

Reduced Mass

$$\frac{1}{2} M^* V^2 = \frac{1}{2} M V^2 + \frac{1}{2} I_f \omega_f^2 + \frac{1}{2} I_e \omega_e^2$$

$$+ \frac{1}{2} I_s \omega_s^2 + \frac{1}{2} I_p \omega_p^2 + \frac{1}{2} I_n \omega_n^2$$

$$M^* V^2 = M V^2 + I_f \omega_f^2 + I_e \omega_e^2 + I_s \omega_s^2 + I_p \omega_p^2 + I_n \omega_n^2$$

Now $V = r\omega$ so $\omega = \frac{V}{r}$ so $\omega^2 = V^2 \times \frac{1}{r^2}$

we call $\frac{1}{r}$ or $\frac{\omega}{V}$ the Velocity Ratio

$$\frac{\omega}{V} = \frac{1}{r} = \delta$$

we write $\omega^2 = \delta^2 V^2$

The individual Velocity Ratios are:

$$\gamma_f = \frac{1}{r_f}$$

$$\gamma_R = \frac{1}{r_R}$$

$$\gamma_s = \text{Sprocket ratio} \times \frac{1}{r_R}$$

$$= \gamma_{s,R} \times \frac{1}{r_R}$$

Now Primary shaft

$$\gamma_P = \gamma_{P,S} \times \gamma_{s,R} \times \frac{1}{r_R}$$

Now Crankshaft:

$$\gamma_M = \gamma_{M,P} \times \gamma_{P,S} \times \gamma_{s,R} \times \frac{1}{r_R}$$

$\Rightarrow$ Substituting $\omega^2 = \gamma^2 V^2$ into equation

$$M^* \cancel{V^2} = M \cancel{V^2} + I_f \cancel{\gamma_f^2 V^2} + I_R \cancel{\gamma_R^2 V^2} + I_s \cancel{\gamma_s^2 V^2} \\ + I_p \cancel{\gamma_p^2 V^2} + I_n \cancel{\gamma_n^2 V^2}$$

$$M^* = M + I_f \gamma_f^2 + I_R \gamma_R^2 + I_s \gamma_s^2 + I_p \gamma_p^2 + I_n \gamma_n^2$$

Example: Mass = 165 kg

$$R_f = 0.3 \text{ m} \quad R_R = 0.35$$

Wheels:

$$I_f = 0.5 \text{ kgm}^2 \quad I_R = 0.6 \text{ kgm}^2$$

Engine:

$$I_n = 0.06 \text{ kgm}^2$$

$$I_p = 0.05 \text{ kgm}^2$$

$$I_s = 0.02 \text{ kgm}^2$$

Also need transmission ratios

$$\begin{array}{ll} \text{Sprockets} & \gamma_{S,R} = 2.6 \\ \text{Prim} \rightarrow \text{Sec} & \gamma_{P,S} = 1.5 \end{array}$$

$$\text{Engine} \quad \gamma_{M,P} = 3.3$$

Ratio between vehicle speed and Engine speed is:

$$\gamma_m = \gamma_{S,R} \times \gamma_{P,S} \times \gamma_{P,M} \times \frac{1}{R_R}$$

$$\gamma_m = 2.6 \times 1.5 \times 3.3 \times \frac{1}{0.35}$$

$$\underline{\gamma_m = 36.77}$$

Also need :

$$\gamma_p = \gamma_{P,S} \times \gamma_{S,R} \times \frac{1}{R_R} = 1.5 \times 2.6 \times \frac{1}{0.35} = 11.14$$

$$\gamma_s = \gamma_{s,R} \times \frac{1}{R_R} = 2.6 \times \frac{1}{0.35} = 7.43$$

