

1809812

by Liam George

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1. Executive Summary

Within this assignment a number of sections were carried out where different methods were used to calculate/ simulate and test various control systems.

In section 2 it was proven that a transient response of a second order control system can be calculated and simulated, and proven using matlab without any discrepancies.

Section 3 a differential equation was used to represent an open loop control system and derived to find the poles and zeros and produce an argand diagram. The system response was proven using a step input, proving to be stable. The effects of adding gain was then shown and a root locus diagram proving critical stability. Using Simulink 3 different values of K were used to show stability, critical stability and instability.

In section 4 the block reduction method was used to reduce the transfer blocks to 1 simple block. The system was analysed to find the poles and zeros and an argand diagram produced which was proven using matlab. The block reduction method was further proven by using Simulink and measuring the step input response on both the original block diagram and the reduced block diagram proving the reduction technique was correct.

The frequency response method in [section 5](#) used a control system with a transfer function where individual values were chosen for p, q, r and k chosen and calculations carried out and proven using the bode frequency response method on matlab. The magnitude and phase of the transfer function was plotted using the bode frequency response method and analysed. The Phase-margin method was then used to determine the stability of the system.

Section 6 comprises of an equivalent block diagram of a digital control system, calculations were carried out and proven using Simulink which showed critical stability, stability and unstable responses above and below the calculated minimum value.

2. Transient Response of a Second order system

Below shown in figure 1 is a second order control system with the chosen values for K shown in the transfer function.

$$K = 18$$

$$a = 4$$

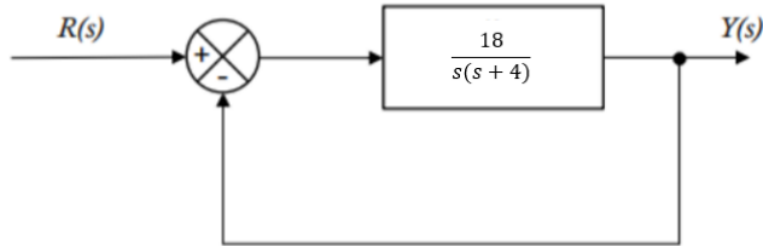


Figure 1 - Control system

As shown in appendix 8.1 calculations, the transfer function reduced to a single transfer function is :-

$$\frac{18}{s^2 + 4s + 18}$$

Using the single transfer function above, the Natural frequency values are shown in appendix 8.1 and added to equation below

$$\frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{18}{s^2 + 4s + 18}$$

Further calculations were carried out as shown in appendix 8.1 and results shown in table 1 below.

Calculated Values	
Damping factor	0.47
Damped frequency	3.85 rad/s
Rise time 0 to 100% of final value	0.535 seconds
Peak Time	0.86 seconds
Percentage Overshoot	18.77 %
Settling time to within +2%	2.0 seconds
Number of oscillations during settling time	1.196

Table 1 - Transient response calculated results table

Using the Matlab code shown below in figure 2 the response to the unit step input is shown below on the graph with simulated values shown.

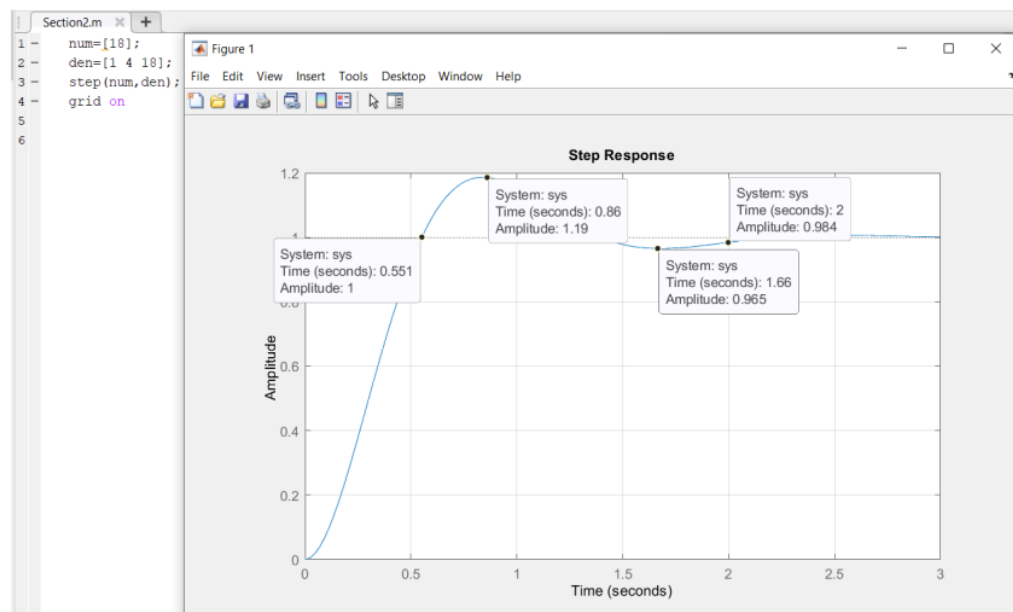


Figure 2 - Step input response

From the Graph shown above in figure 2 the simulated values are shown in table 2 below alongside the calculated values for comparison.

