



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

Complete, fold and seal
(see note 2 below)

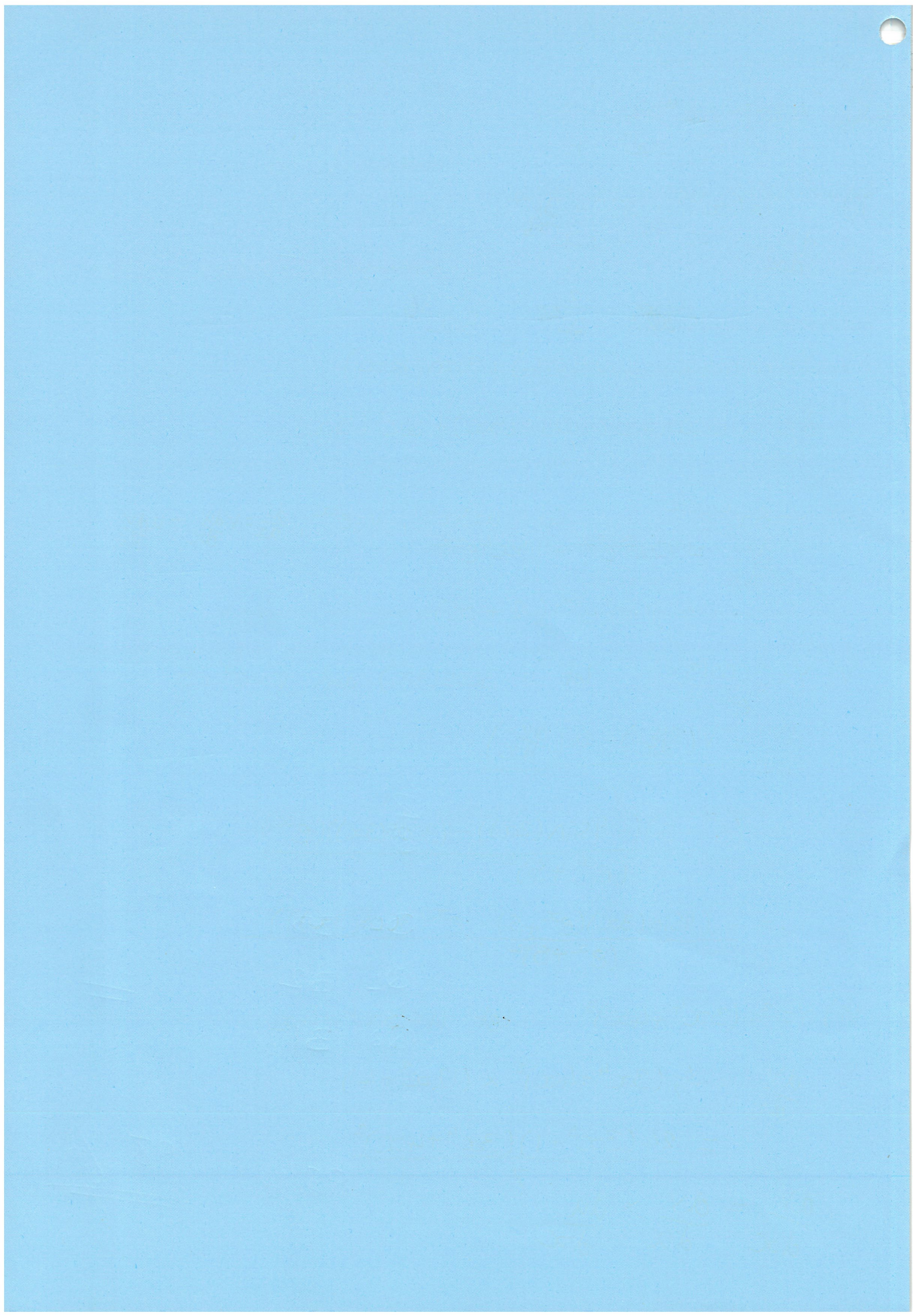
SURNAME: Webb

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1	Student Number	WEB25887285
2	Title of Examination Paper	Mathematics Test 1
3	Module Code	GOEN4001
4	Date and Time of Examination	19/01/2026 1400Hrs - 1600Hrs
5	Examination Venue	Tycoch
6	Seat Number	E5
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
1	
2	
3	
Q1	35 ✓
Q2	32 ✓
Q3	31 ✓
Total	98/100

