



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

Complete, fold and seal
(see note 2 below)

SURNAME: Sueic

FORENAME(S): Aleksandar

1	Student Number	
2	Title of Examination Paper	TCA 2
3	Module Code	PLFE 4000/ PCPD 4000
4	Date and Time of Examination	17 th May 2023 16.00
5	Examination Venue	Pembroke College
6	Seat Number	
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
1	22
2	6
3	8
4	16
5	12
6	11
7	14
8	8
Total	97 ✓

Second Marker
JD
18/05/23

$$(1) ay = 5e^{2x} \cos x$$

$$u = 5e^{2x}$$

$$v = \cos x$$

$$du = 10e^{2x}$$

$$dv = -\sin x$$

$$\frac{dy}{dx} = -5e^{2x} \sin x + 10e^{2x} \cos x$$

$$= 5e^{2x} (-\sin x + 2 \cos x)$$

(6) ✓

$$ii) y = \frac{2-3x}{x+1}$$

$$u = 2-3x$$

$$v = x+1$$

$$du = -3$$

$$dv = 1$$

$$\frac{dy}{dx} = \frac{-3(x+1) - 1(2-3x)}{(x+1)^2} = \frac{-3x-3-2+3x}{(x+1)^2}$$

$$= \frac{-5}{(x+1)^2}$$

(6) ✓

$$c) y = 5 \sin(x^3)$$

$$u = x^3$$

$$du = 3x^2$$

$$\frac{dy}{du} = 5 \cos(u) \quad \checkmark \quad \frac{dy}{dx} = 5 \cos(u) \times 3x^2$$

$$= 15x^2 \cos(x^3) \quad \checkmark$$

(6) ✓

$$d) y = \frac{2}{x^3} + \sqrt{x} - x$$

$$= 2x^{-3} + x^{1/2} - x \quad \checkmark$$

$$\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-1/2} - 1 \quad \checkmark$$

$$= \frac{-6}{x^4} + \frac{1}{2\sqrt{x}} - 1 \quad \checkmark$$

(4) ✓

$$e) v = 12(1 - 7e^{-3t})$$

$$= 12 - 84e^{-3t} \quad \checkmark$$

$$\frac{dv}{dt} = -3 \times -84e^{-3t} = 252e^{-3t} \quad \checkmark$$

$$\text{when } t = 0 \quad \frac{dv}{dt} = 252 v/s \quad \checkmark$$

(6) ✓

③ $y = 2x^3 - 15x^2 - 84x + 16$ $\frac{dy}{dx} = 6x^2 - 30x - 84 \hat{=} 6$

$y = 2(-2)^3 - 15(-2)^2 - 84(-2) + 16 = 108$ $= x^2 - 5x - 14$
 $y = 2(7)^3 - 15(7)^2 - 84(7) + 16 = -621$ $(x+2)(x-7)$
 $x = -2, x = 7$

a) $(-2, 108), (7, -621)$

b) $\frac{d^2y}{dx^2} = 12x - 5$

at $(-2, 108)$ $\frac{d^2y}{dx^2} = -9 < 0 \therefore \text{maximum}$

at $(7, -621)$ $\frac{d^2y}{dx^2} = 9 > 0 \therefore \text{minimum}$

④ a) $\int_1^4 (1 + 3x + 5x^2) dx = \left[x + \frac{3x^2}{2} + \frac{5x^3}{3} \right]_1^4$

$= \left[4 + \frac{3(4)^2}{2} + \frac{5(4)^3}{3} \right] - \left[1 + \frac{3(1)^2}{2} + \frac{5(1)^3}{3} \right]$

$= \frac{404}{3} - \frac{15}{6} = \frac{261}{2} = 130.5$

b) $\int_0^{\pi/2} (5\cos(\frac{x}{3}) - 2\sin(x)) dx = \left[\frac{5}{1/3} \sin(\frac{x}{3}) + 2\cos(x) \right]_0^{\pi/2}$

$= \left[\frac{5}{1/3} \sin\left(\frac{\pi/2}{3}\right) + 2\cos\left(\frac{\pi}{2}\right) \right] - \left[\frac{5}{1/3} \sin\left(\frac{0}{3}\right) + 2\cos\left(\frac{0}{3}\right) \right]$

$= 15 \cdot \frac{\sqrt{3}}{2} - 2 = \frac{11}{2} = 5.5$

⑧

~~$$\frac{19x-5}{2x(x-1)} = \frac{A}{2x} + \frac{B}{x-1} = \frac{A(x-1) + B(2x)}{2x(x-1)}$$~~

~~$$\therefore 19x-5 = A(x-1) + B(2x)$$~~

~~$$\begin{array}{lcl} x=-1 & -19-5=0 & -2B \\ x=0 & -5=A & +0 \end{array} \quad \therefore B=12 \\ \therefore A=-5$$~~

~~$$\int_3^5 \frac{-5}{2x} + \frac{12}{x-1} = \left[-\frac{5}{2} \ln(2x) + 12 \ln(x-1) \right]_3^5$$~~

~~$$= \left[-\frac{5}{2} \ln(2 \times 5) + 12 \ln(5-1) \right] - \left[-\frac{5}{2} \ln(2 \times 3) + 12 \ln(3-1) \right]$$~~