Calculated and Simulated Values		
	Calculated	Simulated
Damping factor	0.47	0.5
Damped frequency (rad/s)	3.85	3.79
Rise time 0 to 100% of final value (seconds)	0.535	0.551
Peak Time (seconds)	0.86	0.86
Percentage Overshoot (%)	18.77	19
Settling time to within +2% (seconds)	2.0	2.0
Number of oscillations during settling time	1.196	1.19

Table 2 - Transient response comparison table

As table 2 shows above, the Simulated results are the same as the calculated results, the very minor differences are due to rounding errors. This proves that the transient response of a control system can be determined throughout calculations and simulation and provide accurate predictions without causing any disruption to the operational process within industry and allowing the control engineer to ensure the response is correct.

3. Control System Modelling

Shown below is an open loop control system represented by a differential equation.

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = \frac{d^2x}{dt^2} - p \frac{dx}{dt} + qx$$

From the equation above and appendix 8.2, calculations were carried out assuming all initial conditions are zero. the equation is translated into the frequency domain and an open loop transfer function is derived as shown below :-

$$G(s) = \frac{s^2 - ps + q}{s^2 + as + b}$$

Values for 'a','b','p' and 'q' were chosen as shown below and calculations carried out in appendix 8.2, The location of the calculated poles and zeros as shown in the argand diagram below in figure 3.

$$a = 6 \qquad b = 4 \qquad p = 4 \qquad q = 6$$

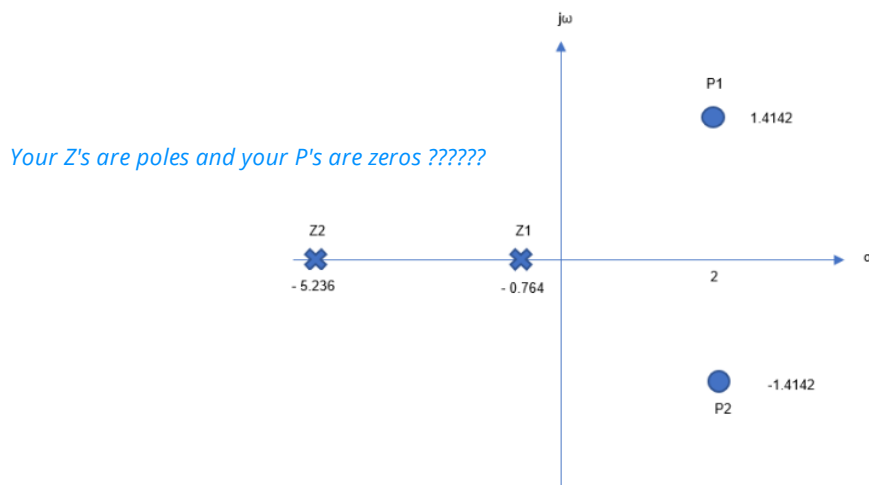


Figure 3 - Control system argand diagram

As shown in figure 4 above, the calculated poles and zeros from appendix 8.2 are plotted on the argand diagram, showing the poles are on the left hand plane on the real axis and the zeros on the right-hand plane as complex values. proving that the system will be stable and overdamped from the argand diagram and calculations in appendix 8.2.

With a calculated damping factor of 1.5. `

$$G(s) = \frac{s^2 - 4s + 6}{s^2 + 6s + 4}$$

The transfer function above was inputted on matlab to produce a step response as shown in appendix 8.2.1, showing the command window, step input response shown in appendix 8.2.2 and Root locus in appendix 8.2.3 to verify the calculations as shown below in table 3 comparing calculated with simulated results.

Poles and Zeros		
	Calculated	Simulated
Poles	-5.236	-5.2361
	-0.764	-0.7639
Zeros	2	2
	1.4142	1.4142

Table 3 - Poles and Zeros comparison table

As shown in table 3 above there is no difference between the calculated and simulated results, verifying the calculations are correct.

By adding negative feedback on the argand diagram then both poles P1 and P2 will move along to zero Z1 and Z2 as shown below in figure 4 below.