12
98%



i) a) i) $y = 2x^6 - 5x^3 + 4x - 4$

$$\frac{dy}{dx} = 12x^5 - 15x^2 + 4$$

2

Q1

ii) $y = 4x^{-1/2} - x^{1/5} + 3e^{-2}$

$$\frac{dy}{dx} = -2x^{-3/2} - \frac{1}{5}x^{-4/5}$$

3

35/35

$$= \cancel{\frac{-2}{2x^{3/2}}} - \frac{1}{5x^{4/5}}$$

Q1
a

iii) $y = 4\sin(4x) - 2\cos(5x) + 3\tan(\frac{1}{3}x)$

$$\frac{dy}{dx} = 16\cos(4x) + 10\sin(5x) + \sec^2(\frac{1}{3}x)$$

3

22/22

iv) $y = 5x^4 e^{2x}$

let $u = 5x^4$, $v = e^{2x}$, $\frac{dv}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

$$\frac{dy}{dx} = 20x^3 e^{2x} + 10x^4 e^{2x}$$

$$= 10x^3 e^{2x} (2 + x)$$

4

v) $y = \frac{3\cos(3x)}{e^x + 5}$ let $u = 3\cos(3x)$ and $v = e^x + 5$

$$\frac{dy}{dx} = \frac{v(\frac{du}{dx}) - u(\frac{dv}{dx})}{v^2}$$

4

$$= \frac{(e^x + 5)(-9\sin(3x)) - (3\cos(3x))(e^x)}{(e^x + 5)^2}$$

$$= \frac{-(9\sin(3x)(e^x + 5) + e^x(3\cos(3x)))}{(e^x + 5)^2}$$

vi) $y = \frac{2}{7}(3x^4 + 4x)^3$ let $u = 3x^4 + 4x$

3

$$\frac{dy}{dx} = \frac{6}{7}(3x^4 + 4x)^2 \times (12x^3 + 4)$$

$$= \frac{24(3x^3 + 1)(3x^4 + 4x)^2}{7}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

vii) $y = \frac{1}{5} (5x^4 + \sin(2x))^{\frac{1}{5}}$

Let $u = 5x^4 + \sin(2x)$, $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\therefore \frac{dy}{dx} = \frac{1}{5} (5x^4 + \sin(2x))^{-\frac{4}{5}} \times (20x^3 + 2\cos(2x))$

