

Trigonometry

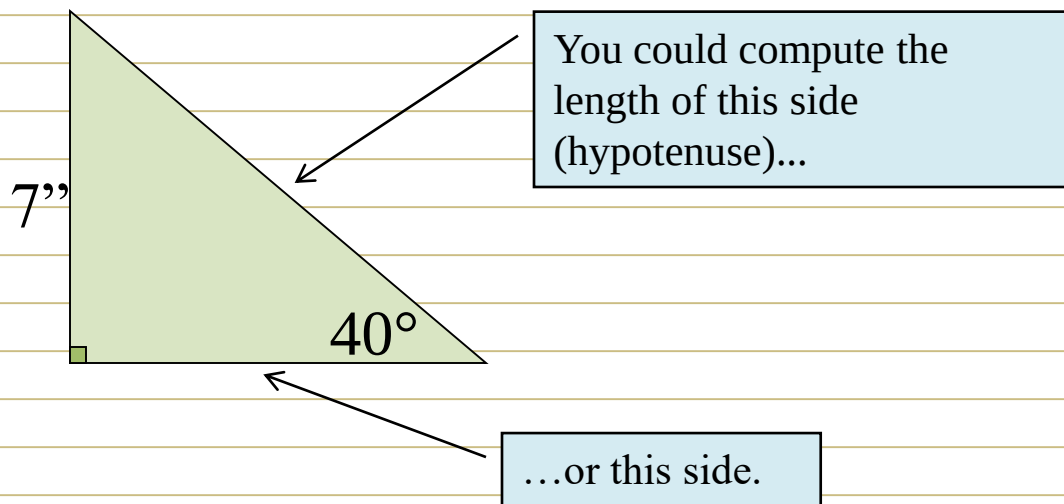
- (i) Basic Calculations of Angles and Sides of Right Triangles
- (ii) The sine law
- (iii) The cosine law

Introduction

- You can use the three trig functions (*sin*, *cos*, and *tan*) to solve problems involving right triangles.
- You will be able to use law of sine and cosine to solve a triangle is to find the lengths of each of its sides and all its angles.

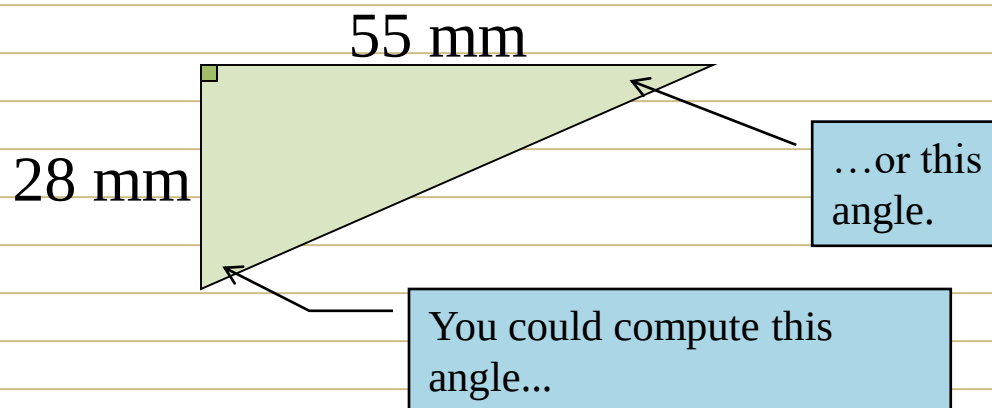
Introduction

- If you have a right triangle, and you know an acute angle and the length of one side, you have enough info to compute the length of either remaining side.



Introduction

- If you have a right triangle, and you know the lengths of two sides, you have enough info to compute the size of either acute interior angle.



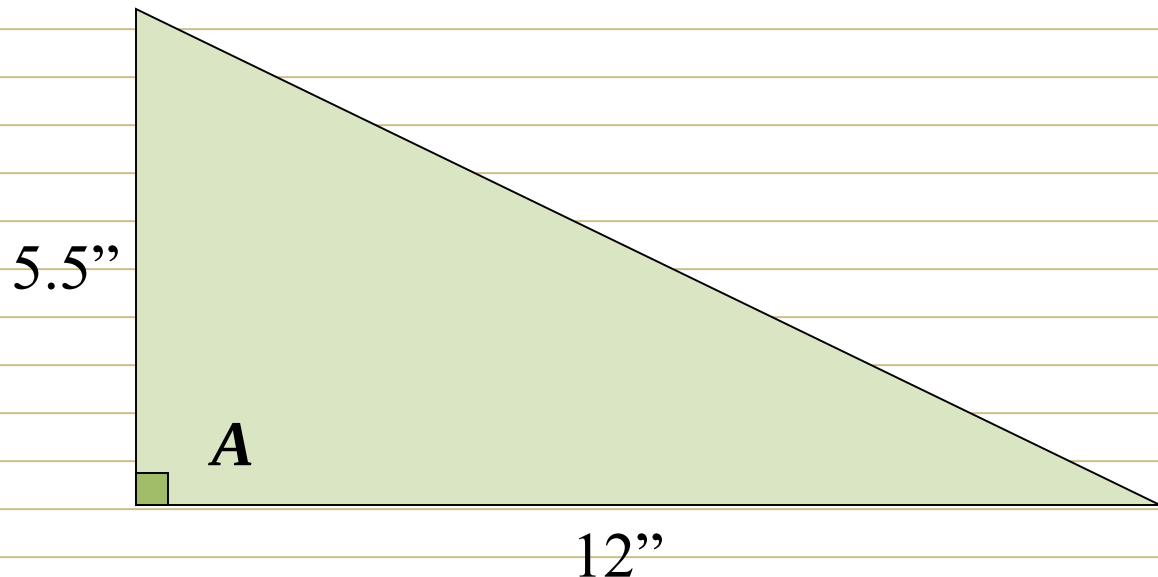
Part I

Use trigonometry to determine the size of an angle.

Determine an unknown angle

Example 1

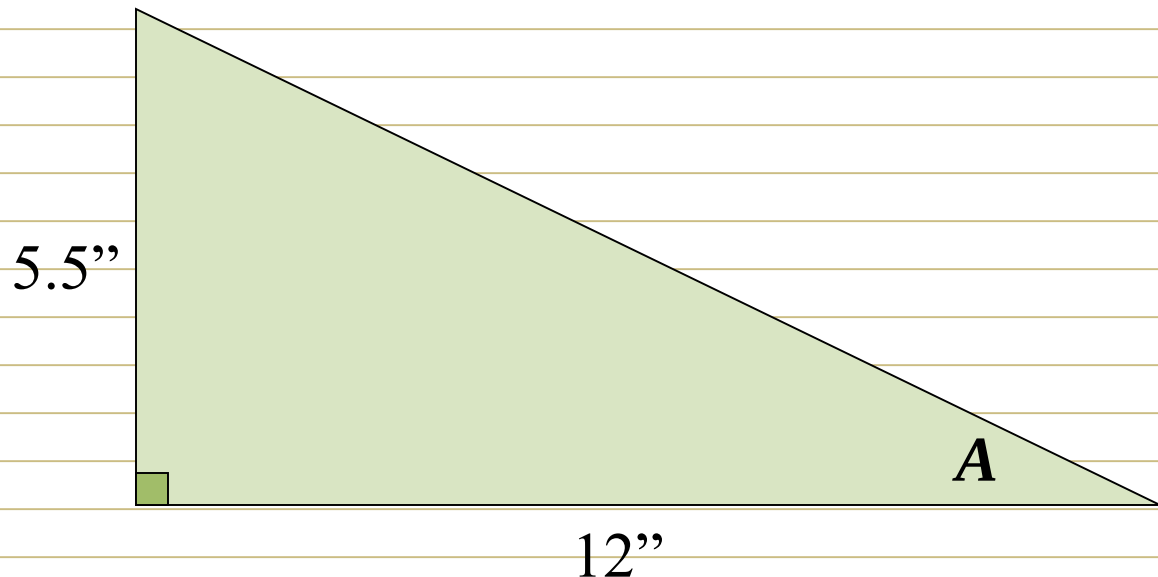
- To start, we will determine the size of an unknown angle when two sides of the right triangle are known.



Determine an unknown angle

Example 1

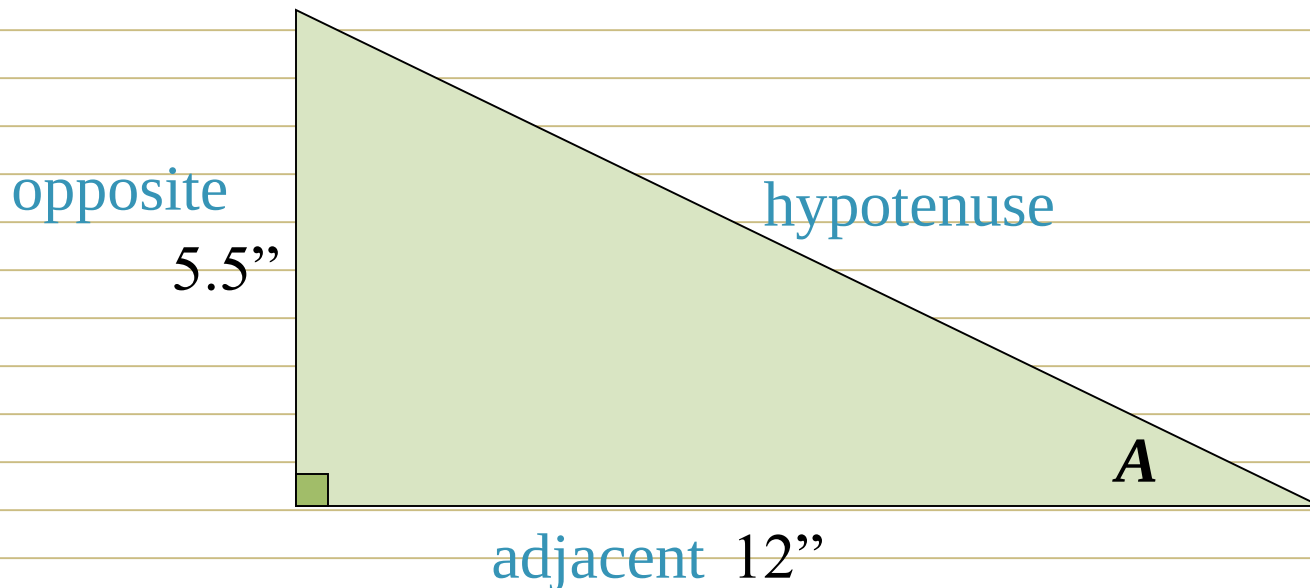
- Let the unknown angle **A** be the reference angle.



Determine an unknown angle

Example 1

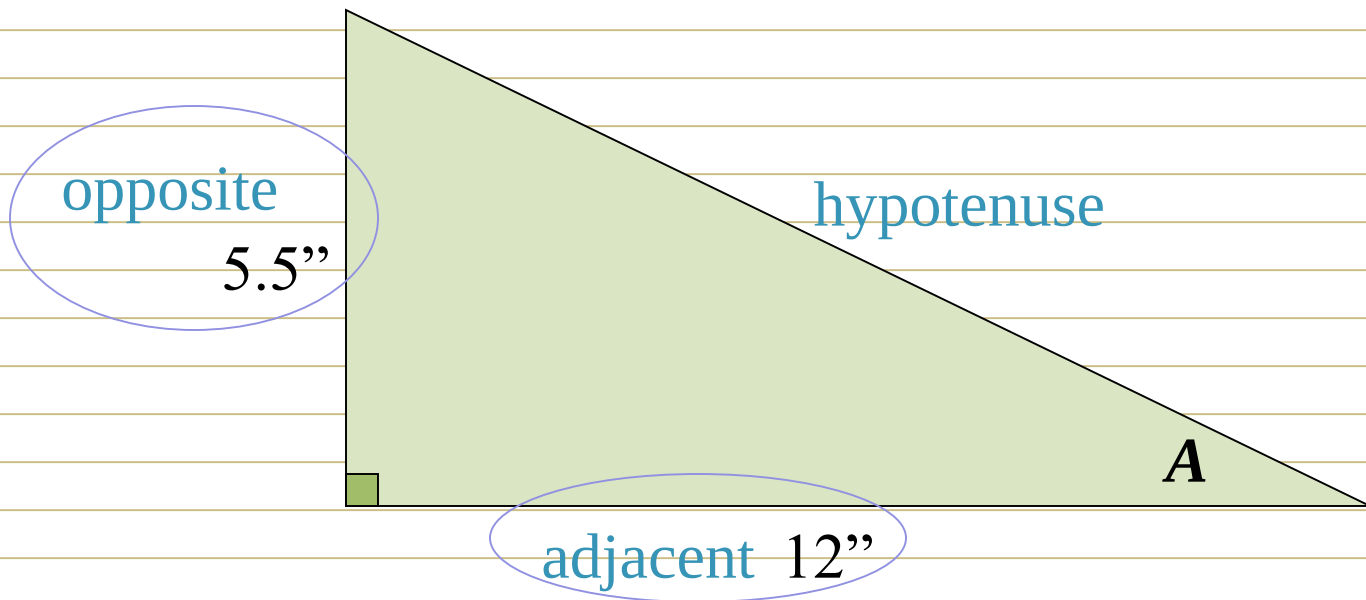
- Now label the sides of the right triangle...



