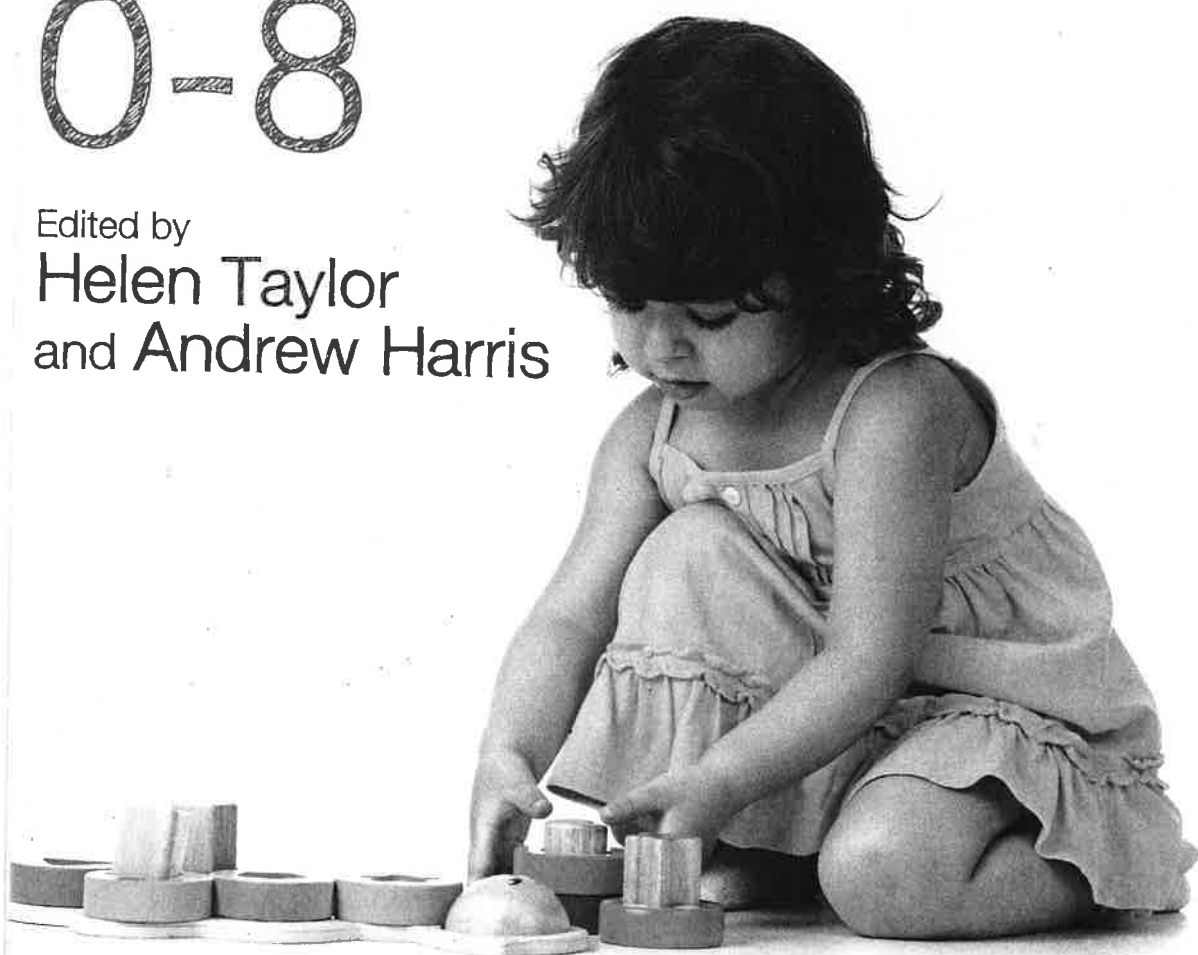


Learning and Teaching Mathematics 0-8

Edited by
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Developing an understanding of place value

Fundamental principles of the place value system

We identified earlier that counting is very useful for exploring small quantities. However, for numbers larger than 20, counting as a means of quantifying and of comparing quantities is relatively inefficient and prone to errors. Thus, for exploring multi-digit numbers we use a place value system. While our place value system is a very powerful tool, using it requires a sophisticated understanding of four fundamental properties identified by Ross (2002: 419):

- 'Positional': the position ('place') of a digit within the number determines its value
- 'Base-ten': working from right to left, successive positions ('places') represent increasing powers of 10 (... 10^3 , 10^2 , 10^1 , 10^0 , 10^{-1} , 10^{-2} , 10^{-3} , ...)
- 'Multiplicative': the value of a particular digit within a number is determined by multiplying the digit by its positional value (for example, in 256 the two is worth 2×100 , the five is worth 5×10 and the six is worth 6×1)
- 'Additive': the value of the overall number is the total of the values of the individual digits (for example, 256 is worth $200 + 50 + 6$).