$= \frac{2(10x^3 + \cos(2x))}{25(5x^4 + \sin(2x))^{\frac{4}{5}}}$

b) i) $W = 3x^4y^3 + 5x^3 + 6y^2 + e^{2x} - \cos(3y)$

$\frac{\partial W}{\partial x} = 12x^3y^3 + 15x^2 + 2e^{2x}$

$\therefore \frac{\partial^2 W}{\partial x^2} = 36x^2y^3 + 30x + 4e^{2x}$

(ii) $\frac{\partial W}{\partial y} = 9x^4y^2 + 12y + 3\sin(3y)$

$\therefore \frac{\partial^2 W}{\partial y^2} = 18x^4y + 12 + 9\cos(3y)$

(iii) $\frac{\partial^2 W}{\partial x \partial y} = 36x^3y^2$

iv) $\frac{\partial^2 W}{\partial y \partial x} = 36x^3y^2$

Q11

6/6

c) $y = 3e^x + x^2$

Show that $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 - 6x + 2$

$$\frac{dy}{dx} = 3e^x + 2x$$

$$\frac{d^2y}{dx^2} = 3e^x + 2$$

$$\Rightarrow (3e^x + 2) - 3(3e^x + 2x) + 2y = 2x^2 - 6x + 2$$

$$3e^x + 2 - 9e^x - 6x + 2y = 2x^2 - 6x + 2$$

$$1 - 6e^x - 6x + 2(3e^x + x^2) = 2x^2 - 6x + 2$$

$$1 - 6e^x - 6x + 6e^x + 2x^2 = 2x^2 - 6x + 2$$

$$2x^2 - 6x + 2 = 2x^2 - 6x + 2$$

$$\therefore \text{LHS} = \text{RHS}$$

Q1C

4/4

4

d) $y = x \sin(x)$

Let $u = x$ and $v = \sin(x)$

$$\frac{dy}{dx} = x \cos(x) + \sin(x) \quad \frac{dy}{dx} = u \left(\frac{dv}{dx} \right) + v \left(\frac{du}{dx} \right)$$

$$\frac{d^2y}{dx^2} = -x \sin(x) + \cos(x) + \cos(x) = -x \sin(x) + 2\cos(x)$$

$$\frac{d^3y}{dx^3} = -x \cos(x) - \sin(x) - 2\sin(x)$$

$$= -x \cos(x) - 3\sin(x)$$

Q1d

3/3

3

Q2) a)
$$\frac{5x+7}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$$

Q2

$$\frac{5x+7}{(x+1)(x-3)} = \frac{A(x-3)}{(x+1)(x-3)} + \frac{B(x+1)}{(x-3)(x+1)}$$

29/33

$$\Rightarrow 5x+7 = A(x-3) + B(x+1)$$

Q2 a

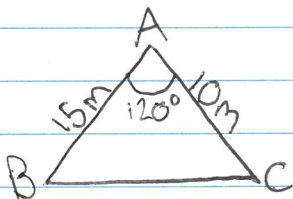
Let $x=3$, $15+7 = A(0) + B(4)$
 $22 = 4B$
 $\therefore B = \frac{22}{4} = \frac{11}{2}$

9/9

Let $x=-1$, $-5+7 = A(-4) + B(0)$
 $2 = -4A$
 $\therefore A = -\frac{2}{4} = -\frac{1}{2}$

$$\therefore \frac{5x+7}{(x+1)(x-3)} = \frac{-1}{2(x+1)} + \frac{11}{2(x-3)}$$

b) i)

Q2 b5/5

$$(BC)^2 = (AB)^2 + (AC)^2 - (2 \times AB \times AC \times \cos(A))$$

$$(BC)^2 = 15^2 + 10^2 - (2 \times 15 \times 10 \times \cos(120))$$

$$BC = \sqrt{475} = 5\sqrt{19} \approx 21.79 \text{ m}$$

$$2) \frac{AB}{\sin C} = \frac{AC}{\sin B} = \frac{BC}{\sin A} = \frac{21.79}{\sin 120} \approx 25.166$$

$$C = \sin^{-1}\left(\frac{15}{25.166}\right) \approx 36.59^\circ$$

$$B = \sin^{-1}\left(\frac{10}{25.166}\right) \approx 23.41^\circ$$

$$c) x^2 - 11x + 24 = (x-8)(x-3)$$

$$\begin{aligned} -8 + -3 &= -11 \\ \text{Factors of } 24 &= 8, 3, -8, -3, \\ &\pm 24, \pm 1, \pm 12, \pm 2 \end{aligned}$$

$$\therefore \tan^2(x) - 11\tan(x) + 24 = (\tan(x) - 8)(\tan(x) - 3) = 0$$

$$\begin{aligned} \therefore \tan(x) - 8 &= 0 \rightarrow x = \tan^{-1}(8) = 82.87^\circ \text{ and } 262.87^\circ \\ \text{and } \tan(x) - 3 &= 0 \rightarrow x = \tan^{-1}(3) = 71.57^\circ \text{ and } 251.57^\circ \end{aligned}$$