Determine an unknown angle

Example 1

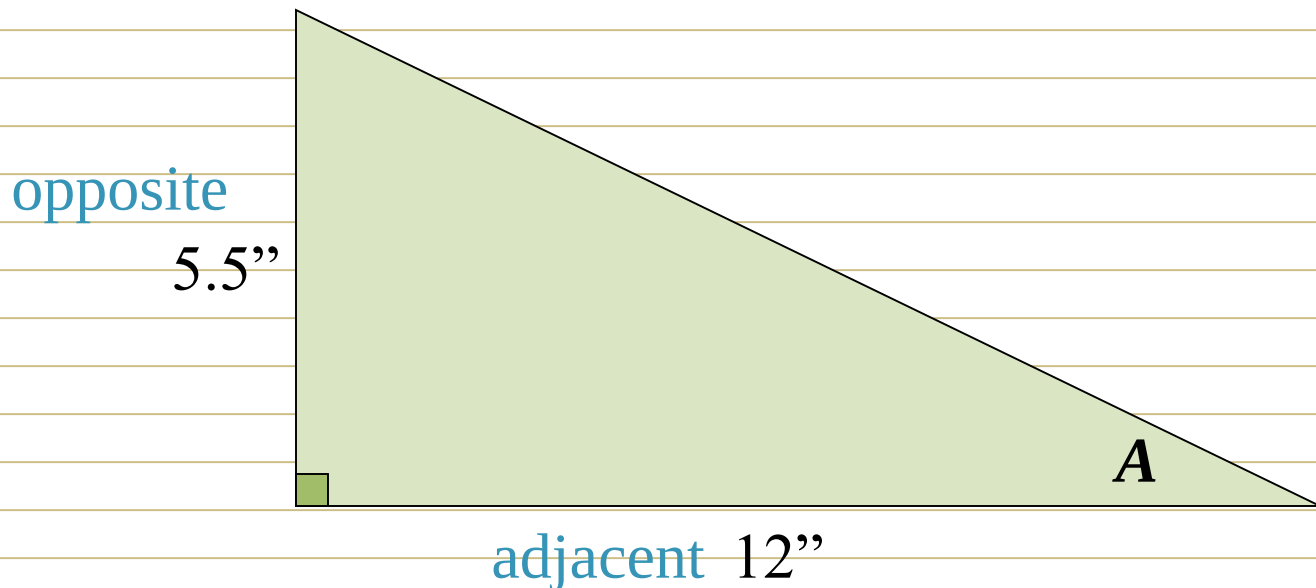
- Note that we only know the lengths of the *opposite* and *adjacent* sides.



Determine an unknown angle

Example 1

- So we need to pick a trig function that has the opposite and adjacent sides in it...



Determine an unknown angle

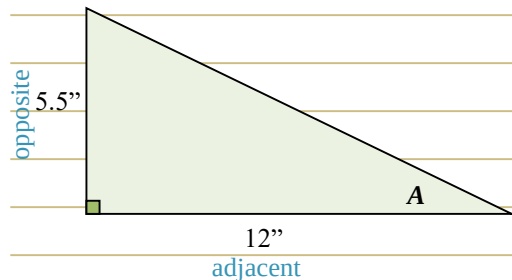
Example 1

- Which trig function should you pick?

$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$



You need to pick the tangent function since it is the only one that has both *opposite* and *adjacent* sides in it.

Determine an unknown angle

Example 1

- Now plug-in the numbers you have into the tangent function...

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

$$\tan A = \frac{5.5}{12}$$

$$\tan A = 0.458333...$$

opposite

5.5"

$$A = 24.6^\circ$$

A

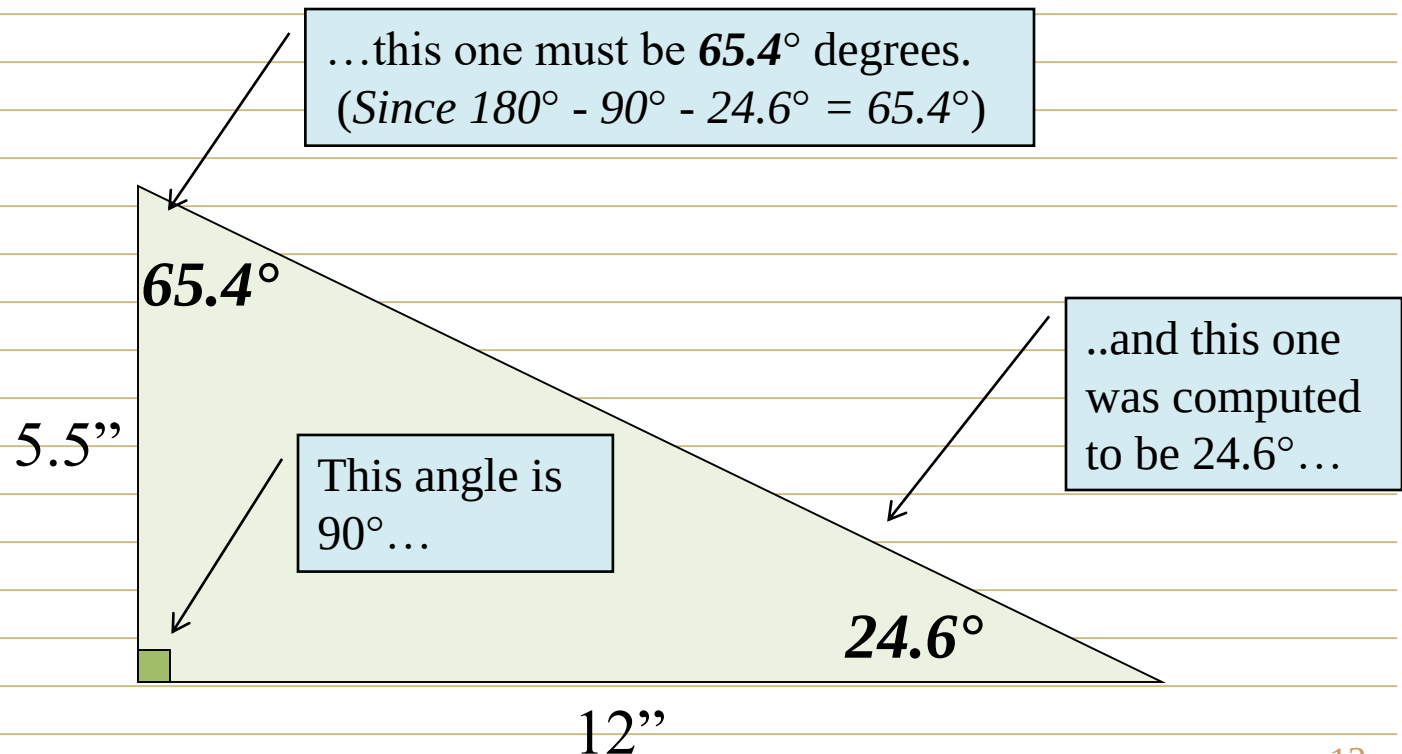
adjacent 12"

Now use your calculator to solve. Type-in .458333, press the 2nd function key, then press the *tan* key

Determine an unknown angle

Example 1

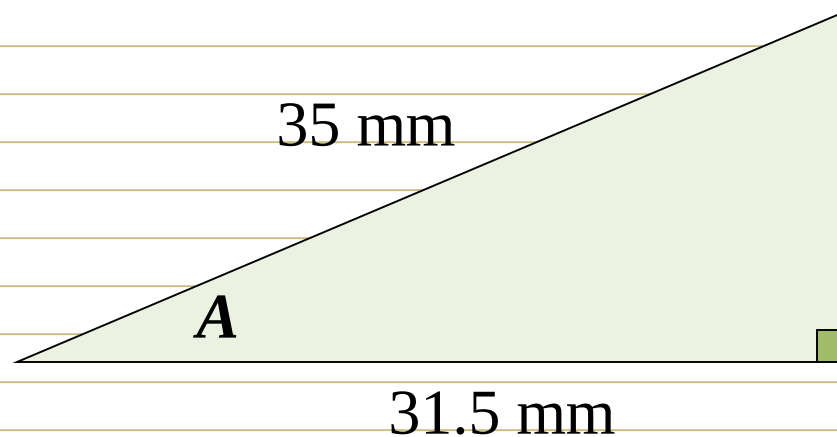
- How could you determine the size of the remaining angle?



Determine an unknown angle

Example 2

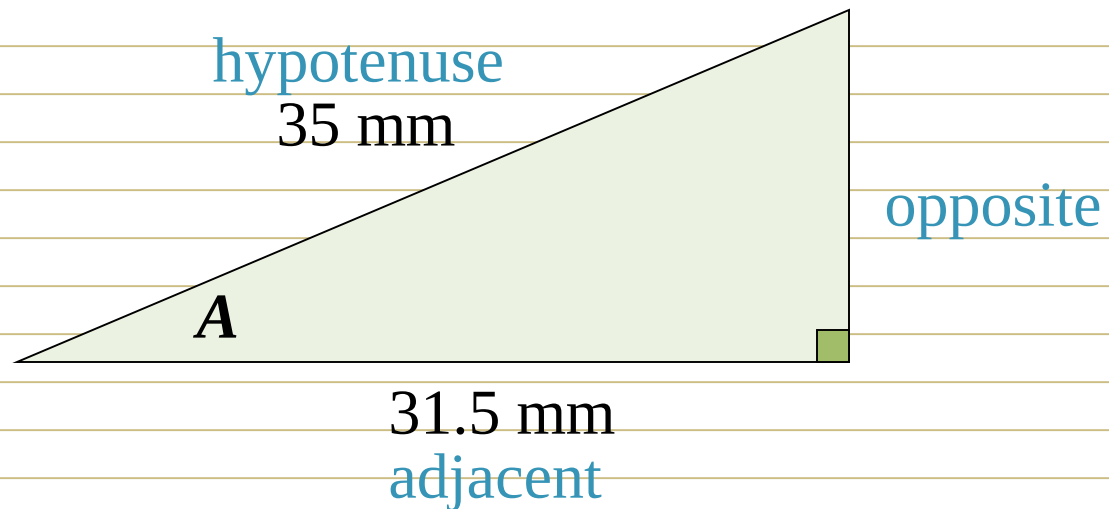
- Let's try another one...
- Determine the size of angle A.



Determine an unknown angle

Example 2

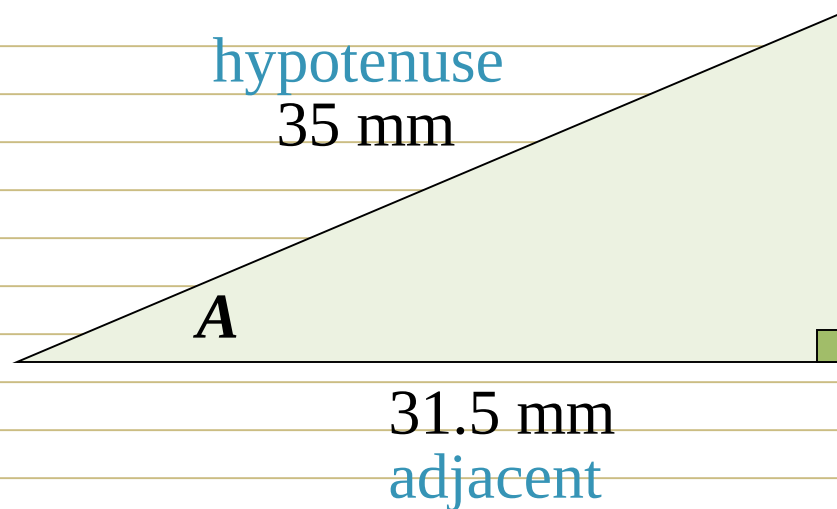
- First, label the sides of the triangle...



Determine an unknown angle

Example 2

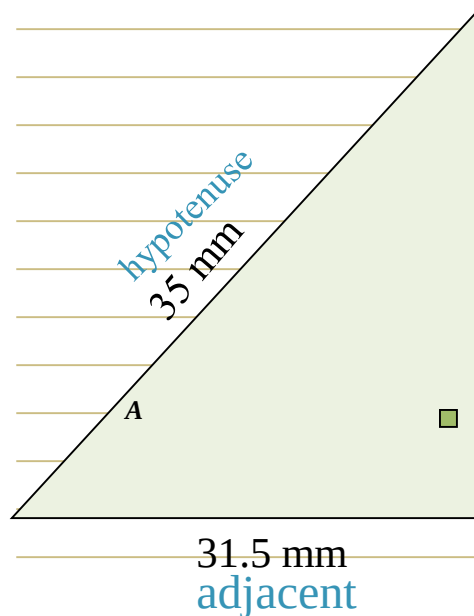
- Since you know the lengths of the adjacent side and the hypotenuse, pick a trig function that has both of these...



Determine an unknown angle

Example 2

- Which trig function should you pick?



$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$

You need to pick the cosine function since it is the only one that has both the *adjacent* side and *hypotenuse* in it.

Determine an unknown angle

Example 2

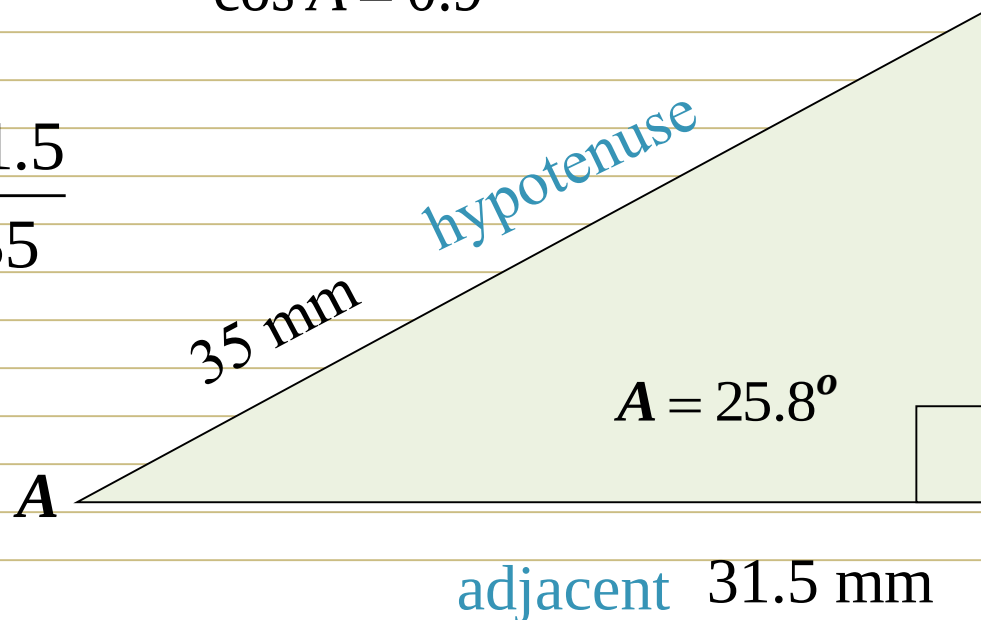
- Now plug-in the numbers you have into the cosine function...

$$\cos A = \frac{adj}{hyp}$$

$$\cos A = 0.9$$

Now use your calculator to solve. Type-in 0.9, press the 2nd function key, then press the cos key

