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Theories of Elastic Failure

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Declaration of Authenticity

This report is the result of the Authors original research. It has been composed by the Author and has not been previously submitted for examination which has led to the award of a degree.

Signed:

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Abstract

When designing the orthopaedic implants one of the critical considerations would include calculating the mechanical properties of the material to predict how it will perform under stresses, therefore establishing allowable working stresses, and determining its suitability for purpose. The yield stresses can be calculated by carrying out a simple uniaxial test to ensure that the material complies to the required parameters set by the customer.

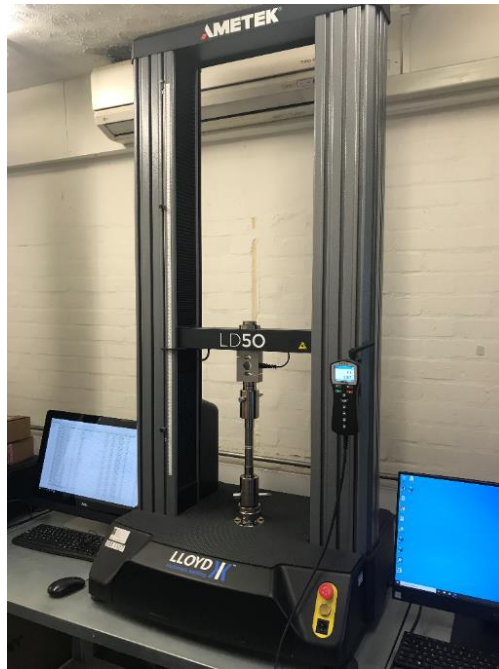


Figure 1 - Ametek LD50

This destructive testing accelerates changes in material behaviour by applying excessive tensile stresses on the material samples, resulting in permanent changes in the materials microstructure which can then be studied to determine when the elastic limit is reached. If the material owns the required elastic properties it will retain the same length before and after the load is applied, despite the fact that it will be longer whilst the load is being applied.

However, this is only true if the load is relatively small and within the elastic range of the material being tested (Timings, 1998). The theory of elastic failure supposes that failure can occur once the internal stresses reach their critical limit, therefore any elongation, deformation or fracture can then be studied in relation to the load applied.

Theories of Elastic failure

The following three theories of material failure can be used to predict material failure:

Maximum Principal Stress (Rankine) Theory

The Rankin theory states that when the maximum principal stress reaches the elastic limit in simple tension, failure or fracture occurs. The criterion of failure is,

$$\sigma_1 = \sigma_{\text{yield}}$$

However, it should be noted that when under compression, failure may occur if the least principal stress σ_3 is compressive and its value reaches the value of the yield stress in compression for the chosen material before the value σ_y , is reached in tension. And additional criterion is therefore,

$$\sigma_3 = \sigma_{\text{yield}} \text{ (Compressive)}$$

This theory applies well to brittle materials such as ceramic or cold metal which fracture cleanly, however may not be the best method for ductile materials as even under pure tension the failure for ductile materials does not takes place because of direct stresses which are applied but due to stresses applied in shear on planes of 45° to the axis of the test piece. (Hearne, 1997)

In short, material failure occurs as the maximum principal stress exceeds it yield strength.

$$\text{Max} (\sigma_1, \sigma_3) \leq \sigma_{\text{yield}}$$

The maximum shear stress criterion is typically used when predicting failure in brittle materials such as glass or cast iron.

The Maximum Shear Stress (Tresca) Theory

The Tresca theory states that it can be assumed that failure will occur as the maximum shear stress in a complex system becomes equal to that at the yield point in the simple tensile test. As the maximum shear stress is half the greatest difference between two principal stresses the criterion of failure becomes,

$$\frac{1}{2} (\sigma_1 - \sigma_3) = \frac{1}{2} (\sigma_{\text{yield}} - 0)$$

That is, $\sigma_1 - \sigma_3 = \sigma_{\text{yield}}$, where the value σ_3 being algebraically the smallest value, i.e taking account of sign and the fact that one stress may be zero. This produces fairly accurate correlation with experimental design. (Hearne, 1997)

In short, failure occurs when the maximum shear stress in the complex stress state equates to the materials limiting shear strength in the simple tensile test.

$$\sigma_1 - \sigma_3 \leq \sigma_{\text{yield}}$$

* σ_1 and σ_3 are the maximum and minimum principal stresses

The maximum shear stress criterion is typically used when predicting failure in ductile materials and because of its relative simplicity it is often used before Von Mises.