$$d) Y = x^2 - 4x + 3 \text{ and } Y = 2x + 1$$

$$\text{Let } x^2 - 4x + 3 = 2x + 1$$

$$\Rightarrow x^2 - 6x + 2 = 0$$

using quadratic formula with $a=1$, $b=-6$, $c=2$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - (4 \times 1 \times 2)}}{2 \times 1}$$

$$= \frac{6 \pm 2\sqrt{7}}{2} = 3 \pm \sqrt{7}$$

$$\therefore x = 3 + \sqrt{7} \text{ and } x = 3 - \sqrt{7}$$

$$\approx 5.65$$

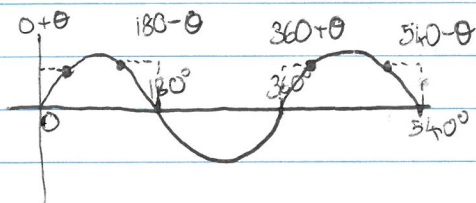
$$\approx 0.354$$

Since $Y = 2x + 1$, substitute x for values: show workings.

coordinates are at $(0.354, 1.71)$ and $(5.65, 12.3)$

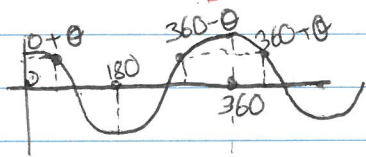
$$e) i) 2\sin(x) = 0.75 \rightarrow x = \sin^{-1}\left(\frac{0.75}{2}\right)$$

$$\therefore x = 22.02^\circ \text{ and } x = 180 - 22.02 = 157.98^\circ$$



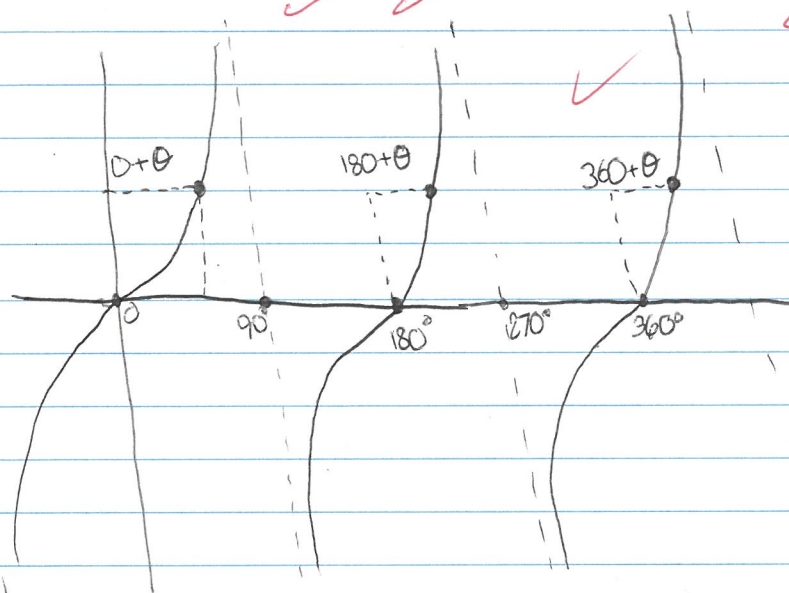
e)ii) $4\cos(x) - 1 = 0 \rightarrow x = \cos^{-1}\left(\frac{1}{4}\right)$ ✓

$\therefore x = 75.52^\circ$ and $x = 360 - 75.52 = 284.48^\circ$ ✓ ✓



iii) $-5\tan(x) = 1 \rightarrow x = \tan^{-1}\left(-\frac{1}{5}\right) = -11.31^\circ$ ✓ ✓

$\therefore x = 180 - 11.31 = 168.69^\circ$ and $x = 360 - 11.31 = 348.69^\circ$ ✓ ✓



Q3a)i) $\ln(2x-3) + \ln(3) = \ln(5x+9)$
 $\ln(3(2x-3)) = \ln(5x+9)$
 $\therefore 6x-9 = 5x+9$
 $\therefore x = 18$