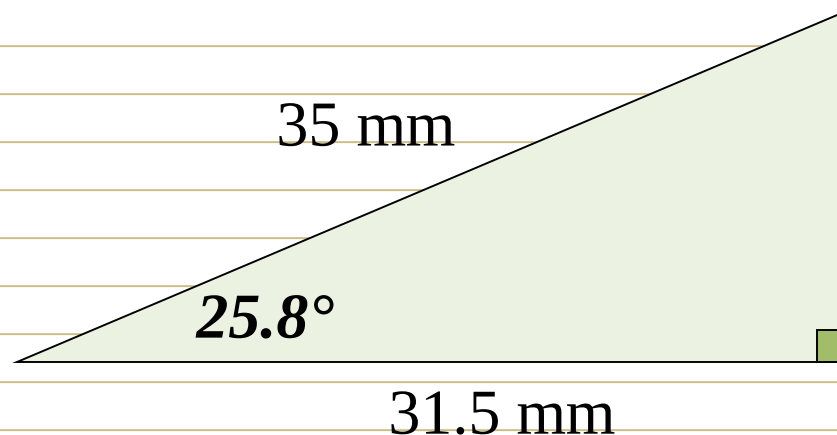
$$\cos A = \frac{31.5}{35}$$



Determine an unknown angle

Example 2

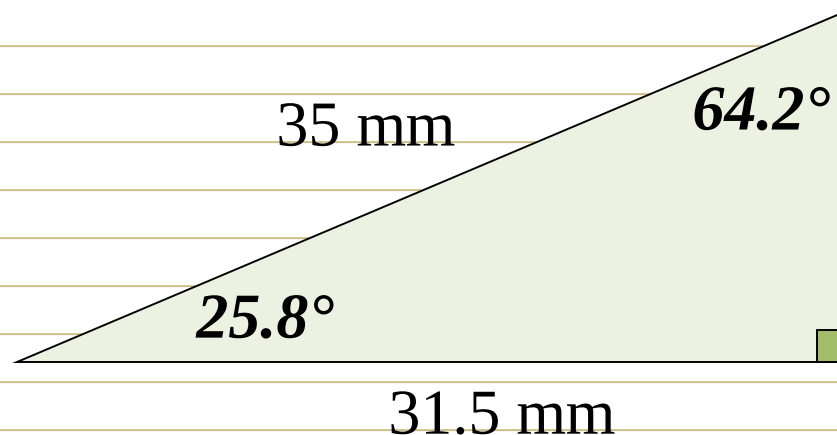
- Now that you know how big angle A is, determine the size of the remaining angle.



Determine an unknown angle

Example 2

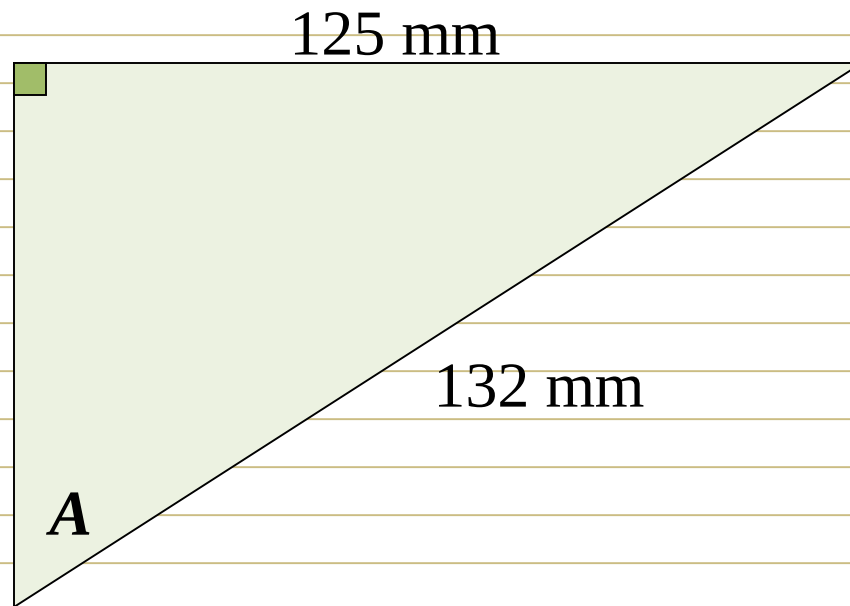
- To determine the other angle:
- $180^\circ - 90^\circ - 25.8^\circ = 64.2^\circ$



Determine an unknown angle

Example 3

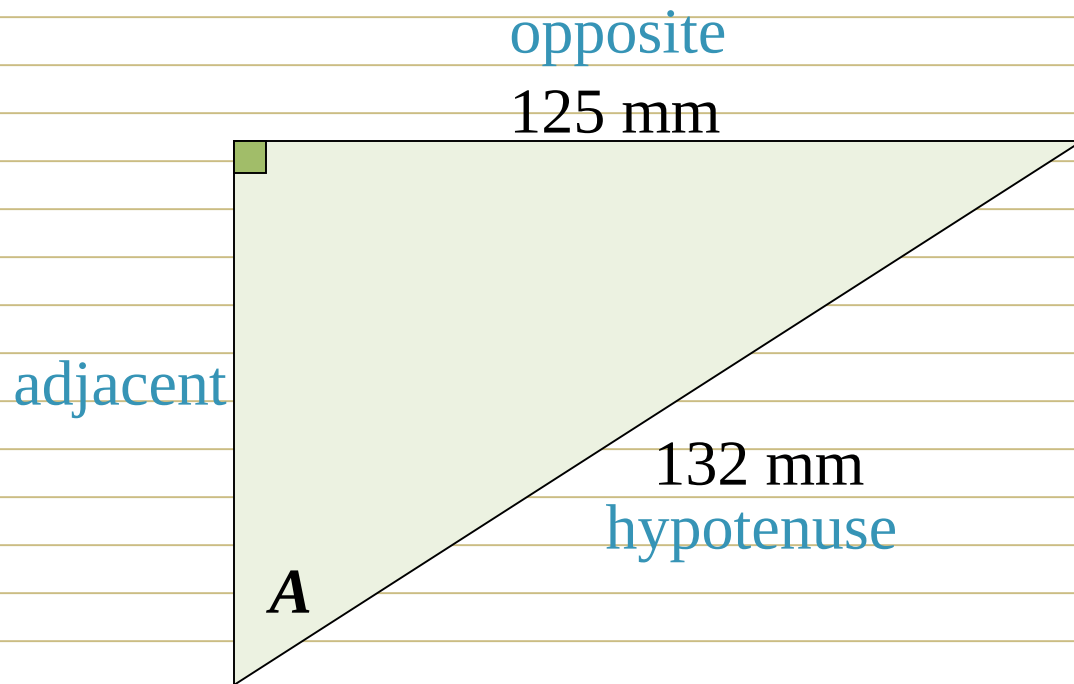
- Let's try one more.
- Determine the size of angle A .



Determine an unknown angle

Example 3

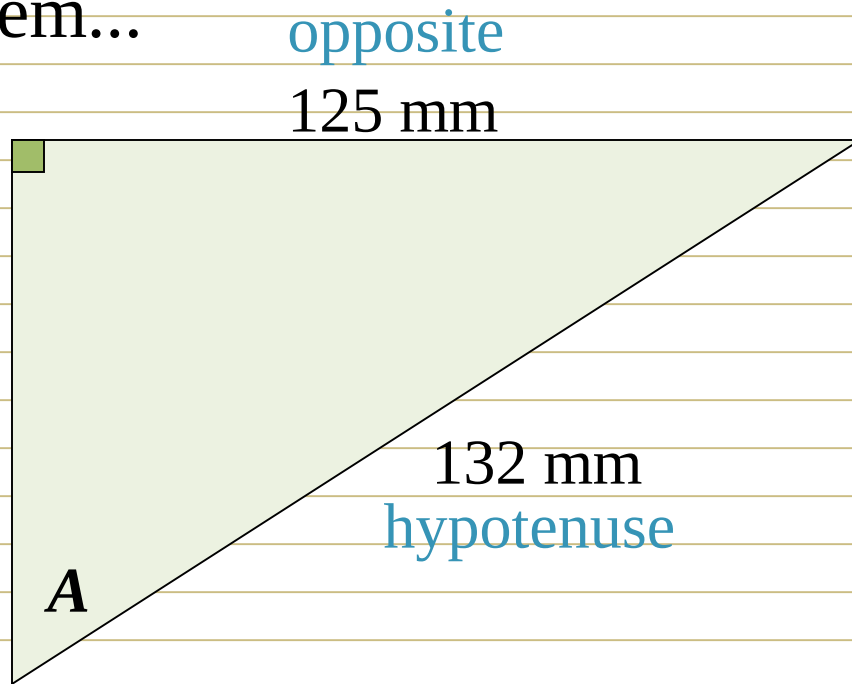
- Label the sides of the triangle...



Determine an unknown angle

Example 3

- Since you know the lengths of the opposite side and the hypotenuse, pick a trig function that contains them...



Determine an unknown angle

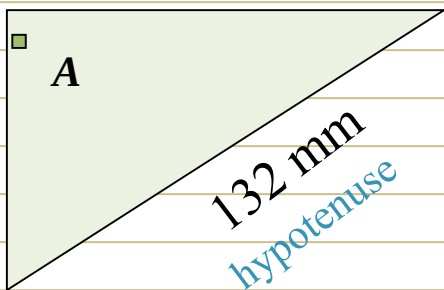
Example 3

- Which trig function should you pick?

$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$



You need to pick the sine function since it is the only one that has both the *opposite* side and *hypotenuse* in it.

Determine an unknown angle

Example 3

- Now plug-in the numbers you have into the sine function...

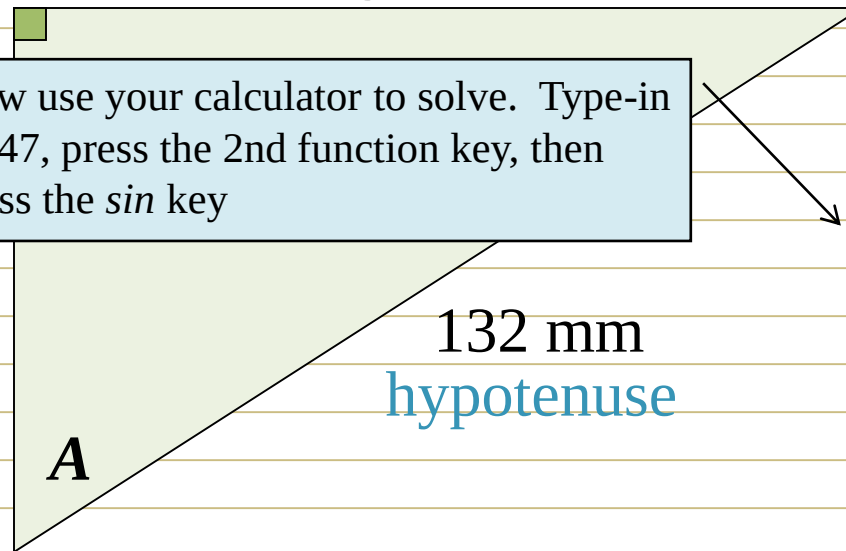
$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin A = \frac{125}{132}$$

Now use your calculator to solve. Type-in 0.947, press the 2nd function key, then press the *sin* key

$$\sin A = 0.947$$

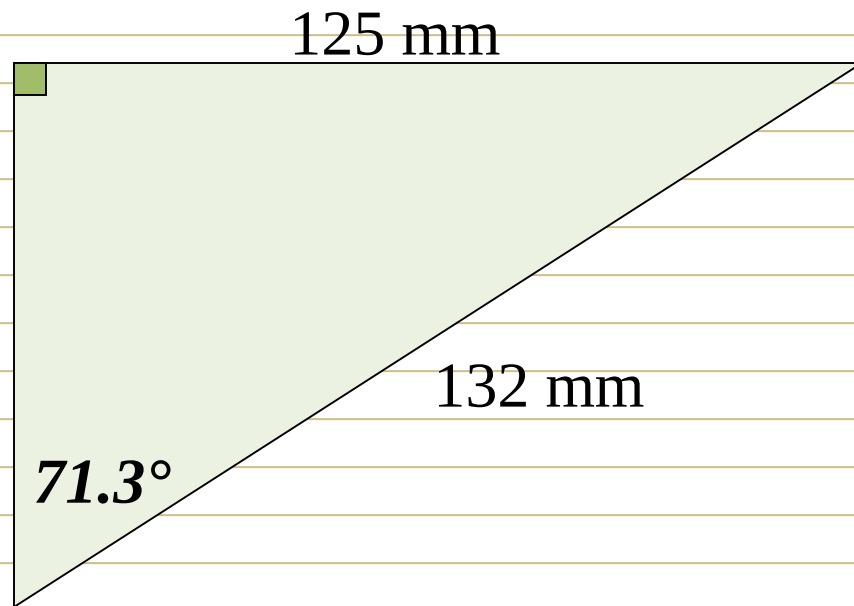
$$A = 71.3^\circ$$



Determine an unknown angle

Example 3

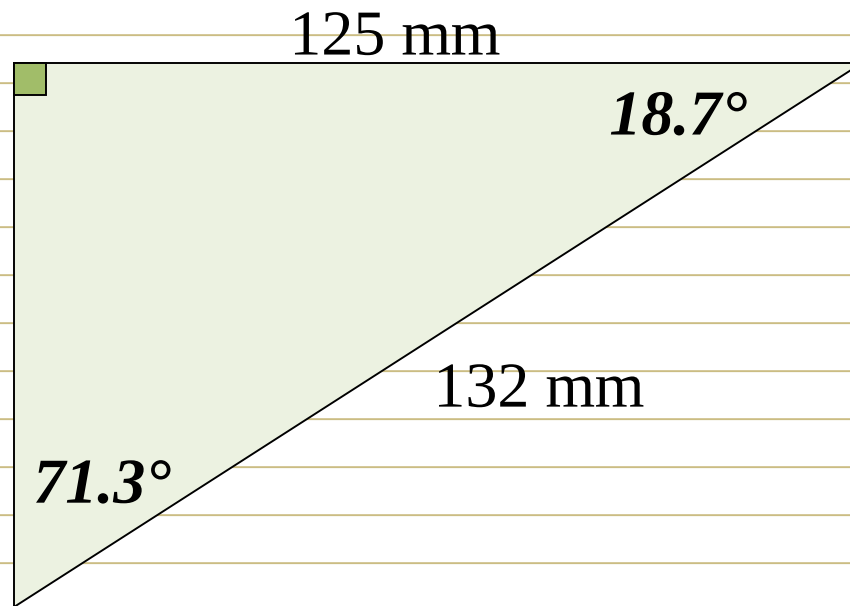
- What is the size of the remaining angle?