The Maximum Shear Strain Energy (Von Mises) Theory

The Von Mises theory predicts that failure of a material which is subjected to any state of stress occurs when the following expression,

$$(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 = \sigma_{\text{yield}}^2$$

Although the equation appears quite different, both Von Mises and Tresca criterion give similar failure predictions. The Tresca criterion will predict material failures at lower stresses, however Von Mises is the preferred method as it is more accurate, especially when calculating failure for ductile metals. (Kelly, 2014, pp. 279 - 280)

In short, failure occurs when the maximum shear energy component in the complex stress system is equal to that at the yield point in a simple tensile test

$$\sigma_{\text{equal}} \leq \sigma_{\text{yield}}$$

The maximum shear stress criterion is typically used when predicting failure in ductile materials.

Graphical Interpretation of the Failure Theories

The three failure theories can be displayed on a simple graph as shown below (figure 2)

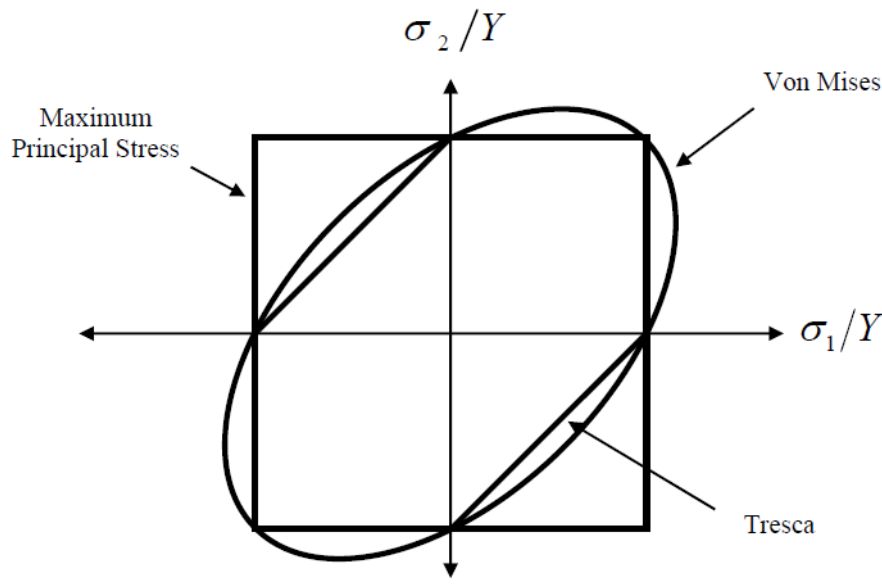


Figure 2 - Failure theory graph

The two principal stresses (σ_1 and σ_2) can be plotted against each other as shown above in what is called a stress space, supposing that the plane stress conditions are $\sigma_3 = 0$. The closed curves shown in the graph are called the failure locus' of their associated failure theories and when the points (σ_1, σ_2) stress states lies inside of their locus then the material is considered to remain elastic. However, if the stress state points reach the locus, failure criterion is satisfied and failure occurs. Failure is typically presented as fracture in brittle materials but in more ductile materials failure can also present as elongation dependant on its plasticity.

Limitations of elastic failure theories

The afore mentioned theories relate to failure which is assumed to occur under elastic loading conditions, however these elastic failure theories would not apply should the component failure occur in a way that cannot be predicted in a simple loading test (failure in service for modes such as creep, impact loading, buckling, under fatigue etc). Should elastic failure be assumed to be relevant it would be necessary to consider which of the theories would be most appropriate for the material in question and for the service loading condition expected. (Hearne, 1997)

Von Mises is generally considered to be the most reliable and relevant theory with the following exceptions:

- For brittle materials the maximum principal stress theory is most suitable, however this is regarded as unsafe for use on ductile materials.
- All theories produce similar results in loading situations where one principal stress is large compared to another.
- The main differences between the theories is found in the second and fourth quadrants of the graphical representations where the principal stresses are of opposite sign but numerically equal.
- The Mohr theory is preferred if the test data for tension, compression and torsion is reliable for bi-axial stress conditions.
- The Tresca maximum shear stress theory is the safest in most bi-axial and tri-axial stress conditions. It is also the simplest, most applicable formula and therefore the theory that is most widely used in the orthopaedic industry.