Q331/32

ii) $\log(3x+2) + \log(x+1) = \log(15)$
 $\log((3x+2)(x+1)) = \log(15)$

Q3a17/18

$\Rightarrow (3x+2)(x+1) = 15$
 $3x^2 + 2x + 3x + 2 = 15$
 $3x^2 + 5x - 13 = 0$

Let $a=3$, $b=5$ and $c=-13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - (4 \times 3 \times -13)}}{2 \times 3}$$

$$= \frac{-5 \pm \sqrt{25 + 156}}{6}$$

$\therefore$ either $x = \frac{-5 + \sqrt{181}}{6} \approx 1.41$

or $x = \frac{-5 - \sqrt{181}}{6} \approx -3.08$ not a solution

iii) $4^{(x+3)} \times 2^{(2-x)} = 8^{(x+1)}$
 $\log_2(4^{(x+3)} \times 2^{(2-x)}) = \log_2(8^{(x+1)})$

$$\log_2(4^{(x+3)}) + \log_2(2^{(2-x)}) = \log_2(8^{(x+1)})$$

$$(x+3)\log_2(4) + (2-x)\log_2(2) = (x+1)\log_2(8)$$

$$2x + 6 + 2 - x = 3x + 3$$

$$3x - x = 6 + 2 - 3$$

$$2x = 5$$

$$\therefore x = \frac{5}{2} = 2.5$$

$$\text{iv) } \frac{6x+1}{3x-2} = \frac{4}{5}$$

$$5(6x+1) = 4(3x-2)$$

$$30x+5 = 12x-8$$

$$18x = -13$$

$$\therefore x = -\frac{13}{18} \approx -0.722$$

$$\text{v) } \frac{(4x+1)}{5} + \frac{(2x-3)}{3} = \frac{11}{6}$$

$$\frac{3(4x+1)}{15} + \frac{5(2x-3)}{15} = \frac{11}{6}$$

$$\frac{12x+3+10x-15}{15} = \frac{11}{6}$$

$$\Rightarrow 22x-12 = \frac{55}{2} = 27.5$$

$$\therefore 22x = 39.5 \Rightarrow x = \frac{39.5}{22} \approx 1.795$$

$$\text{b) i) } p^2 - 13p + 42$$

Factors of 42 = $\pm 21, \pm 2, \pm 7, \pm 6$
 $-6 + -7 = -13$

$$= (p-7)(p-6)$$

$$\text{ii) } 4b^2 - 49$$

Difference of two squares

$$\therefore 4b^2 - 49 = (2b+7)(2b-7)$$

$$\text{iii) } mn + 5mp + 3nq + 15pq = m(n+5p) + 3q(n+5p)$$

$$= (n+5p)(m+3q)$$

Q3b

6/6

c) i) $6m^2 + 17m + 12 = 0$
 $(3m+4)(2m+3) = 0$

Factors of 12:
 $\pm 6, \pm 2, \pm 3, \pm 4$
 $(4 \times 2) + (3 \times 3) = 17$

$\therefore$ $3m+4=0$ or $2m+3=0$
 $\Rightarrow$ either $m = -\frac{4}{3}$ or $m = -\frac{3}{2}$

ii) $y^2 + 13y + 40 = 0$
 $(y+5)(y+8) = 0$

Factors of 40: $\pm 8, \pm 5$
 $8 \times 5 = 40$

$\therefore$ either $y+5=0$ or $y+8=0$
 $\Rightarrow y = -5$ or $y = -8$

d) $3w^2 - 2w - 8 = 0$
 Let $a=3, b=-2, c=-8$

$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - (4 \times 3 \times -8)}}{2 \times 3}$

$\therefore w = \frac{2 \pm \sqrt{100}}{6} = \frac{2 \pm 10}{6} = \frac{1 \pm 5}{3}$

$\therefore$ either $w = \frac{1+5}{3} = 2$ or $w = \frac{1-5}{3} = -\frac{4}{3} \approx -1.33$