Determine an unknown angle

Example 3

- The angle is computed to be 18.7° .



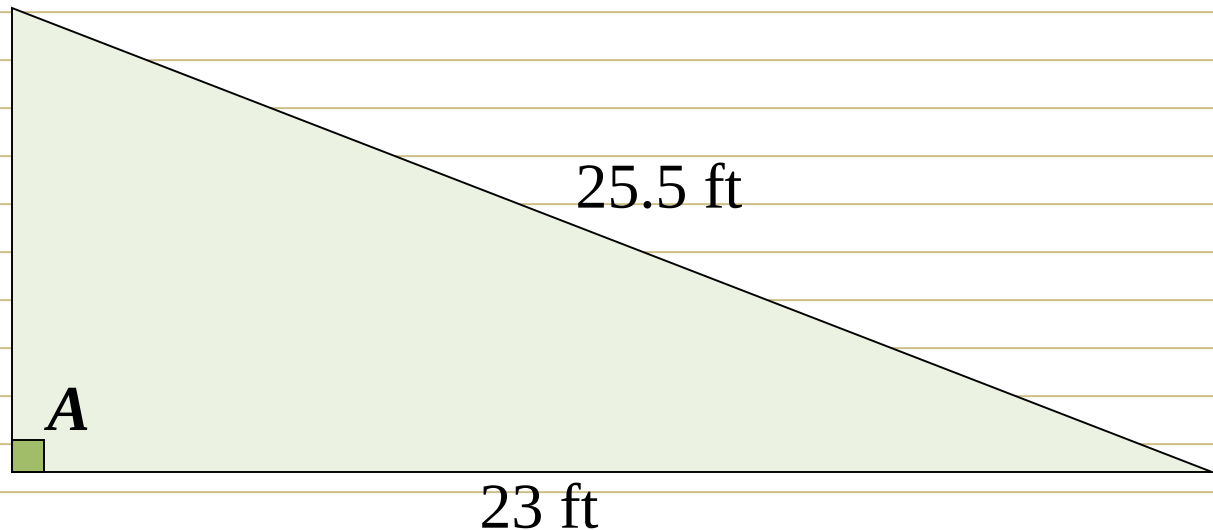
Summary of Part I

- By now you should feel like you have a pretty good chance of determining the size of an angle when any two sides of a right triangle are known.
- Click to see one more problem like the last three you have done...

Summary of Part I

Example 4

- Determine the size of angle A .
- Solve the problem, then click to see the answer.



Summary of Part I

Example 4

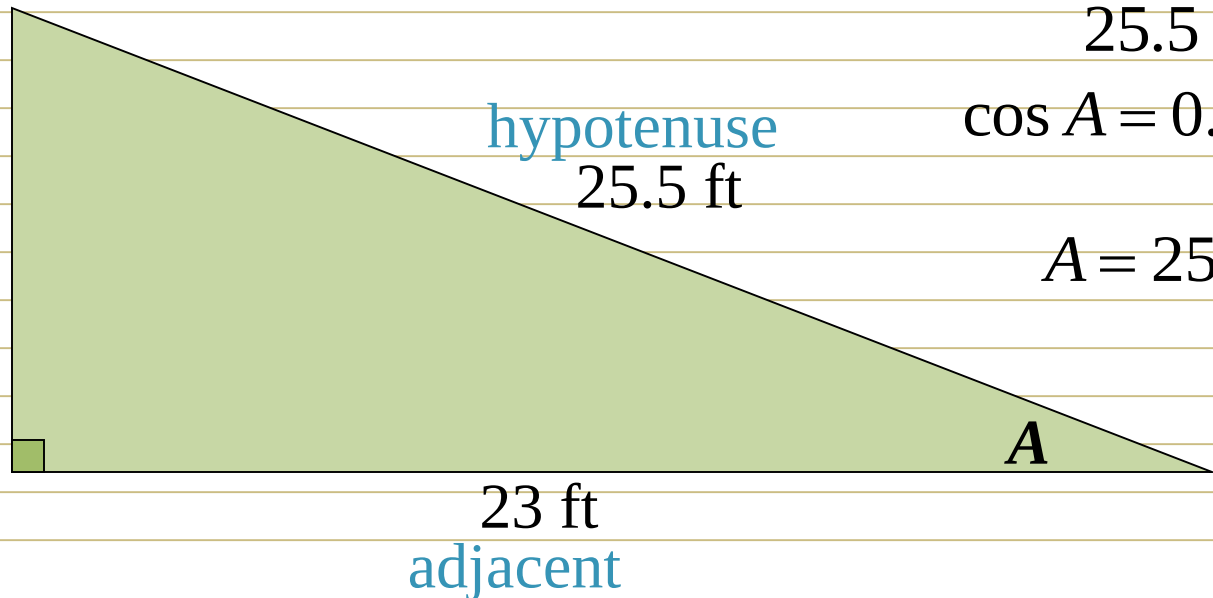
- Selecting the $\cos$ function will allow you to determine the size of angle A .

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos A = \frac{23}{25.5}$$

$$\cos A = 0.902$$

$$A = 25.6^\circ$$

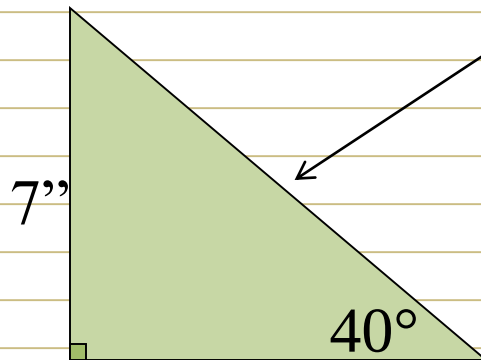


Part II

Use trigonometry to determine the length of a side of a right triangle.

Determining the length of a side

- Recall that if you have a right triangle, and you know an acute angle and the length of one side, you have enough info to compute the length of either remaining side.



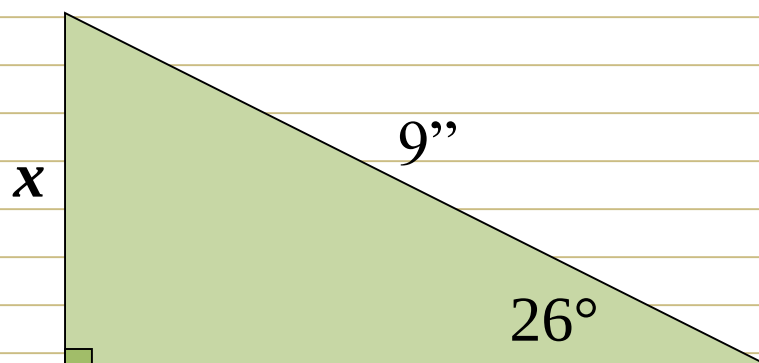
You could compute the length of this side (hypotenuse)...

...or this side.

Determining the length of a side

Example 5

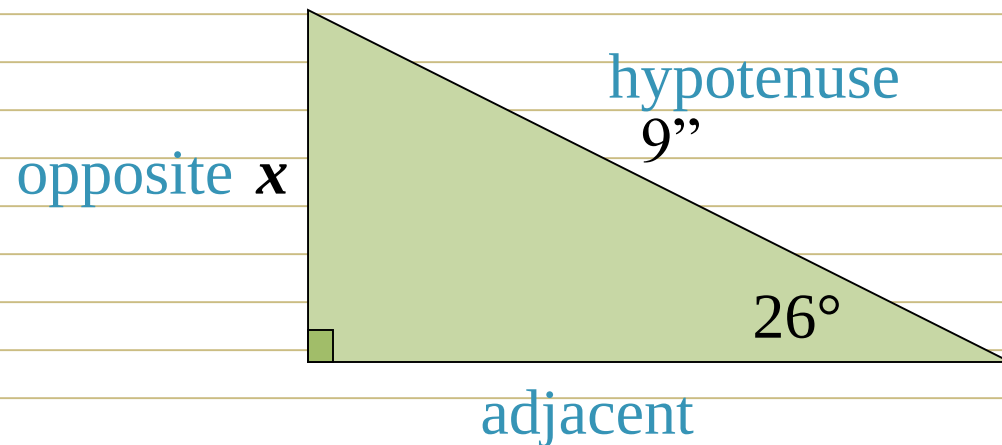
- In this problem, we will determine the length of side x .



Determining the length of a side

Example 5

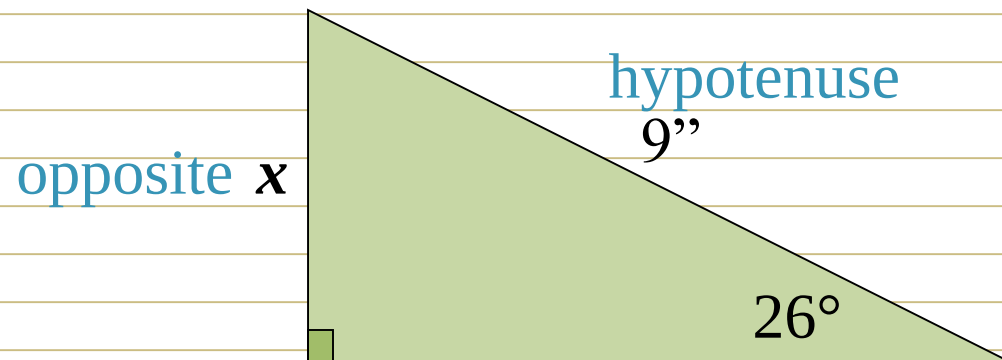
- As always, first label the sides of the triangle...



Determining the length of a side

Example 5

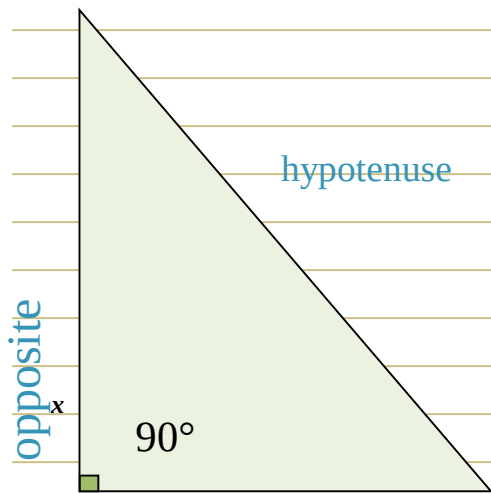
- Since you know the length of the hypotenuse and want to know the length of the opposite side, you should pick a trig function that contains both of them...



Determining the length of a side

Example 5

- Which trig function should you pick?



$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$

You need to pick the sine function since it is the only one that has both the *opposite* side and *hypotenuse* in it.

Determining the length of a side

Example 5

- Now set-up the trig function:

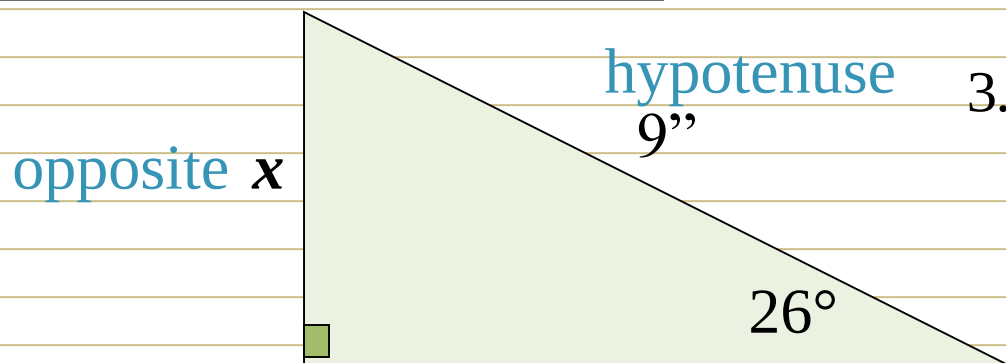
$$\sin A = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 26^\circ = \frac{x}{9}$$

Use basic algebra to solve this equation.
Multiply both sides of the equation by 9 to clear the fraction.

$$9(0.438) = x$$

$$9(\sin 26^\circ) = \frac{x}{9}(9)$$

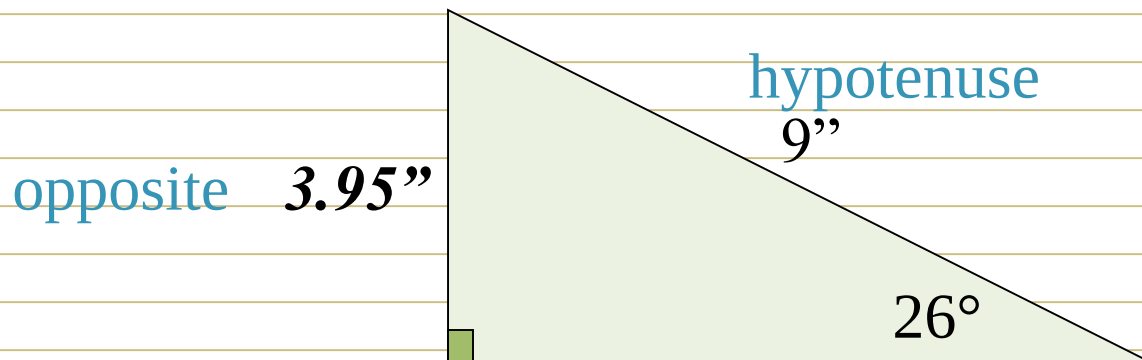
$$3.95'' = x$$



Determining the length of a side

Example 5

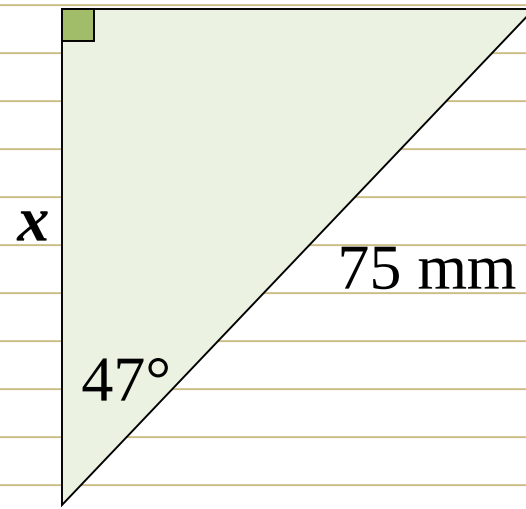
- Now you know the opposite side has a length of 3.95”.