Component development

Since its launch in 1982, the Taperloc Hip system is the industry standard in Cementless hip arthroplasty and is core product at Biomet UK Ltd. The stem is manufactured from Titanium alloy Ti-6AL-4V, which allows stress transfer to preserve cortical density and also incorporates a Titanium porous plasma coated alloy which provides a scratch fit stability to encourage and accelerate bone fixation. The main advantages of using Titanium alloys in the manufacture of orthopaedic implants include:

- The material is strong and robust enough to withhold the expected forces
- The relative density of the material means that it is suitably lightweight.
- The material is bioactive and resists implant rejection post surgery.
- The material accepts various coating applications such as porous titanium & Hydroxyapatite which increase bone adherence, reduced rejection rates and accelerated bonding between cancellous bone and the orthopaedic implant.



Figure 3 - Taperloc Hip stem system

Elastic failure analysis is a critical requirement as a part of initial product development and submission of Design History File for the notified licencing body. Before clinical research is initiated a theoretical study of stress and strain

distributions along the femur is undertaken to predict how the implant will behave under applied forces. Together with metallurgy, the shape of the stem has a direct impact on how the implant withstands both axial and torsional stress which occur during standard movements such as walking or stair climbing. To replicate these movements in a test scenario a standard material test piece is loaded to failure, before the data can be used to plot a stress strain curve, to illustrate the elastic behaviour under loading (as shown in the stress strain diagram example in figure 4).

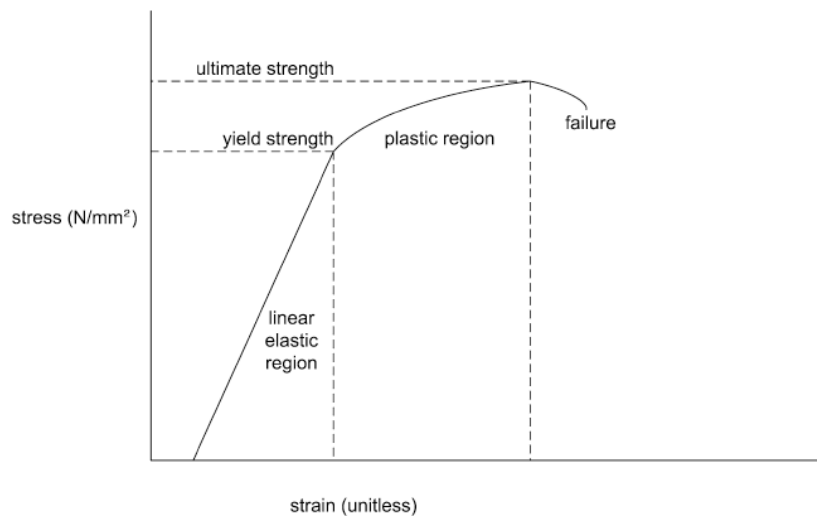


Figure 4 - Stress/Strain diagram for a ductile material

The features within the diagram define the mechanical properties of the implant and with the addition of computational modelling (FEA) the designer to predict failure parameters that could result in an adverse patient outcome (figure 5) when considering stress dispersions between implant and bone. (Abdul H Abdullah, 2010)

