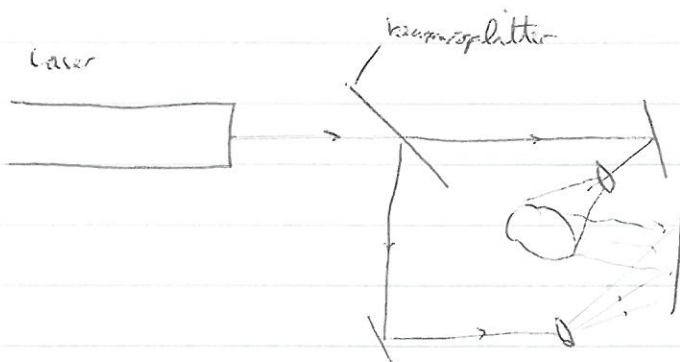
Reconstruction using White light

Try get not too much fringing, put object & reference in from opposite sides of film



Only wavelengths λ with which fit in the fringes are reflected, so can see the image in white light.

Set up to Make Holograms



Need all components stable to $\frac{\lambda}{10}$ throughout the exposure.

Also the path length for object and reference wave should be roughly the same. The coherence length has to be comparable to the size of the object.

$$\delta f \delta t = 1$$

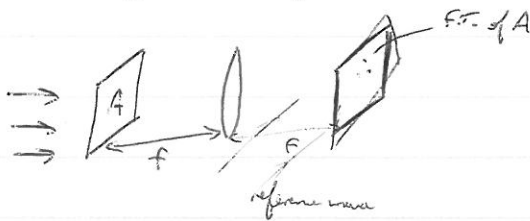
$$c = \frac{\delta x}{\delta t}$$

$$\frac{\delta f \delta x}{c} = 1$$

$$\delta x = \frac{c}{\delta f} = \frac{3 \times 10^8}{10^7} = 30 \text{ cm}$$

Coherence length = length over which the phase of the beam stays essentially constant

(Reference: Applications of Holography : Cantfield & Lee)
~~Example~~ Symbol recognition



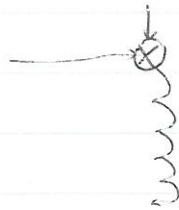
Only if it sees a letter A do you get a dot. So the hologram is a character recognition device to recognize letter A's. And position of the dot ~~is~~ is characteristic of where the A is.

In principle can make holograms in any situation where propagating waves are present - not just optics.

Synthetic Aperture Radar

Microwaves

Photon energy = $10^{-6} - 10^{-3} \text{ eV}$ $\therefore$ very hard to detect a photon - cannot use the distributed kind of receiver used in optics, but can produce extremely coherent sources.



can sample by moving detector.

Holography - Notes

Holography is an interference method of recording the light waves diffracted by a subject illuminated with coherent light. The diffracted waves are caused to interfere with a phase-related reference wave. If the waves are highly coherent, the relative phase between the subject and reference waves remains constant in time, producing an observable effect on the intensity distribution of the resulting interference pattern. The photographic record of this pattern, the hologram, contains sufficient information about both the phase and amplitude of the diffracted waves to permit their reconstruction. Wavefront reconstruction takes place, in a second step, when the hologram is illuminated with the reference wave.

Preservation of relative phase information in a retrievable form is the unique characteristic of the holographic process. In contrast, photography can preserve only the spatial distribution of light intensity in the subject scene. However, the intensity is a quantity averaged over all phases of the light wave and so contains no information about the original phase of the light. By use of a reference wave, holography encodes the unfocused subject light intensity with both amplitude and phase information prior to its encoding.

When the recording medium is silver halide photographic emulsion, the change in transmittance might be due to conversion of silver halide to atoms of silver during exposure and development. Under such circumstances an absorption hologram is obtained. If the same hologram is (bleached) bleached, so as to convert the silver to a transparent compound whose index of refraction differs from that of gelatin, then the record is written in the resulting localized changes in the index of refraction of the emulsion. This is called a phase hologram.

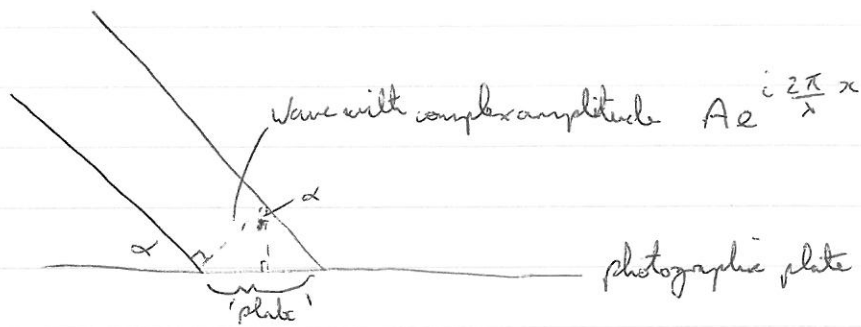
Q Switching

Controls longitudinal modes. The Q switch is a simple liquid cell containing a saturable dye such as crystallogenic dissolved in methanol. As the ruby is optically pumped, creating an inverted population of Cr^{3+} ions, the dye absorbs the radiant energy emitted by the rod and prevents oscillation. This absorption is non-linear. At a sufficiently high emission rate, the populations of the dye energy levels involved in the absorption process equalize, and the dye is said to saturate. It is now transparent to the emission from the ruby rod and laser oscillation occurs in a narrow pulse.