Determining the length of a side

Example 6

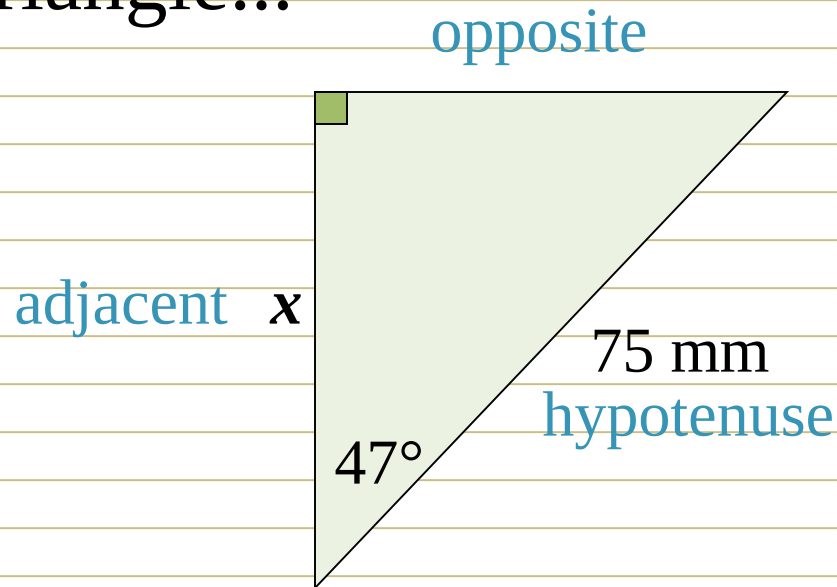
- Let's try another one.
- Determine the length of side x .



Determining the length of a side

Example 6

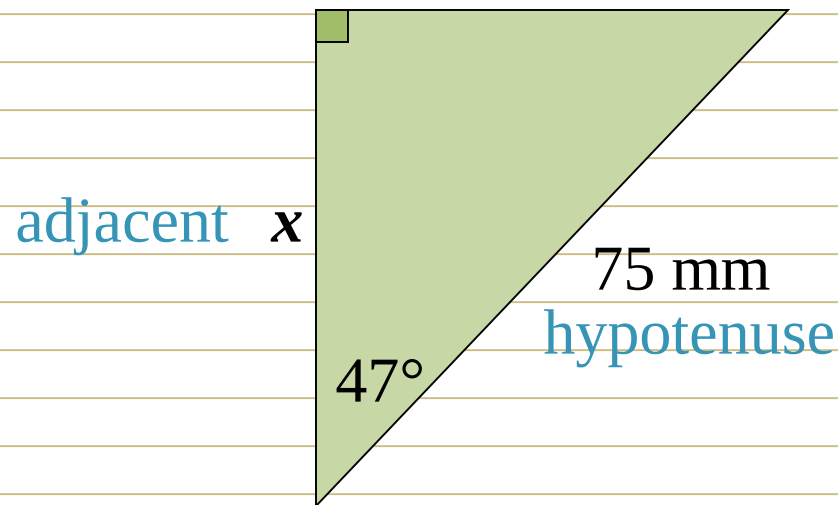
- Since the known angle (47°) will serve as your reference angle, you can label the sides of the triangle...



Determining the length of a side

Example 6

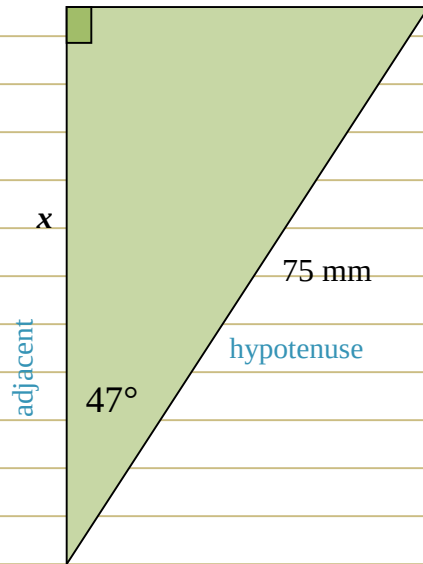
- You know the length of the hypotenuse and want to know the length of the adjacent side, so pick a trig function which contains both of them...



Determining the length of a side

Example 6

- Which trig function should you pick?



$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$

You need to pick the cosine function since it is the only one that has both the *adjacent* side and *hypotenuse* in it.

Determining the length of a side

Example 6

- Set-up your trig function...

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

To finish, evaluate $\cos 47^\circ$ (which is 0.682) and multiply by 75.

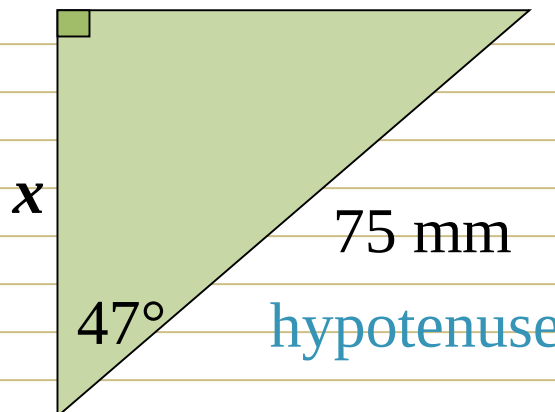
$$\cos 47^\circ = \frac{x}{75}$$

Use basic algebra to solve this equation. Multiply both sides of the equation by 75 to clear the fraction.

$$75(0.682) = x$$

$$75(\cos 47^\circ) = \frac{x}{75}(75)$$

adjacent

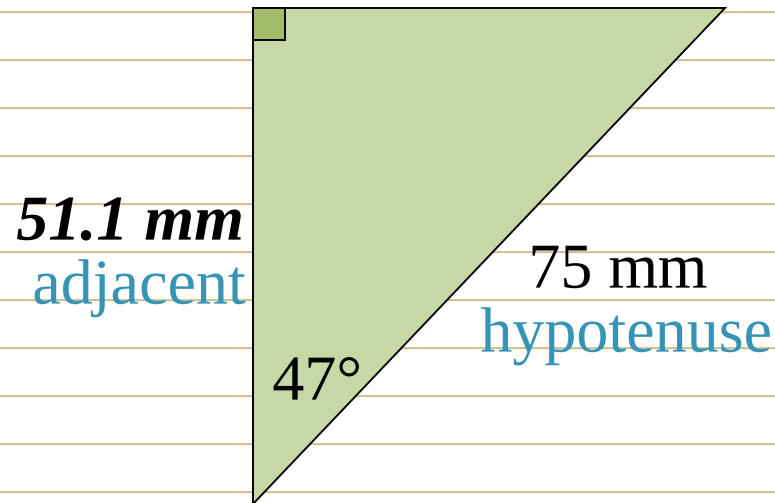


$$51.1 \text{ mm} = x$$

Determining the length of a side

Example 6

- Now you know the length of the adjacent side is 51.1 mm.



Determining the length of a side

Example 7

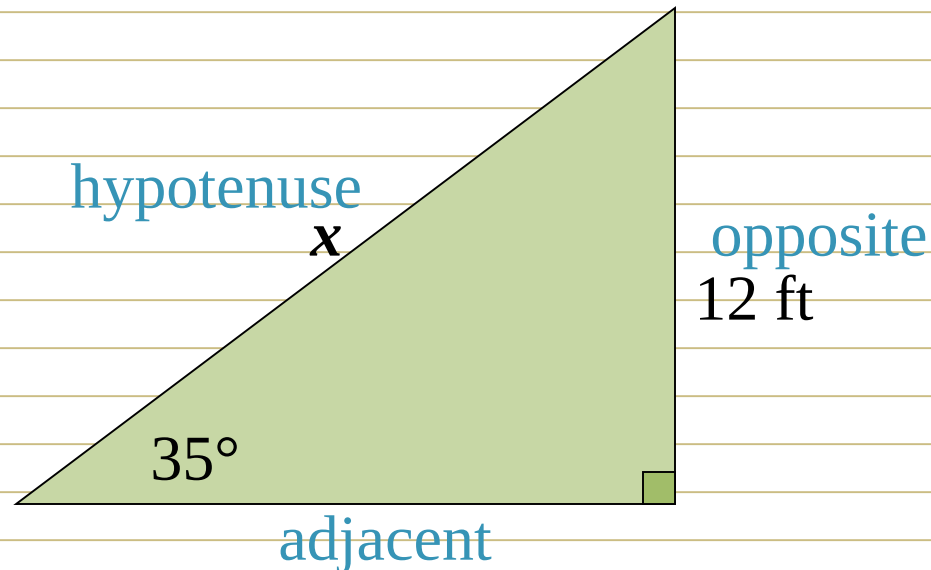
- Let's try a little bit more challenging problem.
- Determine the length of side x .



Determining the length of a side

Example 7

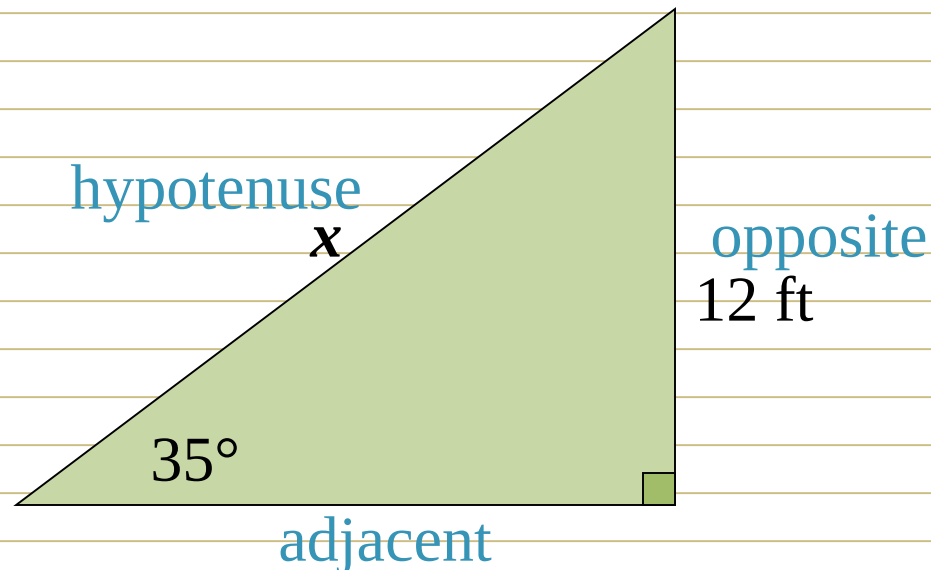
- Label the sides of the right triangle...



Determining the length of a side

Example 7

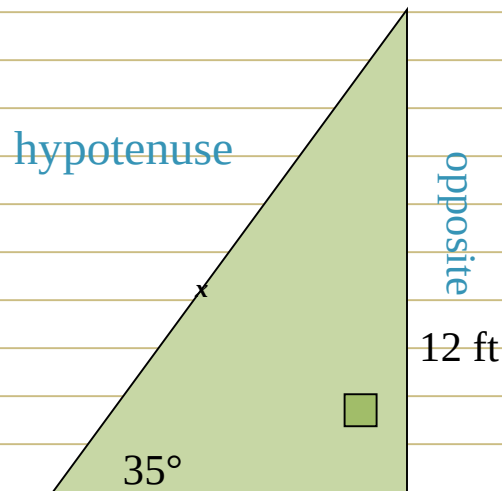
- Which trig function will you pick? You know the length of the side opposite and want to know the length of the hypotenuse.



Determining the length of a side

Example 7

- Which trig function should you pick?



$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$

You need to pick the sine function since it is the only one that has both the *opposite* side and *hypotenuse* in it.

Determining the length of a side

Example 8

- The reason the last problem was a little bit more difficult was the fact that you had an unknown in the denominator of the fraction.
- Keep clicking to see a similar trig function solved.

