



SURNAME: Grough

FORENAME(S): Jordan Marc

12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
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1	Student Number	1708174
2	Title of Examination Paper	Further Engineering Mathematics
3	Module Code	SGEES001
4	Date and Time of Examination	21/05/19 09:30
5	Examination Venue	G43
6	Seat Number	
7	Number of Answer Books handed in	

[illegible]

Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

12 Tudalen

Cyfarwyddiadau i Ymgeiswyr

1. Rhwch yr holl wybodaeth y gofynnir amdani yn y blychau isod.
2. Rhwch eich enw yn y lle a ddarparwyd ar y dde, plynwch ef a'i selio i lawr (gan ddefnyddio'r label a ddarparwyd) i guddio'ch enw.

3. Rhaid i'r atebion fod mewn llawysgriften ddarllenadwy. Gall yr Arholwyr naill ai anwybyddu sgrïpt annarllenadwy, neu drefnu iddi gael ei theipio a chael yr ymgeisydd i dalu.
4. Ysgrifennwch eich atebion mewn inc.
5. Dechreuwch bob ateb ar dudalen newydd. Gwnewch unrhyw fras nodiadau yn y llyfr atebion a thynnu llinell drwyddynt.
6. Ni ddylid rhwgo unrhyw ran o'r llyfr hwn; rhaid ei gyflwyno yn gyfan.
7. Os defnyddir mwy nag un llyfr, rhwymwch hwy gyda'i gilydd gan ddefnyddio'r tagiau sydd ar gael oddi wrth y gwyllywyr/gwylwyr.
8. Rhaid dychwelyd pob llyfr. Ni chaniateir mynd â llyfrau atebion, wedi eu defnyddio neu heb eu defnyddio, allan o'r lleoliad arholi o dan unrhyw amgylchriad.

[illegible]

(See the other side for English version)

Llenwch, plynwch a seliwch
(gweler nodyn 2 isod)

CYFENN:

ENW(AU) CYNTAF:

Section A

a)

x	f	xf	$f(x-\bar{x})^2$
6	7	42	46.96
7	21	147	53.09
8	25	200	8.70
9	19	171	3.19
10	14	140	27.83
11	9	99	52.27
12	5	60	58.14
$\Sigma = 100$		$\Sigma = 859$	$\Sigma = 250.18$

$$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{859}{100}$$

$$\bar{x} = 8.59 \checkmark \approx 9 \text{ hours}$$

$$\sigma = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{250.18}{100}}$$

$$\sigma = 1.58 \checkmark \approx 2 \text{ hours}$$

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b) ~~$p=0.6, q=0.4$~~ $p=0.6, q=0.4$

(i) 1 resistor in 3 $[3=n, 1=k]$

$$\binom{3}{1} = \frac{3!}{1!(3-1)!} = 3$$

$$P(1 \text{ in } 3) = 3 \times 0.6^1 \times 0.4^{(3-1)} \\ = 0.288$$

(ii) At least 3 in 4 ~~$[4=n, 3=k]$~~

$$P(\text{at least } 3 \text{ in } 4) = P(X=3) + P(X=4)$$

$$P(3 \text{ in } 4) = \binom{4}{3} = \frac{4!}{3!(4-3)!} = 4 \quad [4=n, 3=k]$$

$$P(3 \text{ in } 4) = 4 \times 0.6^3 \times 0.4^{(4-3)} \\ = 0.346$$

$$\binom{4}{4} = \frac{4!}{4!(4-4)!} = 1 \quad [4=n, 4=k]$$

$$P(4 \text{ in } 4) = 1 \times 0.6^4 \times 0.4^{(4-4)} \\ = 0.130$$

$$P(\text{At least } 3 \text{ in } 4) = 0.346 + 0.130 \\ = 0.476$$

Section B

1. $R=20\Omega, C=0.05F, E=20e^{-3t}, I(0)=0$

$$\text{E(t)} = RI + \frac{1}{C} \int I \cdot dt$$

$$20e^{-3t} = 20I + 20 \int I \cdot dt$$

Differentiate

$$-60e^{-3t} = 20 \frac{dI}{dt} + 20I$$

$$-3e^{-3t} = \frac{dI}{dt} + I$$

$$\frac{dI}{dt} + I = -3e^{-3t}$$

$$P=1, Q=-3e^{-3t}$$

$$\int P \cdot dt = \int 1 \cdot dt = t$$

$$Ie^t = \int e^t \cdot -3e^{-3t} \cdot dt$$

$$Ie^t = \int -3e^{-2t} \cdot dt$$

$$Ie^t = \frac{3}{2}e^{-2t} + c$$

When $I(0)=0$

$$0e^0 = \frac{3}{2}e^0 + c$$

$$0 = \frac{3}{2} + c$$

$$c = -\frac{3}{2}$$

$$Ie^t = \frac{3}{2}e^{-2t} - \frac{3}{2}$$

$$I = \frac{3}{2}e^{-3t} - \frac{3}{2}e^{-t}$$

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2. $\frac{d^2 y}{dt^2} + 3\frac{dy}{dt} + 2y = e^t$ $y_0 = 0, y_1 = 1, t = 0$