Two consequences of the base-ten property are that only ten digits (0–9) are needed to represent numbers and that the digit zero acts as a 'place-holder' (denoting empty 'places' within numbers and thereby preserving the positional values of other digits).

Given the complexity of these properties, adults must provide a connected experience of practical representations, interpretation, discussion, and symbolic recording in order for children to achieve a full understanding of place value.

Quantity value and column value

Thompson and Bramald (2002) interviewed 144 children aged 6–8. They identified two different ways of thinking about multi-digit numbers using place value, which they called 'quantity value' and 'column value'. For example, we can think about the number 43 either as $40 + 3$ (quantity value) or as 4 tens + 3 ones (column value).

Thompson and Bramald found that children were relatively successful in tasks that required understanding of quantity value (such as two-digit mental calculation

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strategies). By contrast, few of the children had an understanding of column value, which is used in written calculation strategies. They concluded that initially adults should emphasise the quantity value perspective because it is more readily understood by children and because it supports understanding of mental calculation strategies which are developed earlier than written calculation strategies. Furthermore, they recommended that learning about column value be delayed until children begin to use written calculation strategies (when an understanding of column value becomes necessary).

Introducing children to place value

Children are fascinated by big numbers and will talk about them happily even if they have no real conception of their relative size. Often children will have heard adults talking about such numbers and some will have explored them, perhaps on a calculator or a computer game. Storybooks, such as *How Big is a Million?* (Milbourne, 2007) which offers visual images showing the relative sizes of one, ten, one hundred, one thousand, and one million, can provide a stimulus for discussion and an informal way to begin learning about place value.

Adults can build on such informal understandings by helping children to work on grouping items (both everyday objects and mathematical resources) as sets of ten, and subsequently sets of ten plus some additional 'ones'. This can be introduced as a speedier form of counting initially but it is important to make explicit



Figure 9.9 Representing 43 (using Multilink, Numicon, Dienes' Base 10 materials and a bead string)

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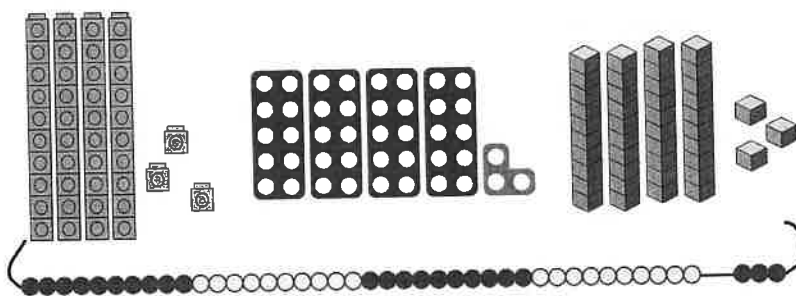


Figure 9.9 Representing 43 (using Multilink, Numicon, Dienes' Base 10 materials and a bead string)

the connections between the physical representations and the digits used to write the numbers. When discussing such representations, adults should refer to the tens quantities using quantity value (ten, twenty, etc.) rather than column value referents (1 ten, 2 tens, etc.). Through experiences of reading, writing and representing numbers and associated mathematical talk, children begin to connect the value of each digit in a number with the position of the digit as well as the digit itself. Figure 9.9 shows possible resources which might be used.

Typical activities to support this development might include:

- Using 0–9 digit cards to make numbers, talking about how the value of each digit changes when their positions are changed and then discussing the impact of the changes on the size of the numbers
- Using a 0–9 dice, base-boards marked with tens and ones columns and a predetermined target (for example, making the largest number or the number closest to 50), children take it in turns to roll the dice and place the digit obtained in either the tens or ones position on their base-board with the winner being the child whose number is closest to the target when everyone has made a two-digit number.