$$\tan A = \frac{\text{opp}}{\text{adj}}$$

$$x(\tan 50^\circ) = \frac{35}{\cancel{x}}$$

$$\tan 50^\circ = \frac{35}{x}$$

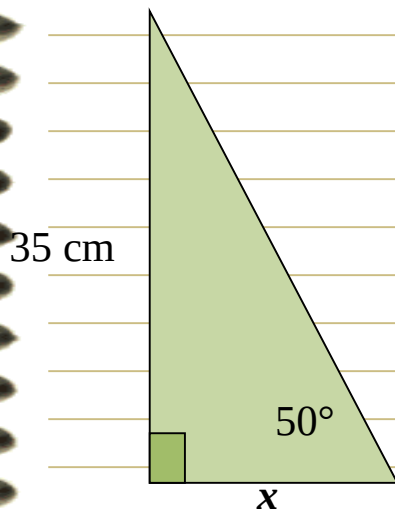
$$x = \frac{35}{\tan 50^\circ}$$

$$x(\tan 50^\circ) = \frac{35}{x}(x)$$

$$x(\tan 50^\circ) = 35$$

$$x = \frac{35}{1.1918}$$

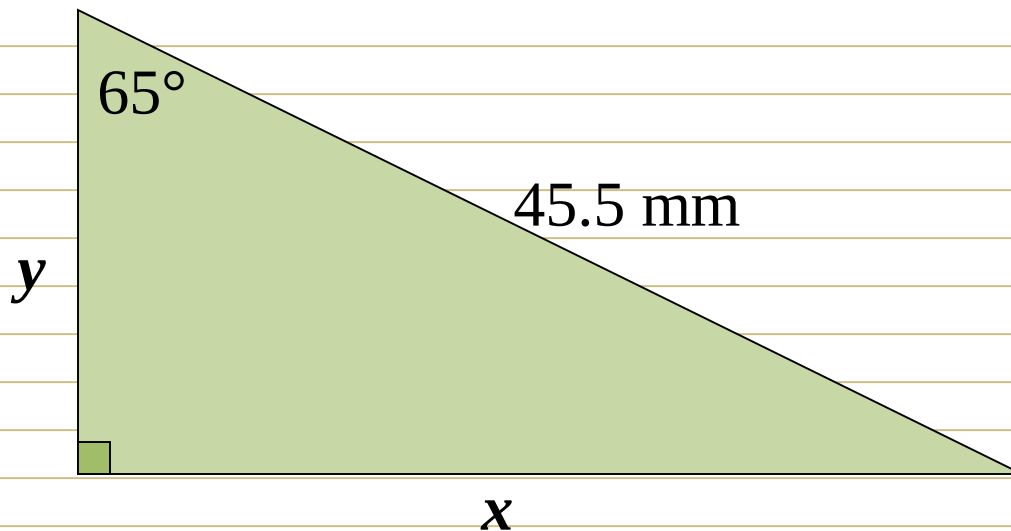
$$x = 29.4$$



Determining the length of a side

Example 9

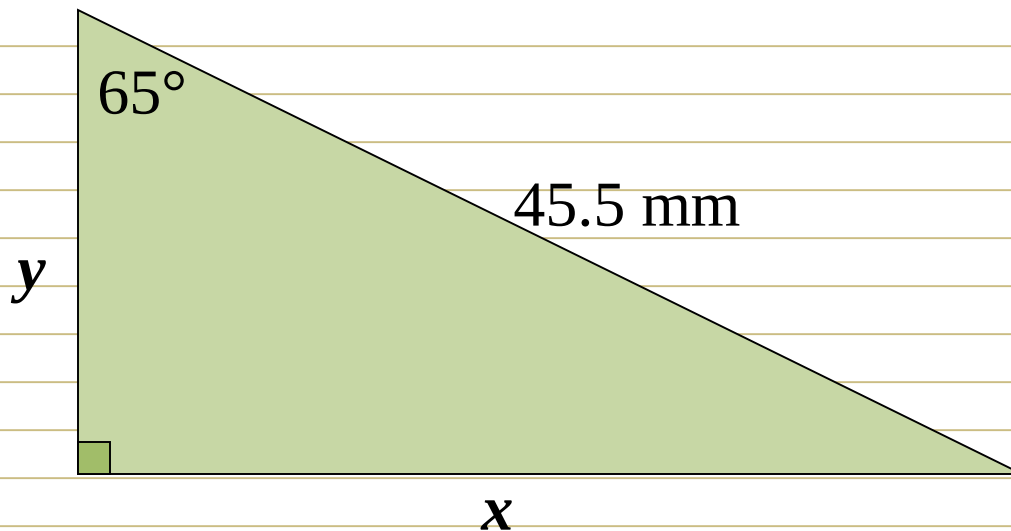
- Let's try one more example.
- Determine the lengths of sides x and y .



Determining the length of a side

Example 9

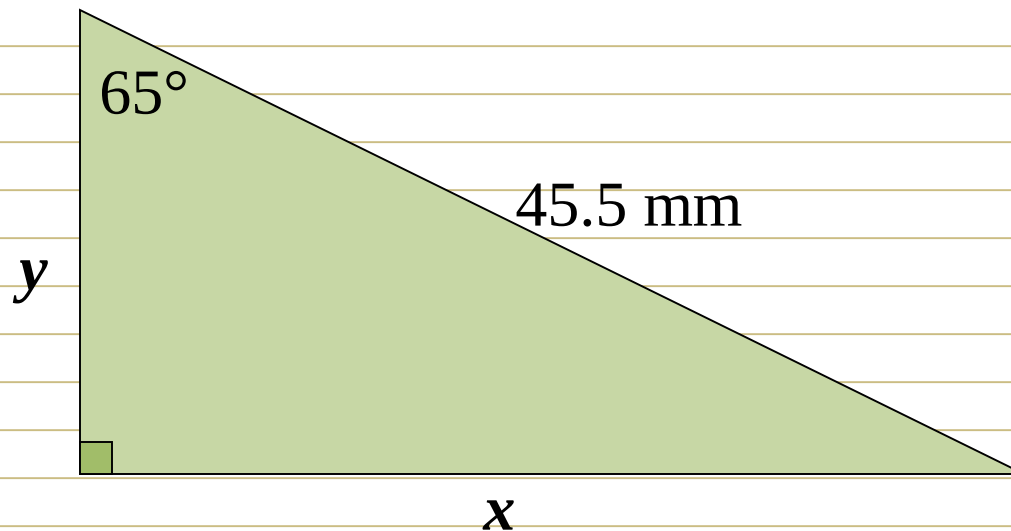
- To start, you must determine *which side* (x or y) you want to solve for first.
- It really doesn't matter which one you pick.



Determining the length of a side

Example 9

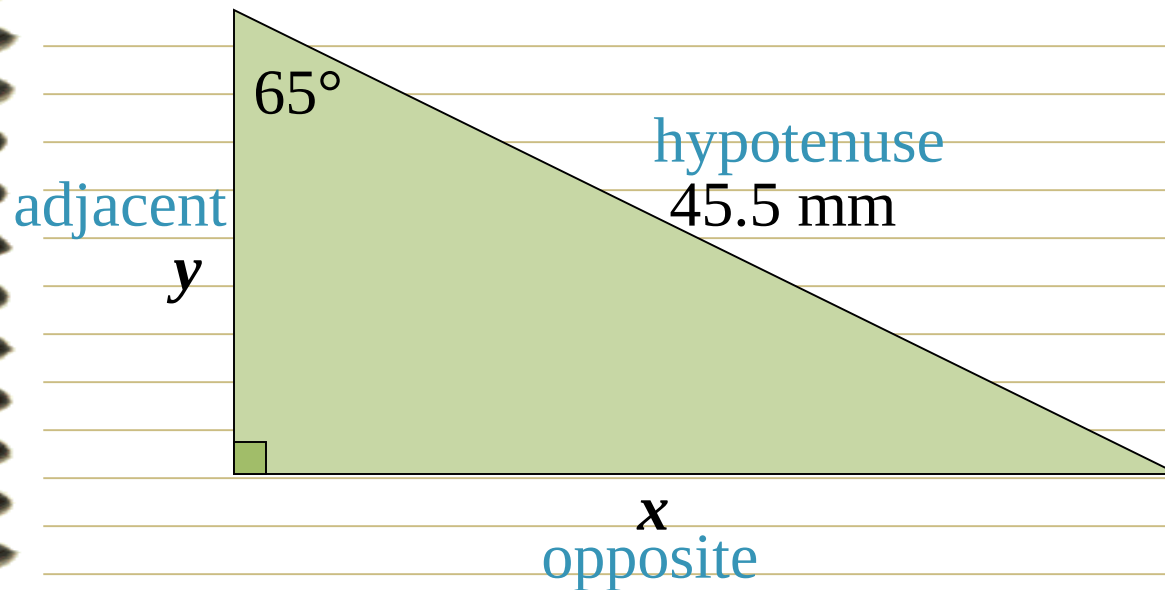
- Let's compute the length of side y first...



Determining the length of a side

Example 9

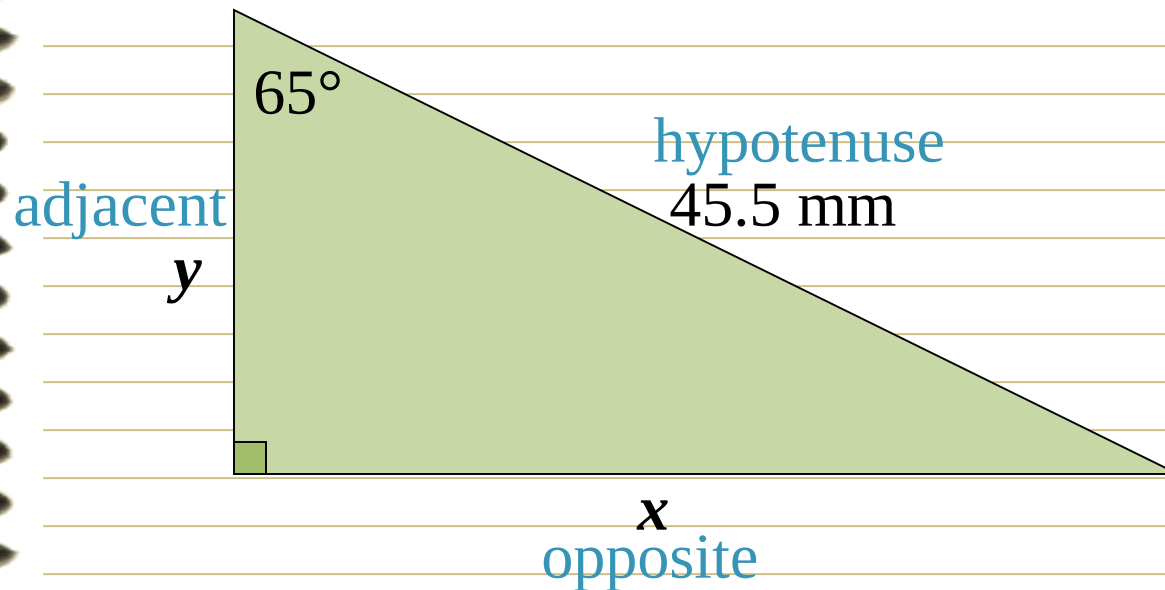
- Label the sides of the triangle...



Determining the length of a side

Example 9

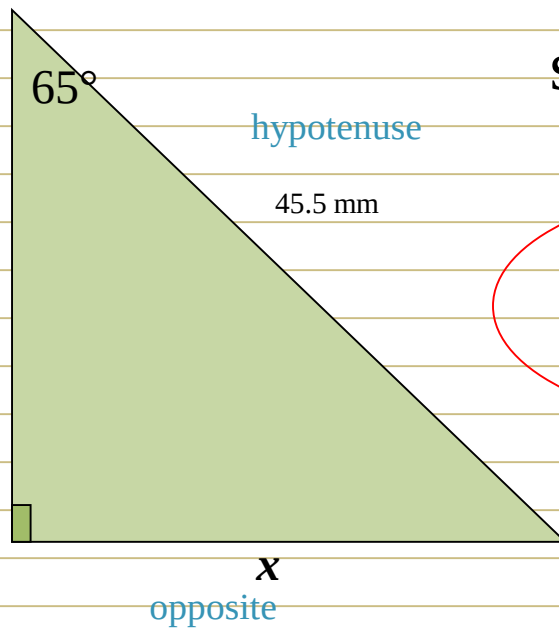
- Since you want to know the length of side y (adjacent) and you know the length of the hypotenuse, which trig function should you select?



Determining the length of a side

Example 9

- Which trig function should you pick?



$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$

You need to pick the cosine function since it is the only one that has both the *adjacent* side and *hypotenuse* in it.

Determining the length of a side

Example 9

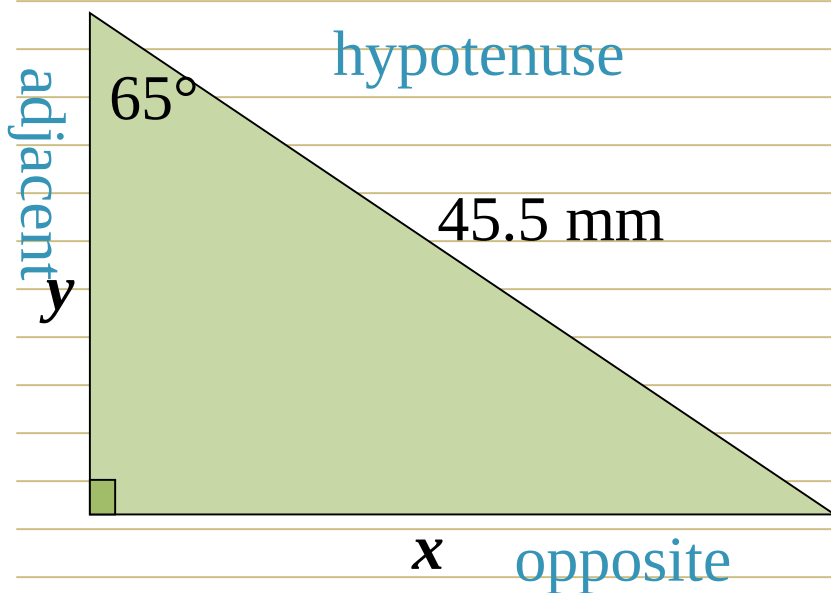
- Now set-up the trig function and solve for y .

$$\cos A = \frac{\text{adj}}{\text{hyp}} \quad \cos 65^\circ = \frac{y}{45.5}$$

$$45.5(0.4226) = y$$

$$45.5(\cos 65^\circ) = \frac{y}{45.5}(45.5)$$

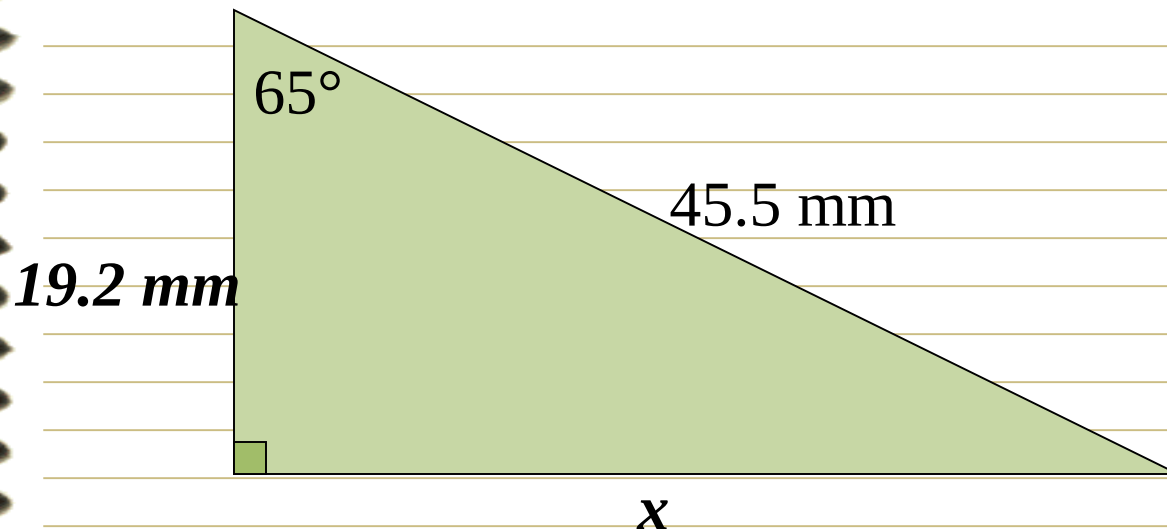
$$19.2 \text{ mm} = y$$



Determining the length of a side

Example 9

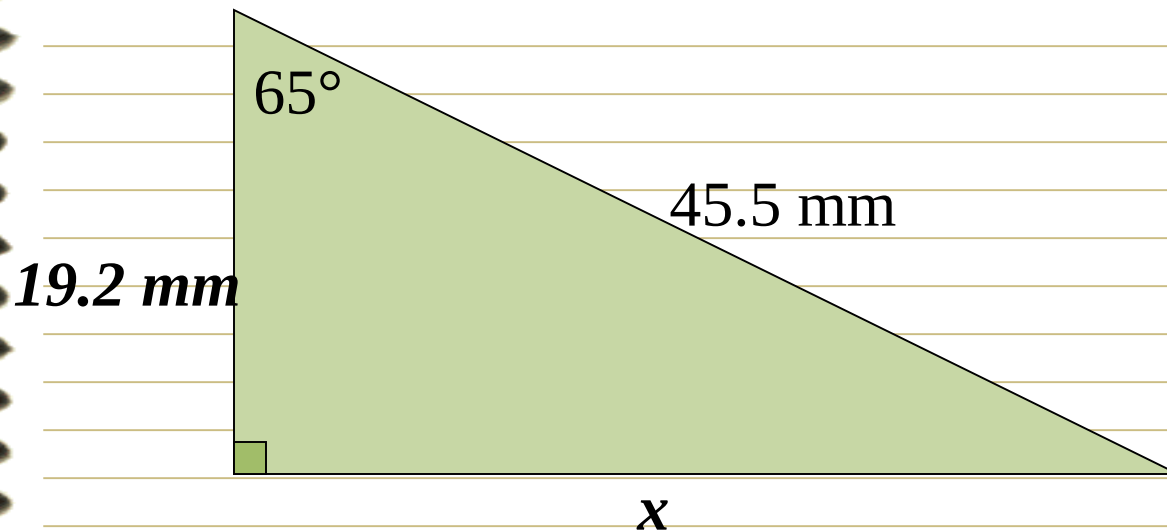
- Now we know side y is 19.2 mm long.
- The next question is, “How long is side x ?”