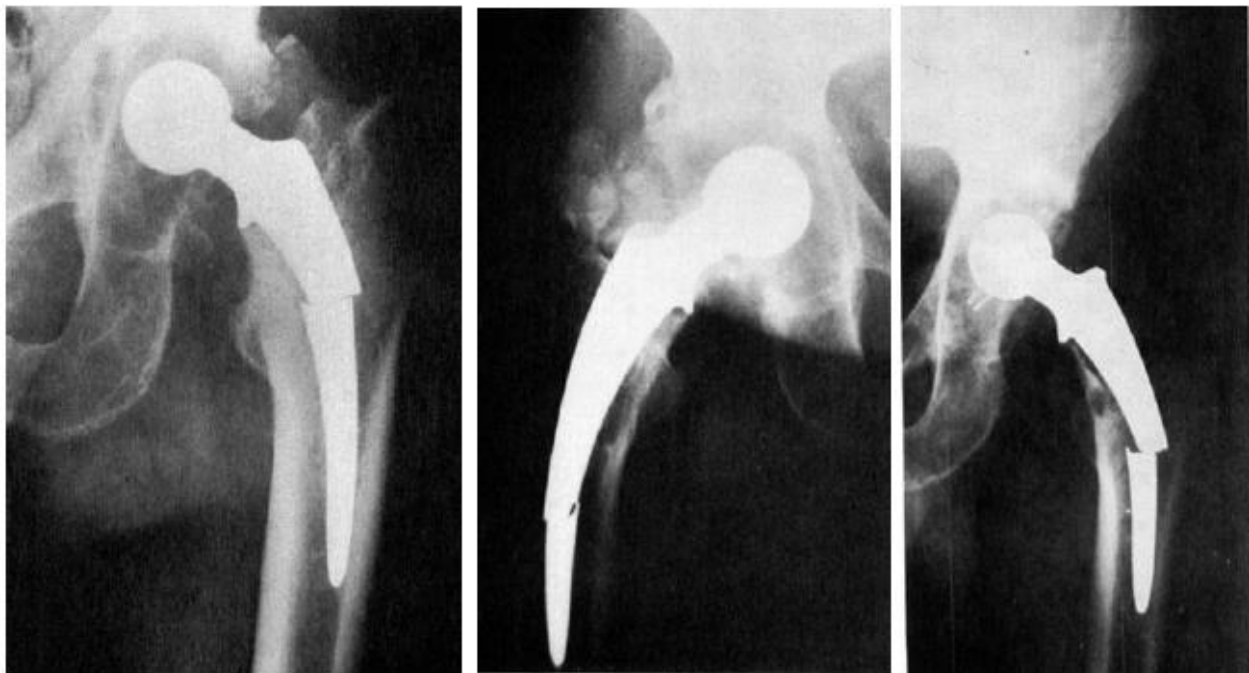


Figure 5 - Orthopaedic implant fractures (Femoral stem)

With this determination and understanding of the mechanical properties of the Titanium the designer is able to compare the properties to bone and make comparison to other usable materials to establish the best comparison (figure 6). Less stress shielding is created with materials that have similar mechanical properties to bone, thus decreasing the risk of component fracture. (S. Raymond Golish, 2011)

Mechanical properties of commonly used alloys

Property	Porous Tantalum	Ti-6Al-4V	Co-Cr-Mo	316L Stainless Steel
Modulus of elasticity (GPa)	2.5-3.9	106-115	210	230
Yield strength (MPa)	35-51	860	825	170-690
Ultimate strength (MPa)	50-110	780-1050	430-1028	515-860
Hardness (on Rockwell C scale)		36	42	60
Fatigue strength (MPa at 10^7 cycles)	18-20	480-590	310	180-300
Mass density (g/cm^3)		4.4	8.3	8.0

Figure 6 - Table of mechanical properties of commonly used alloys

The bone is assumed to consist of cortical (hard) shell or cancellous (spongy) core and the Taperloc stem is made of Ti-6AL-4V alloy (figure 7) and this data is the basis of the FE model.

Mechanical Property	E (GPa)	ν
Cortical bone	17	0.33
Cancellous bone	1.5	0.33
Ti-6Al-4V	110	0.30

Figure 7 - Mechanical properties of model materials

Finite Element Model

Finite Element Analysis is used to examine stress behaviours and calculate stress distribution (with respect to stress shielding) in total hip replacements. The Finite Element models require protocols and boundary conditions to be put in place such as the values for the materials Modulus of Elasticity, Poisson's ratio, number of nodes and elements and applied forces. The outcomes of the FEA study predict the distributed stress around the component without conducting failure analysis and the path of the crack which occurs in failure. (A. Bashiri, 2020)

Various load cases representing such activities as walking, running and stair climbing are investigated with combinations of loads which represent each scenario. The results of each scenario are then presented in terms of Tresca stress distribution and maximum principal stress at three section of the test sample – the proximal re-section, the mid section and the distal section (figure 3).