~~$$= 10.87107 - 3.83837 = 7.0407$$~~

$$\textcircled{5} \quad \frac{19x-5}{2x(x-1)} = \frac{A}{2x} + \frac{B}{x-1} = \frac{A(x-1) + B(2x)}{2x(x-1)}$$

$$19x-5 = A(x-1) + B(2x) \quad \checkmark$$

$$\begin{array}{lcl} x=0 & -5 = -A + 0 & \therefore A=5 \\ x=1 & 19-5 = +2B & \therefore B=7 \end{array} \quad \checkmark$$

$$\int_3^5 \frac{5}{2x} + \frac{7}{x-1} = \left[\frac{5}{2} \ln(2x) + 7 \ln(x-1) \right]_3^5 \quad \checkmark$$

$$= \left[\frac{5}{2} \ln(2 \times 5) + 7 \ln(5-1) \right] - \left[\frac{5}{2} \ln(2 \times 3) + 7 \ln(3-1) \right] \quad \checkmark$$

$$= 15.46052 - 9.3314 = 6.1288 \quad \times$$

12

⑩ $\int_3^5 \frac{19x-5}{2x(x-1)} dx \quad h = \frac{5-3}{4} = 0.5$

$\frac{19x-5}{2x(x-1)}$	3	3.5	4	4.5	5
	y_0	y_1	y_2	y_3	y_n
	$13/3$	$123/35$	$71/24$	$23/9$	$9/4$

Simpson's Rule = $\frac{1}{3} \times \frac{1}{2} \left[\left(\frac{13}{3} + \frac{9}{4} \right) + 4 \left(\frac{123}{35} + \frac{23}{9} \right) + 2 \left(\frac{71}{24} \right) \right]$

= 6.12989 ✓

% error = $\frac{6.1288 - 6.12989}{6.1288} \times 100 = -0.0167\%$ error ✗

Correct Method

⑪ ✓

⑦

t	0 ≤ t < 2	2 ≤ t < 4	4 ≤ t < 6	6 ≤ t < 8	8 ≤ t < 10	Σ
f	1	16	26	5	2	50
f_x	1	3	5	7	9	25
f_x^2	1	48	130	35	18	232
$f(x - \bar{x})^2$	$\frac{8281}{625}$	$\frac{26896}{625}$	$\frac{2106}{625}$	$\frac{3481}{125}$	$\frac{123762}{625}$	$\frac{3138}{25}$

 $\bar{x} = 125.52$

$$\bar{x} = \frac{232}{50} = 4.64 \quad \sigma = \sqrt{\frac{125.52}{49}} = 1.60 \quad (22.1)$$

$$\textcircled{8} \text{ i) } P(\text{connection}) = P(A) \times P(B) = 0.97 \times 0.89 = 0.8633$$

$$\text{ii) } P(\overline{\text{connection}}) = P(A) \times P(\bar{B}) = (1 - 0.97) \times (1 - 0.89) = 33/10,000$$

$$P(\text{connection}) = 1 - P(\overline{\text{connection}}) = 1 - \frac{33}{10,000} = 0.9967$$

$$a) y = 5e^{2x} \cos x \quad \begin{matrix} u = 5e^{2x} & v = \cos x \\ du = 10e^{2x} & dv = -\sin x \end{matrix}$$

$$-5e^{2x} \sin x + 10e^{2x} \cos x$$

$$= 5e^{2x} (-\sin x + 2 \cos x)$$

$$y = \frac{2-3x}{x+1} \quad \begin{matrix} u = 2-3x & v = x+1 \\ du = -3 & dv = 1 \end{matrix}$$

$$\frac{-3(x+1) - 1(2-3x)}{(x+1)^2} = \frac{-3x-3-2+3x}{(x+1)^2} = \frac{-5}{(x+1)^2} = -3-2+3x$$

$$c) y = 5 \sin(x^3) \quad \begin{matrix} u = x^3 & du = 3x^2 \\ dv = \cos u & v = \sin u \end{matrix}$$

$$= 5 \cos u \times 3 = 15x^2 \cos$$

$$d) \frac{2}{x^3} + \sqrt{x} \quad \begin{matrix} u = 2x^{-3} + x^{1/2} - x \\ du = -6x^{-4} + \frac{1}{2}x^{-1/2} - 1 \end{matrix}$$

$$\frac{d}{dx} = 6x^2 - 30x - 84$$

$$\begin{matrix} x^2 & -5x & +14 \\ (x-7)(x+2) \end{matrix}$$

$$x^2 + 2x - 7x - 14$$

$$y = (7, -121) \quad x = -2 \quad (-2, 108)$$

$$\frac{d^2 y}{dx^2} = 12x - 30 = 54$$

$$\int_1^4 (1+3x+5x^2) dx = x + \frac{3x^2}{2} + \frac{5x^3}{3}$$

~~$$\frac{404 - 15}{3} = \frac{46}{2}$$~~

~~$$\int_0^{5\pi/2} (5 \cos x - 2 \sin x) dx = \left[\frac{5x}{1} \sin\left(\frac{x}{3}\right) + 2 \cos(x) \right]$$~~

⑦

F	0	2	2	4	4	6	6	8	10	
x	1	1	3	4	5	7	9	18	23	2
F _{9x}	13.2496	143.08	336	333.696	27.848	38.092	125.52			

4.64