Determining the length of a side

Example 9

- You could use trig to solve for x , but why not use the Pythagorean Theorem?



Determining the length of a side

Example 9

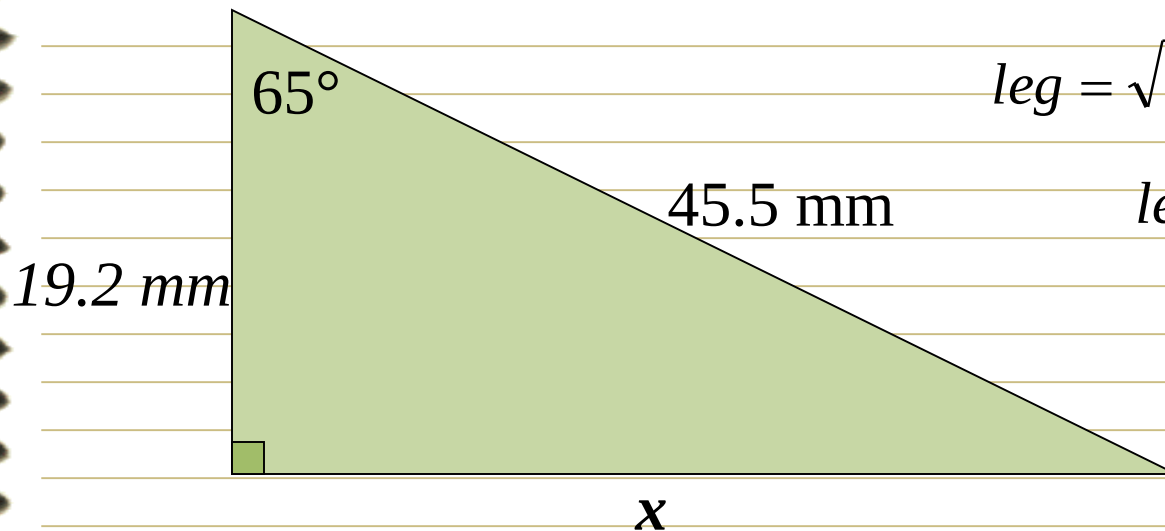
- You know a leg and the hypotenuse of a right triangle, so use this form of the theorem:

$$\text{leg} = \sqrt{\text{hypotenuse}^2 - \text{leg}^2}$$

$$\text{leg} = \sqrt{45.5^2 - 19.2^2}$$

$$\text{leg} = \sqrt{1701.61}$$

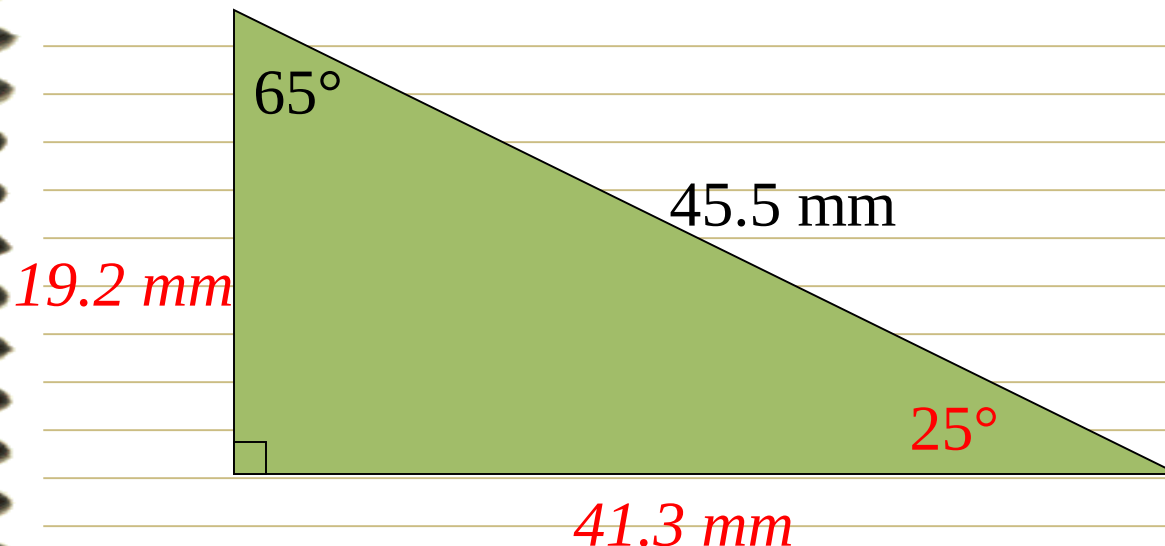
$$\text{leg} = 41.3 \text{ mm}$$



Determining the length of a side

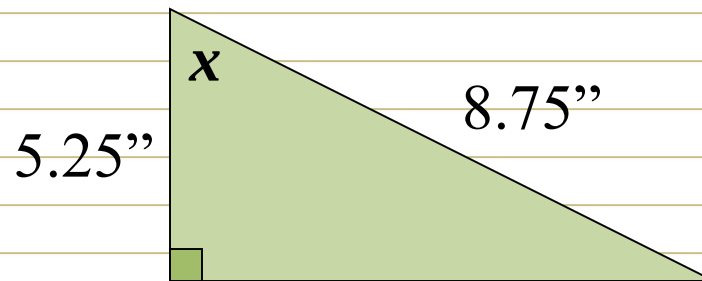
Example 9

- Both sides have been determined, one by trig, the other using the Pythagorean Theorem.
- Also the size of the other acute interior angle is indicated...



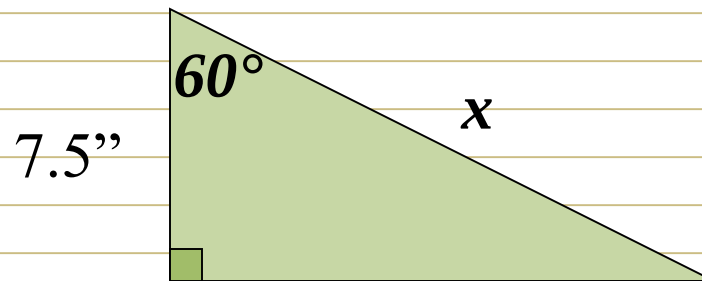
Summary

- After viewing this lesson you should be able to:
 - Compute an interior angle in a right triangle when the lengths of two sides are known.



Summary

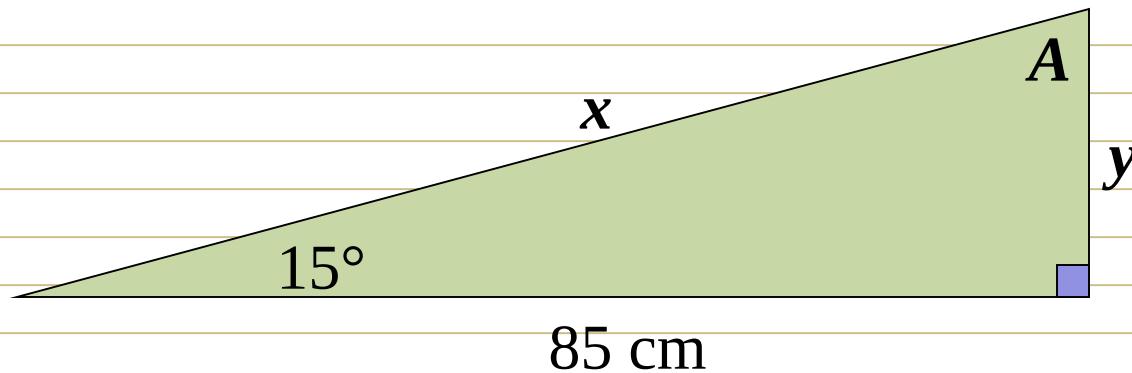
- After viewing this lesson you should be able to:
 - Compute the length of any side of a right triangle as long as you know the length of one side and an acute interior angle.



Final Practice Problem

Example 10

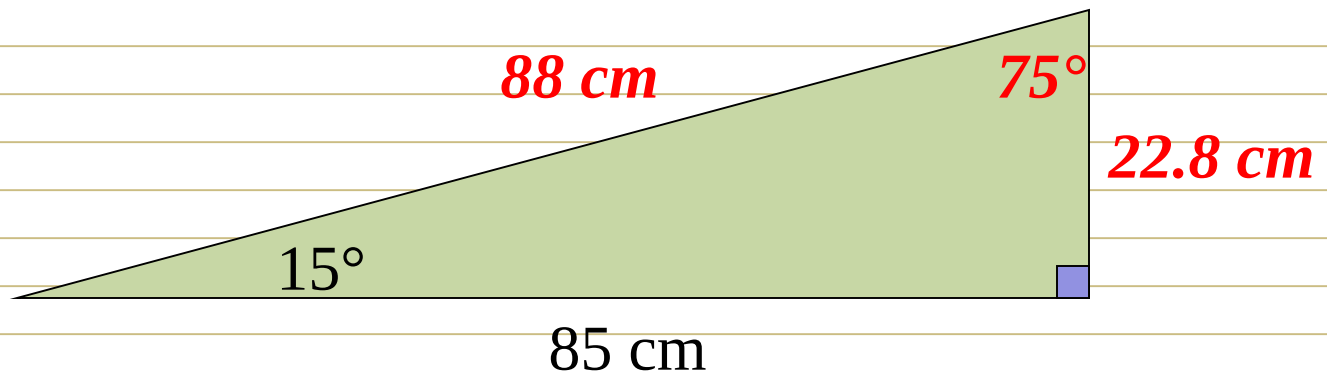
- Determine the lengths of sides x and y and the size of angle A .
- When you are done, click to see the answers on the next screen.



Final Practice Problem

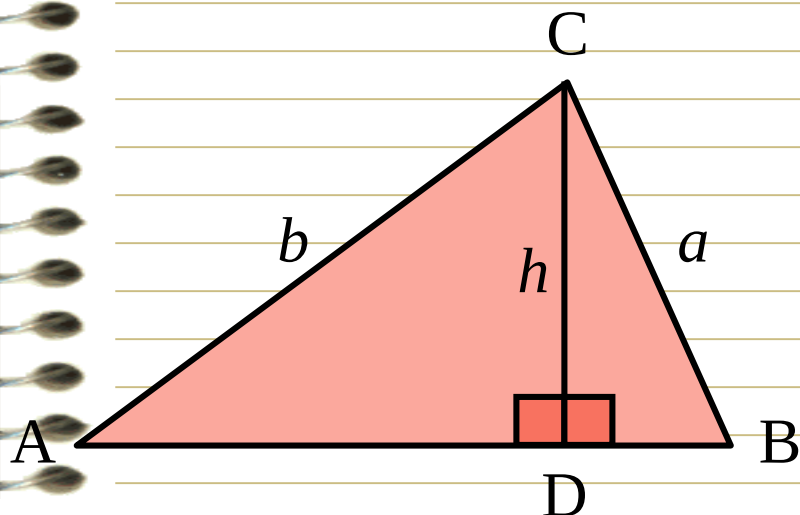
Example 10

- The answers are shown below...