As the dye ~~also~~ absorption is non-linear, the process enhances the differences in the loss characteristics of the various longitudinal and transverse modes. The level of optical pumping can be adjusted to permit the gain of only one mode to exceed the threshold for oscillation. When this occurs, all the energy stored in the ruby pours forth at a single frequency.

Holographic Interferometry

1)



$$K = \frac{2\pi}{\lambda}$$

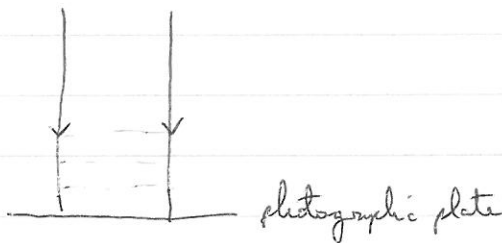
Resolving K gives

$$\sin \alpha = \frac{\text{'Plate'}}{\frac{2\pi}{\lambda}}$$

no complex amplitude distribution

$$R = A e^{i 2\pi \left(\frac{\sin \alpha}{\lambda}\right) x}$$

2)



complex amplitude distribution (for unit amplitude) $O = 1$

for interference $A_{TOT} = \left[1 + A e^{i \left(\frac{2\pi \sin \alpha}{\lambda} \right) x} \right]$

Intensity of resultant fringe in x, y plane

$$\begin{aligned} I &= A_{TOT} A_{TOT}^* \\ &= \left[1 + A e^{i \frac{2\pi \sin \alpha}{\lambda} x} \right] \left[1 + A e^{-i \frac{2\pi \sin \alpha}{\lambda} x} \right] \\ &= 1 + A^2 + A \left(e^{i \frac{2\pi \sin \alpha}{\lambda} x} + e^{-i \frac{2\pi \sin \alpha}{\lambda} x} \right) \\ &= 1 + A^2 + 2A \cos \left(2\pi \left(\frac{\sin \alpha}{\lambda} \right) x \right) \end{aligned}$$

3) Let $O = O(x, y) e^{i\psi(x, y)}$: write as $O = O e^{i\psi}$
 so that

$$A_{TOT} = \left[O e^{i\psi} + A e^{i \frac{2\pi(\sin\alpha)x}{\lambda}} \right]$$

$$= O \left[1 + \frac{A}{O} e^{i \left(\frac{2\pi(\sin\alpha)x}{\lambda} - \psi \right)} \right] e^{i\psi}$$

$$I = |A_{TOT}|^2 = O^2 \left[1 + \left[\frac{A}{O} \right]^2 + \frac{2A}{O} \cos \left(\frac{2\pi(\sin\alpha)x}{\lambda} - \psi \right) \right]$$

ie "carrier fringes" of spatial frequency $\frac{\sin\alpha}{\lambda}$ modulated in intensity and frequency by the object information.

4 Re-illuminate the developed hologram with the reference beam.

$$\text{ie } I.R = R(O+R)(O+R)^*$$

$$= R(OO^* + R R^* + R^*O + R O^*)$$

$$\text{ie } IR = A e^{i \frac{2\pi(\sin\alpha)x}{\lambda}} O^2 \left[1 + \left[\frac{A}{O} \right]^2 + \frac{2A}{O} \cos \left(\frac{2\pi(\sin\alpha)x}{\lambda} - \psi \right) \right]$$

$$IR = (AO^2 + A^3) e^{i \frac{2\pi(\sin\alpha)x}{\lambda}} + A^2 O \left[e^{i \left(\frac{2\pi(\sin\alpha)x}{\lambda} - \psi \right)} + e^{-i \left(\frac{2\pi(\sin\alpha)x}{\lambda} - \psi \right)} \right] e^{i \frac{2\pi(\sin\alpha)x}{\lambda}}$$

$$= (AO^2 + A^3) e^{i \frac{2\pi(\sin\alpha)x}{\lambda}} + A^2 O \left[e^{i \left(\frac{4\pi(\sin\alpha)x}{\lambda} - \psi \right)} + e^{i\psi} \right]$$

$$= (O^2 + A^2) R + A^2 O + A^2 O e^{2i \left(\frac{2\pi(\sin\alpha)x}{\lambda} - \psi \right)}$$

reference beam
 arrives on left
 hologram by a factor
 corresponding to
 the new transmittance
 of the plate

The hologram
 - object beam x const

The pseudoscopic image
 inclination to plane
 $(2 \sin\alpha)$
 so angle is $\sin^{-1}(2 \sin\alpha)$