Partitioning numbers and combining digit values

Partitioning numbers involves splitting numbers into separate positional components based on the value of each digit (for example, 143 is partitioned as $100 + 40 + 3$) and combining is the reverse of this process. Both make use of the 'quantity value' aspect of place value and Ross' (2002) additive property. To be able to partition numbers into tens and ones (and later hundreds, tens and ones) children need to appreciate how the value of any digit is determined by its position within a number (Ross' (2002) positional property). Arrow cards and Gattegno place value charts (see Figure 9.10) are useful resources for supporting both partitioning and combining activities (Anghileri, 2006).

The partitioned form $100 + 40 + 3$ indicates how the number is spoken (one hundred and forty three) whereas the combined form 143 indicates how the number is written. Later, it becomes important for children to explore alternative partitionings of numbers because this will support flexibility when calculating (Anghileri, 2006) (see Chapter 10). For example, 59 can be considered as $40 + 19$ or $60 - 1$ as well as $50 + 9$.

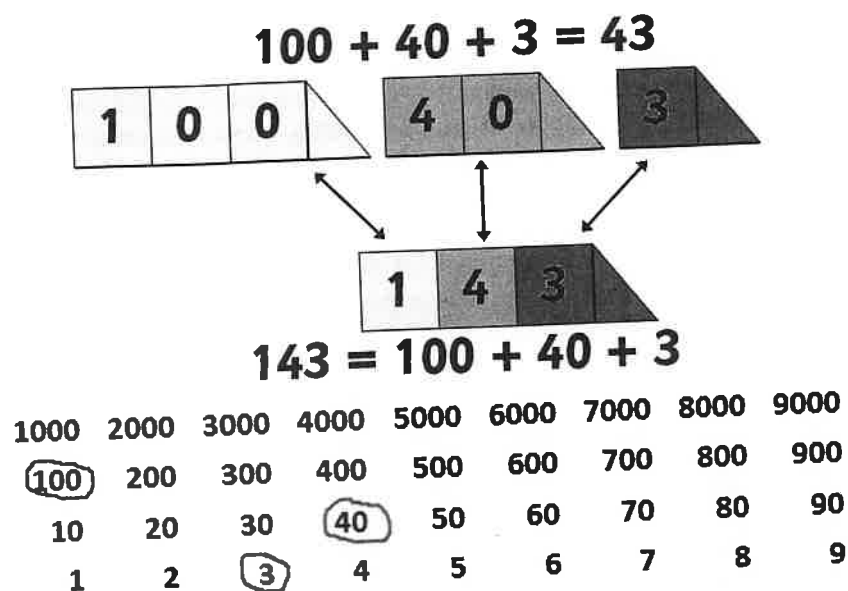


Figure 9.10 Using arrow cards and Gattegno chart to model partitioning and combining

- When do you use the 'quantity value' and 'column value' perspectives in everyday life? Which were you taught at school?
- Which of the place value resources discussed in this chapter reflect the quantity value perspective of 43? Which are better used to model the column value interpretation?

Learning to use place value

There are several key applications of place value which children encounter, namely:

- Ordering numbers
- Rounding numbers
- Supporting calculation with multi-digit numbers (discussed in Chapter 10).

Ordering numbers

Initially, memory of the counting sequence is used for ordering numbers. Later, children can be helped to perceive the patterns that exist in our number system and to then make use of these patterns when ordering numbers. The arrangement of the numbers in a hundred square makes the patterns in the written numbers up to 100 more apparent (see case study in Chapter 1, pp. 8–9). Within each row the recurring ones-digit pattern advances in 1s while the tens digit remains constant until it changes at the end of the row. Within each column the tens digit advances in 1s while the ones digit remains constant. A hundred-square jigsaw provides a good opportunity for children to use these patterns to reconstruct the hundred square.

For bigger numbers more sophisticated strategies are needed. Children need to learn that to order numbers they must compare the values of digits of equal significance (hundreds digits with hundreds digits, tens digits with tens digits, etc.) in turn starting with the most significant digits (so hundreds digits are compared before tens digits and tens digits before ones digits). For example, for the numbers 128 and 135 the hundreds digits are both worth 100 (and so are equal in value) but the tens digit 3 (worth 30) in 135 is larger than the corresponding tens digit 2 (worth 20) in 128, so 135 is larger than 128. Children need to be familiar with the order of 10s and 100s numbers to enable them to compare the values of corresponding digits.

- What developing strategies for ordering numbers are being used by each of the children in the following case study?
- Identify the nature of the children's misconceptions and uncertainties about place value which become evident in their discussion.
- How could you help the children overcome their difficulties?