The sine rule

Consider any triangle ABC:



If we drop a perpendicular h from C to AB, we can divide the triangle into two right-angled triangles; ACD and BDC.

a is the side opposite A and b is the side opposite B.

$$\sin A = \frac{h}{b}$$

$$h = b \sin A$$

$$\sin B = \frac{h}{a}$$

$$h = a \sin B$$

So:

$$b \sin A = a \sin B$$



The sine rule

$$b \sin A = a \sin B$$

Dividing both sides of the equation by $\sin A$ and then by $\sin B$ gives:

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

If we had dropped a perpendicular from A to BC we would have found that:

$$b \sin C = c \sin B$$

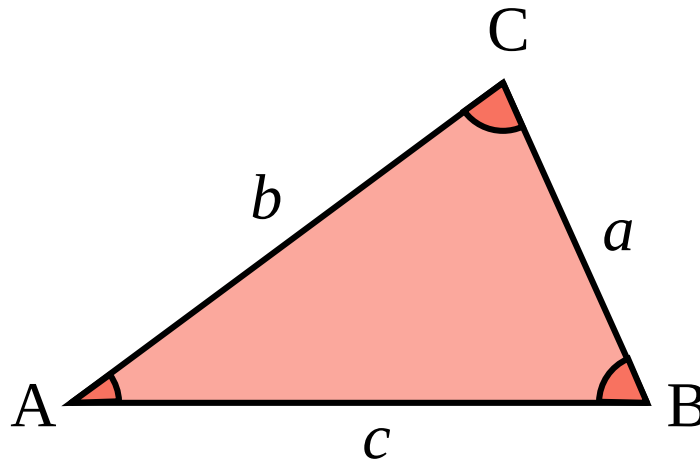
Rearranging:

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$



The sine rule

For any triangle ABC:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

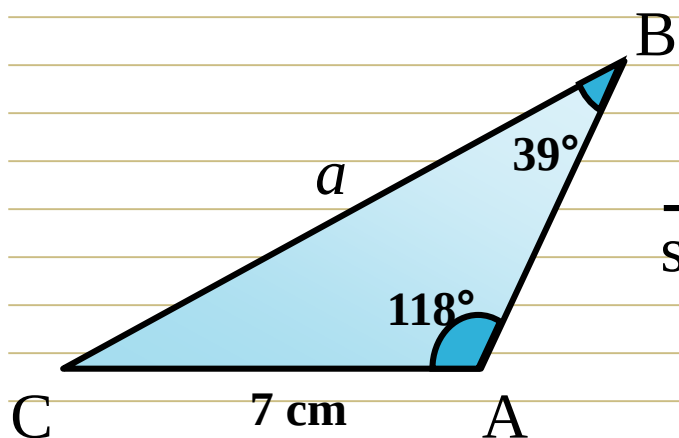
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Using the sine rule to find side lengths

If we are given two angles in a triangle and the length of a side opposite one of the angles, we can use the sine rule to find the length of the side opposite the other angle. For example:

Find the length of side a



Using the sine rule:

$$\frac{a}{\sin 118^\circ} = \frac{7}{\sin 39^\circ}$$

$$a = \frac{7 \sin 118^\circ}{\sin 39^\circ}$$

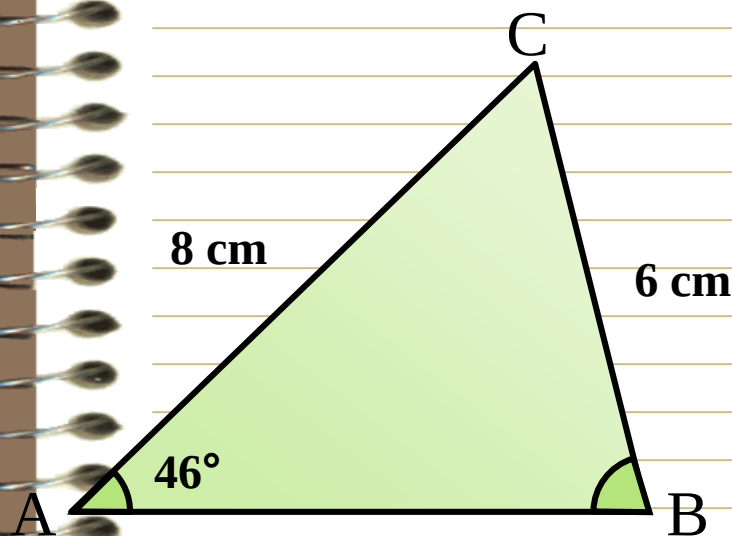
$$a = 9.82 \text{ cm (to 2 d.p.)}$$



Using the sine rule to find angles

If we are given two side lengths in a triangle and the angle opposite one of the given sides, we can use the sine rule to find the angle opposite the other given side. For example:

Find the angle at B



Using the sine rule:

$$\frac{\sin B}{8} = \frac{\sin 46^\circ}{6}$$

$$\sin B = \frac{8 \sin 46^\circ}{6}$$

$$B = \sin^{-1} \frac{8 \sin 46^\circ}{6}$$

$$B = 73.56^\circ \text{ (to 2 d.p.)}$$



Finding the second possible value

Suppose that in the last example we had not been given a diagram but had only been told that $AC = 8$ cm, $CB = 6$ cm and that the angle at $A = 46^\circ$.

There is a second possible value for the angle at B .

Instead of this triangle ...

... we could have this triangle.

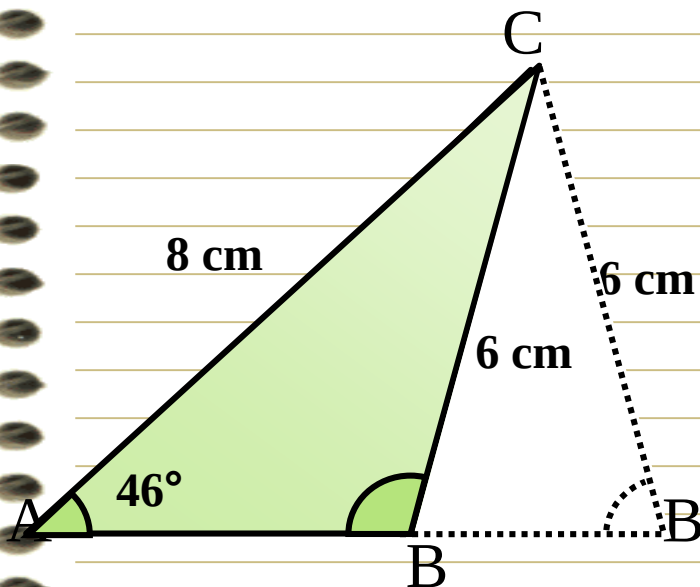
Remember: $\sin \theta = \sin (180^\circ - \theta)$

So for every acute solution, there is a corresponding obtuse solution.

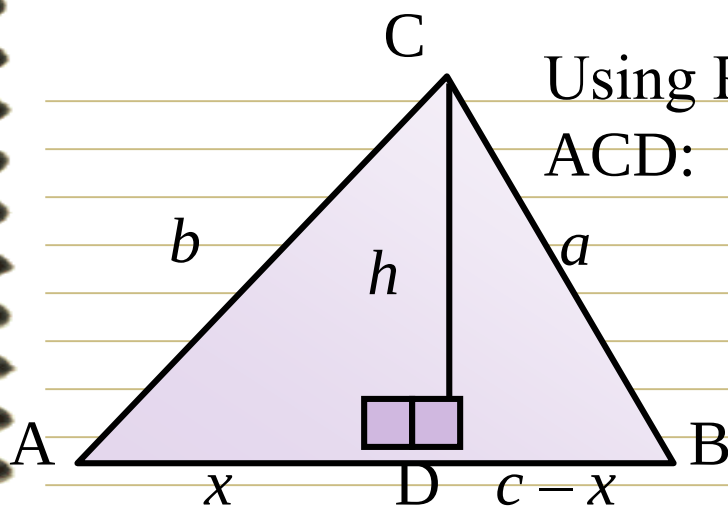
$$B = 73.56^\circ \text{ (to 2 d.p.)}$$

or

$$\begin{aligned} B &= 180^\circ - 73.56^\circ \\ &= 106.44^\circ \text{ (to 2 d.p.)} \end{aligned}$$



The cosine rule



Using Pythagoras' theorem in triangle ACD:

$$b^2 = x^2 + h^2 \quad (1)$$

Also:

$$\cos A = \frac{x}{b}$$

$$x = b \cos A \quad (2)$$

In triangle BCD:

$$a^2 = (c - x)^2 + h^2$$

$$a^2 = c^2 - 2cx + x^2 + h^2$$

Substituting (1) and (2) gives:

$$a^2 = c^2 - 2cb \cos A + b^2$$

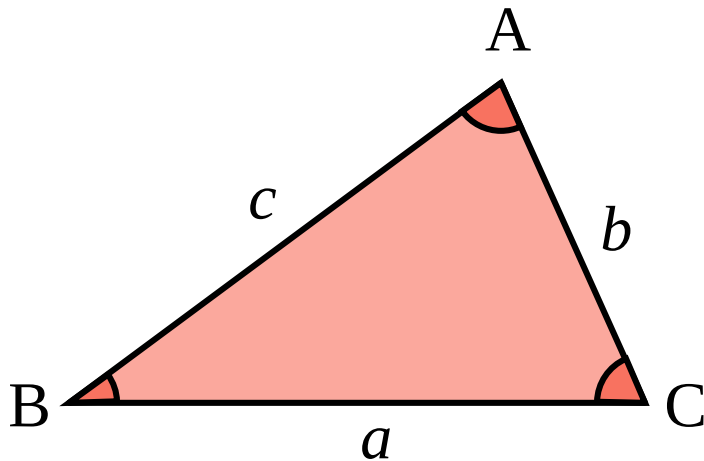
$$a^2 = b^2 + c^2 - 2bc \cos A$$

This is the **cosine rule**.



The cosine rule

For any triangle ABC:



$$a^2 = b^2 + c^2 - 2bc \cos A$$

or

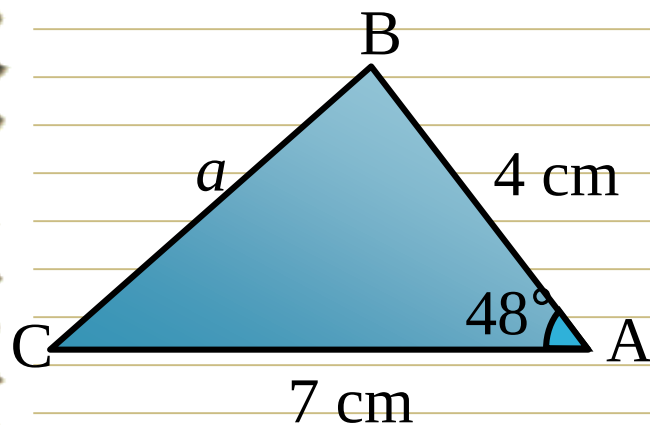
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



Using the cosine rule to find side lengths

If we are given the length of two sides in a triangle and the size of the angle between them, we can use the cosine rule to find the length of the other side. For example:

Find the length of side a



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 7^2 + 4^2 - (2 \times 7 \times 4 \times \cos 48^\circ)$$

$$a^2 = 27.53 \text{ (to 2 d.p.)}$$

$$a = 5.25 \text{ cm (to 2 d.p.)}$$

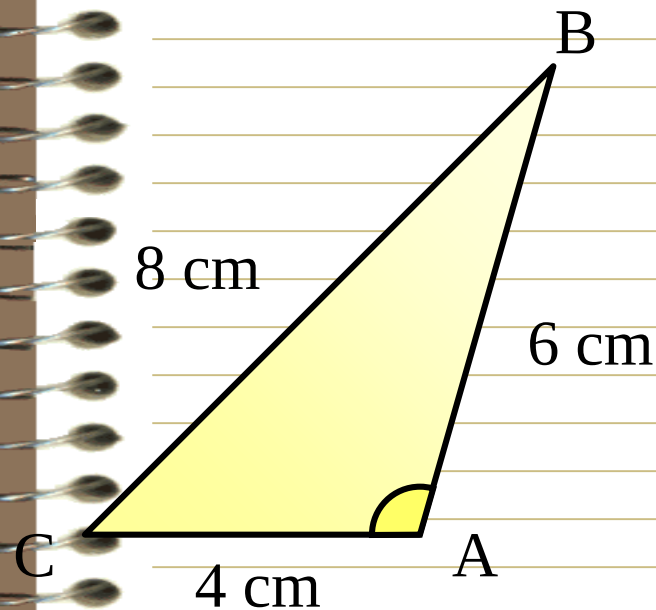


Using the cosine rule to find angles

If we are given the lengths of all three sides in a triangle, we can use the cosine rule to find the size of any one of the angles in the triangle. For example:

Find the size of the angle at A

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



$$\cos A = \frac{4^2 + 6^2 - 8^2}{2 \times 4 \times 6}$$

$$\cos A = -0.25$$

← This is negative so A must be obtuse.

$$A = \cos^{-1} -0.25$$

$$A = 104.48^\circ \text{ (to 2 d.p.)}$$

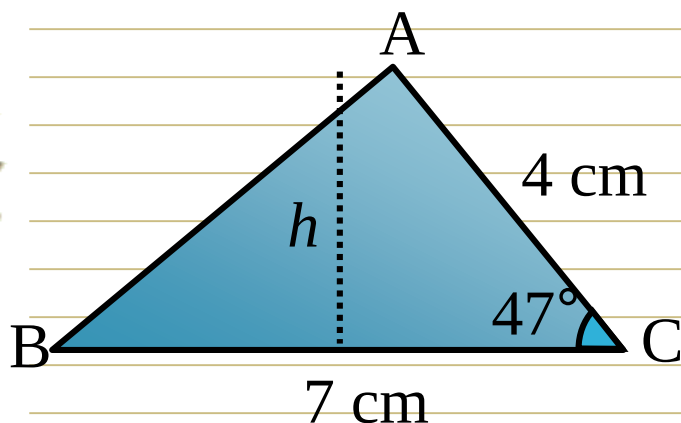


The area of a triangle

The area of a triangle is given by $\frac{1}{2} \times \text{base} \times \text{height}$.

Suppose that instead of the height of a triangle, we are given the base, one of the sides and the included angle. For example:

What is the area of triangle ABC?



We can find the height h using the sine ratio.

$$\frac{h}{4} = \sin 47^\circ$$

$$h = 4 \sin 47^\circ$$

$$\text{Area of triangle ABC} = \frac{1}{2} \times \text{base} \times \text{height}$$

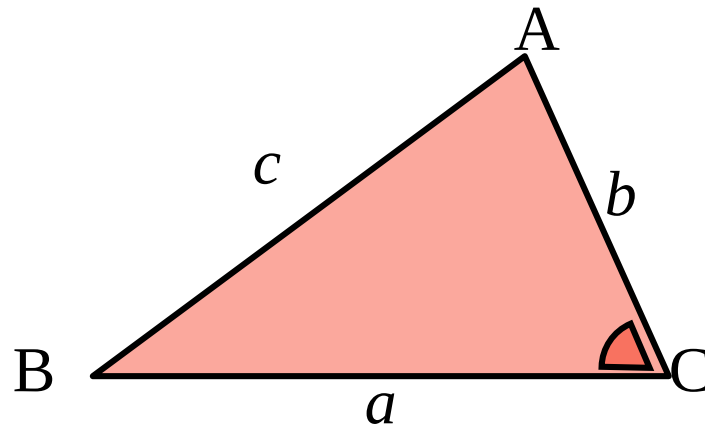
$$= \frac{1}{2} \times 7 \times 4 \sin 47^\circ$$

$$= 10.2 \text{ cm}^2 \text{ (to 1 d.p.)}$$



The area of a triangle using $\frac{1}{2} ab \sin C$

In general, the area of a triangle is equal to half the product of two of the sides and the sine of the included angle.



$$\text{Area of triangle ABC} = \frac{1}{2} ab \sin C$$





End of Presentation