$$(s^2 \bar{y} - s y_0 - y_1) + 3(s \bar{y} - y_0) + 2 \bar{y} = \frac{1}{s-1}$$

When $t=0, y_0=0, y_1=1$

$$s^2 \bar{y} - 1 + 3s \bar{y} + 2 \bar{y} = \frac{1}{(s-1)}$$

$$s^2 \bar{y} + 3s \bar{y} + 2 \bar{y} = \frac{1}{(s-1)} + \frac{1(s-1)}{(s-1)}$$

$$\bar{y}(s^2 + 3s + 2) = \frac{1}{(s-1)} + \frac{(s-1)}{(s-1)}$$

$$\bar{y}(s+1)(s+2) = \frac{s}{(s-1)}$$

$$\bar{y} = \frac{s}{(s-1)(s+1)(s+2)}$$

$$\frac{s}{(s-1)(s+1)(s+2)} = \frac{A}{(s-1)} + \frac{B}{(s+1)} + \frac{C}{(s+2)}$$

$$s = A(s+1)(s+2) + B(s-1)(s+2) + C(s-1)(s+1)$$

Let $s = 1$

$$1 = A(2)(3)$$

$$1 = 6A$$

$$A = \frac{1}{6}$$

Let $s = -1$

$$-1 = B(-2)(1)$$

$$-1 = -2B$$

$$B = \frac{1}{2}$$

Let $s = -2$

$$-2 = C(-3)(-1)$$

$$-2 = 3C$$

$$C = -\frac{2}{3}$$

s	1
s	s ² s
2	2s 2

$$\frac{s}{(s-1)(s+1)(s+2)} = \frac{1/6}{(s-1)} + \frac{1/2}{(s+1)} - \frac{2/3}{(s+2)}$$

$$y = \mathcal{L}^{-1} \left\{ \frac{1/6}{(s-1)} + \frac{1/2}{(s+1)} - \frac{2/3}{(s+2)} \right\}$$

$$y = \frac{1}{6}e^t + \frac{1}{2}e^{-t} - \frac{2}{3}e^{-2t}$$

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3. $R=60\Omega$, $C=0.02F$, $L=2H$, $E(t)=50V$, $I(0)=0$, $I'(0)=10$

$$E = RI + L \frac{dI}{dt} + \frac{1}{C} \int I \cdot dt$$

$$50 = 60I + 2 \frac{dI}{dt} + 50 \int I dt$$

Differentiate

$$0 = 60 \frac{dI}{dt} + 2 \frac{d^2 I}{dt^2} + 50I$$

$$2 \frac{d^2 I}{dt^2} + 60 \frac{dI}{dt} + 50I = 0$$

$$\frac{d^2 I}{dt^2} + 30 \frac{dI}{dt} + 25I = 0$$

$$m^2 + 30m + 25 = 0$$

$$m = \frac{-30 \pm \sqrt{30^2 - (4 \times 1 \times 25)}}{2 \times 1}$$

Roots are:

$$m = -29.14$$

$$m = -0.86$$

Real and different.

$$\therefore I = Ae^{-0.86t} + Be^{-29.14t} \quad (\text{G.S.})$$

When $I(0)=0$

$$0 = Ae^0 + Be^0 \quad [\text{Equation 1}]$$

Differentiate the general solution

$$\frac{dI}{dt} = -0.86Ae^{-0.86t} - 29.14Be^{-29.14t}$$

When $I'(0)=10$

$$10 = -0.86Ae^0 - 29.14Be^0$$

$$10 = -0.86A - 29.14B \quad [\text{Equation 2}]$$

multiply Equation 1 by 0.86

$$\begin{aligned} 0 &= 0.86A + 0.86B \\ + \quad 10 &= -0.86A - 29.14B \\ \hline 10 &= 0A - 28.28B \end{aligned}$$

$$B = \frac{10}{-28.28} = -0.354$$

$$\begin{aligned} 0 &= A - 0.354 \\ A &= 0.354 \end{aligned}$$

$$\therefore I = 0.354Ae^{-0.86t} - 0.354e^{-28.28t}$$

$$\therefore I = 0.354e^{-0.86t} - 0.354e^{-28.28t} \quad (\text{Particular Solution})$$

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