CASE STUDY

Ordering numbers on a washing line

Tony, an adult, was listening to three children, Megan, Yassim, and Carlos (all age 6), who were trying to reorder some number cards pegged on a washing line (see Figure 9.11).

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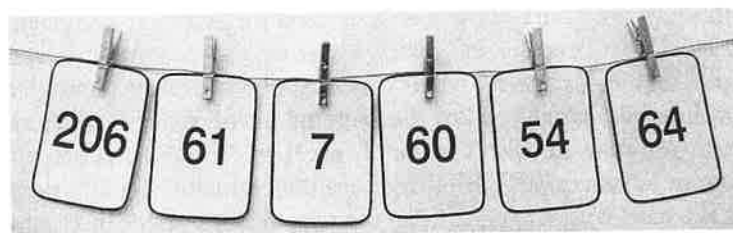


Figure 9.11 Number cards on a washing line

Carlos: This number [he picks up the 7] is bigger than that one [points to 64] 'cos it has a 7. Seven is bigger than six.

Yassim: No, it can't be because it only has one number. It's the only card with one number so it has to be the smallest [she repositions the 7 at the left end of the line].

Megan: I think the sixteen is next [she places the 61 to the right of the 7].

Yassim: Silly, it's sixty-one, not sixteen 'cos it starts with a 6.

Megan: Oh, OK. I think this one [she points at 54] must come after 7.

Carlos: And the 6 [he points to 60] comes before the 7 because 6 is less than 7.

Megan: No, it has a zero in it so it can't be just 6. It must be sixty-something.

Yassim: It's 60, like we did in counting earlier: 10, 20, 30, 40, 50, 60. So it's bigger than 54 'cos 60 comes after 50 [she moves the 60 to the right of the 54].

Carlos: What about the 26 [he picks up the 206 card]?

Megan: It's not 26. It's really big 'cos it has three numbers.

Yassim: Yes, it must be the biggest. I don't know which of these is bigger, though [she points to the 61 and 64 cards]. They both start with 6 ...

Rounding numbers

To be able to round whole numbers to the nearest ten or hundred successfully children need to know the order of the tens and hundreds numbers. Additionally, they need to learn the convention that, should the number to be rounded be equidistant from the

previous and the next tens or hundreds numbers, the number should be rounded up and not down. A number line provides a good representation to support children in determining which tens or hundreds number is nearest to the number to be rounded.

Place value errors and misconceptions

In the earlier case study the children exhibited a number of common place value misconceptions and errors:

- Reversing digits when reading or writing teen numbers (Lawton and Hansen, 2011; MaLT, 2005; Thompson, 2008a). For example, in the case study Megan knew that sixteen is written with a 1 and a 6 but was unsure about the order in which the digits are written and so mistakenly read 61 as sixteen. Children also commonly write sixteen as 61. This is a particular issue with teen numbers (and their corresponding reversals) because of the way in which we say the 'teen' numbers in English.
- Not understanding the role of 'zero as a placeholder' which maintains the values of other digits (Lawton and Hansen, 2011). In the case study, Carlos read 206 as 26 (a 20 followed by 6) and 60 as 6. Similar errors are made by children when writing numbers, for example 26 is written as 206.
- Believing that numbers with more digits are larger (MaLT, 2005). For example, Megan and Yassim believed 206 was the biggest number because it had three digits. While it is true that whole numbers with more digits are larger, it is not necessarily true for decimal numbers which the children will experience later. At this stage in development, adults can accept this as prototypical reasoning which will later be refined.
- Comparing values of digits of unequal significance when ordering numbers. In the case study, Carlos compared the values of the initial digits of the numbers 7 and 64 rather than comparing the values of digits of equal significance.

You might also have noticed in the case study that Yassim and Megan both use the word 'number' when the term 'digit' would have been more accurate (Lawton and Hansen, 2011).

Some further common place value errors include:

- Decade transition errors when counting (Thompson, 2008a), for example, '... thirty-eight, thirty-nine, thirty-ten ...'. Similar counting errors occur when reaching the next hundreds number.
- Not understanding that the position of digits determines their value, for example, 2 tens + 7 ones is interpreted as 9 items (MaLT, 2005)
- Putting more than 9 items in a particular position or 'place' within a number when representing numbers or when performing number calculations
- Recording each ten as a separate digit, for example, writing 24 as 114.