Q3c

4/4

Q3d

4/4

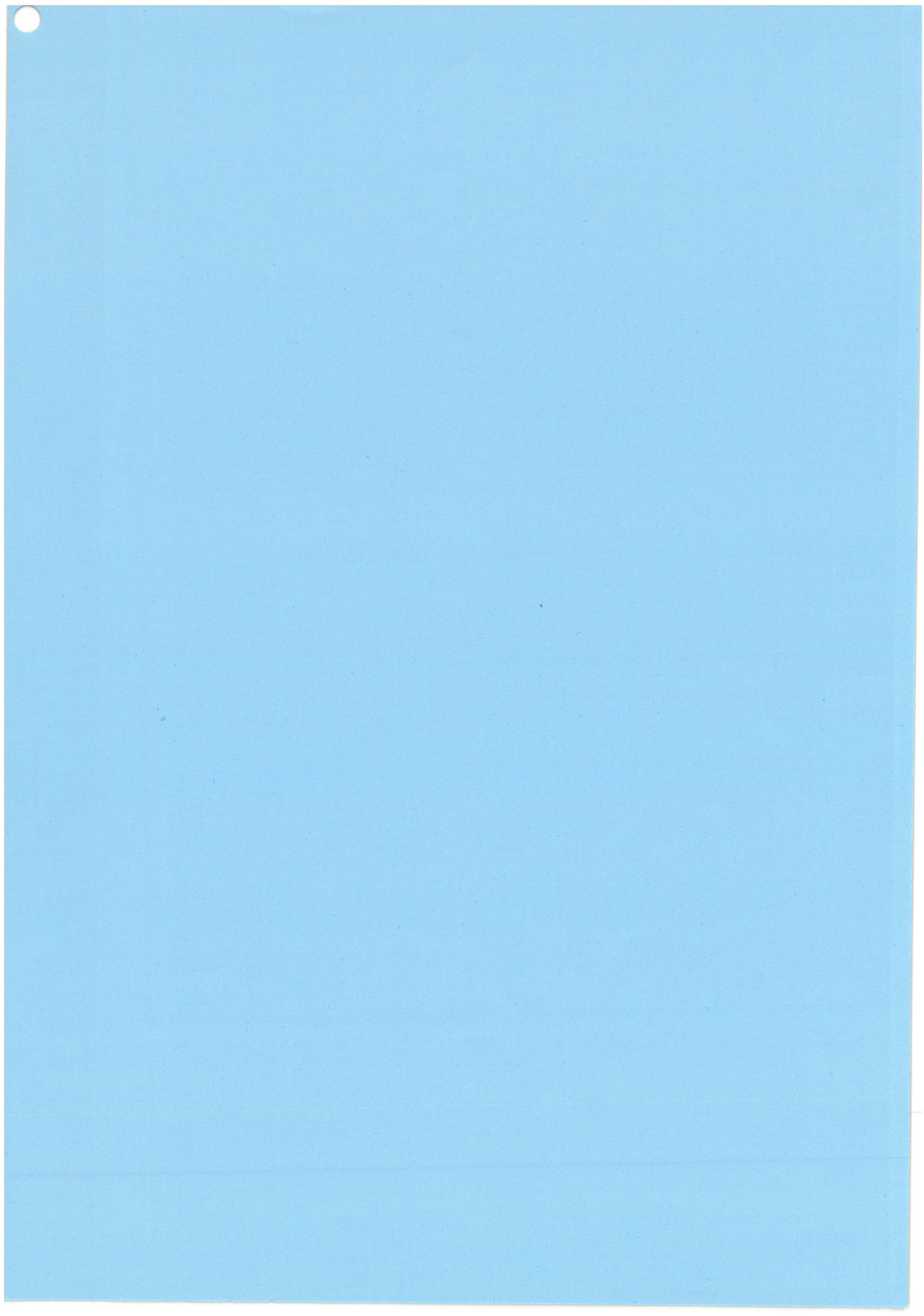


Question No.

Examiner Only

Arholwr yn unig

Rhif y Cwestiwn



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