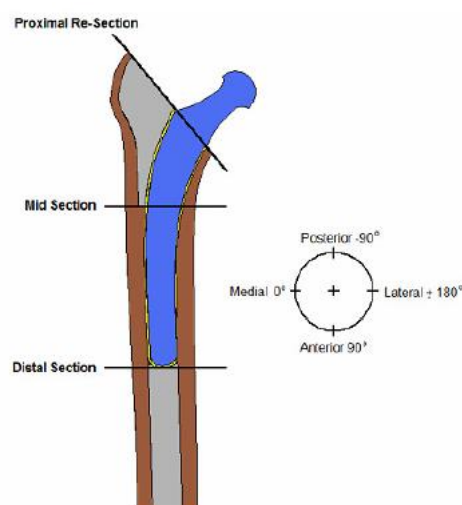


Figure 8 – respective levels of hip femur

Both walking and stair climbing scenarios experience similar stress distributions along the cortical bone with stress being distributed (figure 9).

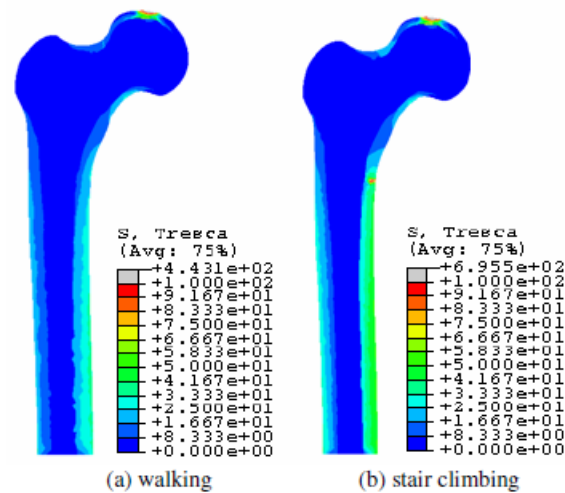


Figure 9 - Tresca stress distribution in femur under different loading conditions

The trends from the two scenarios display stress trends with similar distributions on both but with higher magnitudes of stress in the stair climbing scenario, which are to be expected (figure 10). With the implementation of hip stem orthopaedic the stress state and load transfer is altered within the femur with high stress concentrations being identified in the distal of the orthopaedic implant, particularly in the stair climbing scenario were greater loads are experienced. This will present itself as a decreased load transfer in the medial region of the cortical and encourage implant failure. (Abdul H Abdullah, 2010)

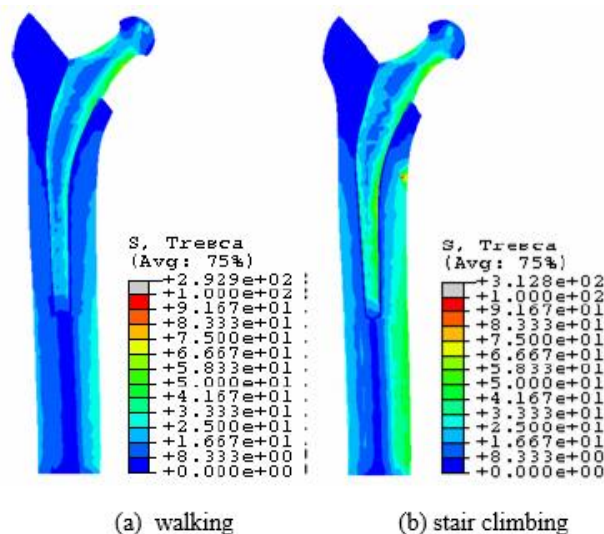


Figure 10 - Tresca stress distribution of implant to femur under different loading conditions

Research paper review

A review of “Finite Element Analysis of Cemented Hip Arthroplasty: Influence of Stem Tapers” by Abdul H. Abdullah, Mohd N. Mohd Asri, Mohd S. Alias and Tardan Giha, shows an investigation and discussion around the importance of minimising loosening problems, mechanical failure and ultimately risk to patient safety by examining stress behaviours using Tresca stress distribution. The Tresca theory is the most conservative theory when calculating elastic failure in ductile materials and therefore the most widely used, especially in the Orthopaedic industry.

The paper discusses the phenomenon of stress shielding effect, where specific regions of the femur are not being stressed properly, leading to weakness and propensity to fracture. Their research further presents the stress distributions along the femur when the orthopaedic implant is introduced with the stem carry much of the load, which is transferred down along the implant to the distal tip before being transferred to the cortical bone.

Conclusion

Although the Tresca criterion has similarities with the predictions given, in ductile materials, using the Von Mises criterion, the Tresca theory applies suitably to the lower loads that are experienced by Orthopaedic implants and also is the most conservative.

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