$$\gamma_R = \frac{1}{R_R} = \frac{1}{0.35} = 2.86$$

$$\gamma_f = \frac{1}{0.3} = 3.33$$

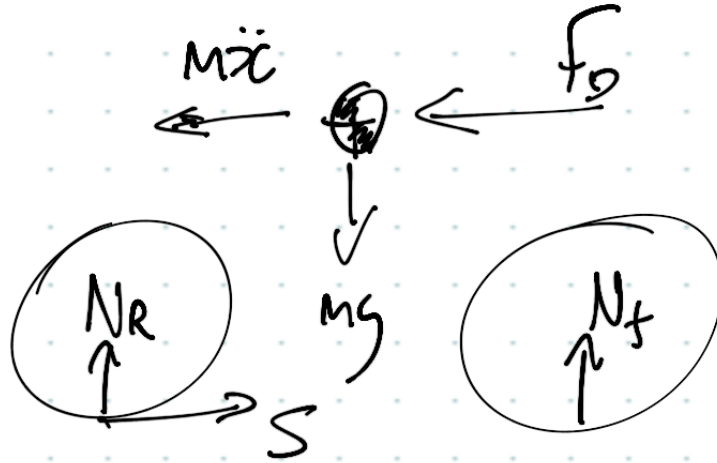
Now we can calculate M^*

$$\begin{aligned} M^* &= M + I_f \gamma_f^2 + I_R \gamma_R^2 + I_s \gamma_s^2 + I_p \gamma_p^2 + I_n \gamma_n^2 \\ &= 165 + (0.6 \times 2.86^2) + (0.5 \times 3.33^2) + \\ &\quad (0.06 \times 36.77^2) + (0.05 \times 11.14^2) \\ &\quad + (0.02 \times 7.43^2) \end{aligned}$$

$$M^* = 165 + 4.9 + 5.54 + 81.12 + 6.2 + 1.1$$

$$M^* = 263.8 \text{ kg}$$

Traction limited Acceleration :



for traction, S must be less than or equal to the product of the driving traction coefficient and the vertical load.

$$S \leq \mu N_R$$

We will also need

$$N_R = mg \frac{(P-b)}{P} + S \frac{h}{P}$$

$$S = M^* \ddot{x} + f_D$$

Resolved Horizontal

$$\left(\begin{array}{l} x = ? \text{ Position} \\ \dot{x} = \frac{dx}{dt} = \text{Speed} \\ \ddot{x} = \frac{d^2x}{dt^2} = \text{acceleration} \end{array} \right)$$

So

$$S = M^* \ddot{x} + f_D \quad \text{also} \quad S \leq \mu N_R$$

$$M^* \ddot{x} + f_D \leq \mu N_R$$

So

$$N_R = \frac{M^* \ddot{x} + f_D}{\mu}$$

$$N_R = Mg \left(\frac{P-b}{P} \right) + S \frac{h}{P}$$

So we get

$$M^* \ddot{x} + f_D = \mu \left[\left(mg \frac{(P-b)}{P} \right) + (M^* \ddot{x} + f_D) \frac{h}{P} \right]$$

$$M^* \ddot{x} + f_D = \mu mg \frac{(P-b)}{P} + \frac{\mu h}{P} (M^* \ddot{x} + f_D)$$

$$(M^* \ddot{x} + f_D) - \frac{\mu h}{P} (M^* \ddot{x} + f_D) = \mu mg \frac{(P-b)}{P}$$

$$(M^* \ddot{x} + f_D) \left(1 - \frac{\mu h}{P} \right) = \mu mg \frac{(P-b)}{P}$$

$$M^* \ddot{x} + f_D = \frac{\mu mg \frac{(P-b)}{P}}{\left(1 - \frac{\mu h}{P} \right)}$$

$$M^* \ddot{x} = \frac{\mu mg \frac{(P-b)}{P}}{\left(1 - \frac{\mu h}{P} \right)} - f_D$$

$$\ddot{x} = \frac{M \frac{M}{M^*} \left(\frac{P-b}{P} \right)}{\left(1 - \frac{Mh}{P} \right)} - \frac{F_D}{M^*}$$

The arrangement does not matter, as long as it is correct:

$$\text{eg. } \ddot{x} = \frac{Mmg(P-b) - F_D P + MF_D h}{PM^* - MM^*h}$$

Wheelie limited Acceleration:

$$N_f = mg \frac{b}{P} - S \frac{h}{P} \quad \text{we say } = 0$$

So we get

$$mg \frac{b}{P} = S \frac{h}{P}$$

So again, get rid of S

$$Mg^{b/p} = (M^*\ddot{x} + F_D)^{h/p}$$

$$Mg^{b/p} = M^*\ddot{x}^{h/p} + F_D^{h/p}$$

$$Mgb = M^*\ddot{x}h + F_Dh$$

$$Mgb = h(M^*\ddot{x} + F_D)$$

$$Mg^{b/h} = M^*\ddot{x} + F_D$$

$$M^*\ddot{x} = Mg^{b/h} - F_D$$

$$\ddot{x} = \frac{M}{M^*} g^{b/h} - \frac{F_D}{M^*}$$