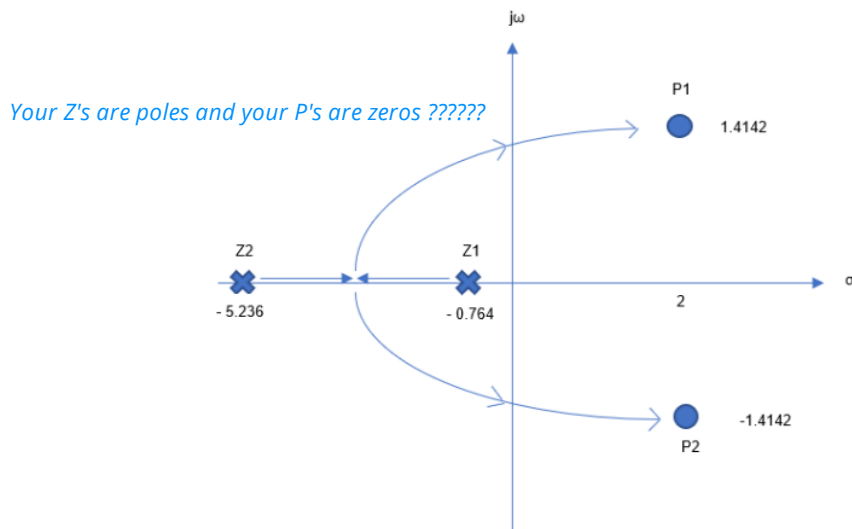


Figure 4 - Negative feedback argand diagram

Gain added into the feedback with a variable gain of K as shown in figure 5 below :-

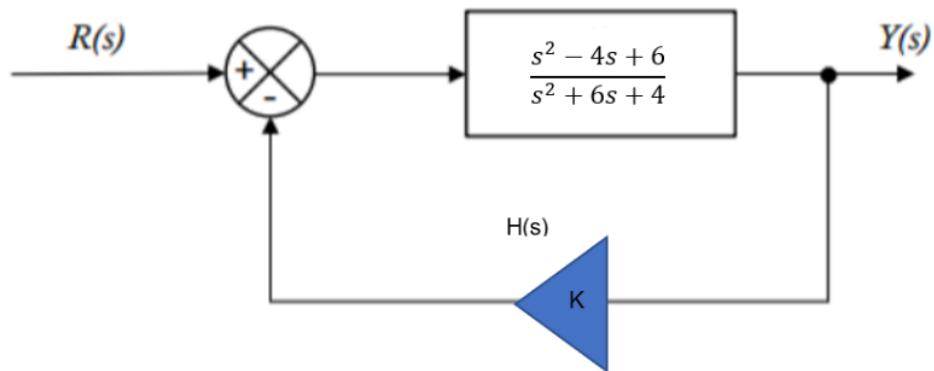


Figure 5 - Control system with added gain

The new transfer function is derived for the closed loop system as shown in appendix 8.2 and below for $T(s)$

$$T(s) = \frac{1}{1+k} \times \frac{(s^2 - 4s + 6)}{s^2 + \frac{(6 - 4k)s}{(1+k)} + \frac{(4 + 6k)}{(1+k)}}$$

Using the Transfer function $T(s)$ the Value for K to give critical damping is calculated as shown in appendix 8.2 and shows critical damping to be 1.5. This is proven using the rlocus function on matlab which is also shown in appendix 8.2.4, showing critical stability to be at 1.5.

Which means that :-

< than 1.5 would be stability

= to 1.5 would be critical stability

> than 1.5 would be unstable

Which is proven below on the Simulink models shown below with 3 different values of K to prove each state.

what about critical damping?

The 3 different values of K chosen to prove the 3 states are :-

Stability = 0.8

Critical Stability = 1.5

Instability = 2.8

Below shown in figure 6 is the graph with the K rating 0.8 proving Stability is below 1.5.

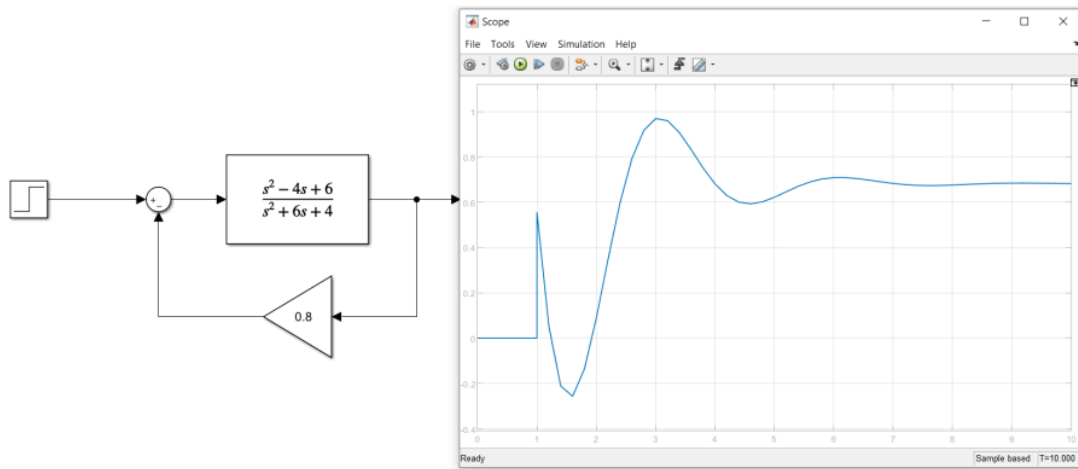


Figure 6 - simulink graph K rating 0.8

In figure 7 below proves that the calculated value of 1.5 is critical Stability.

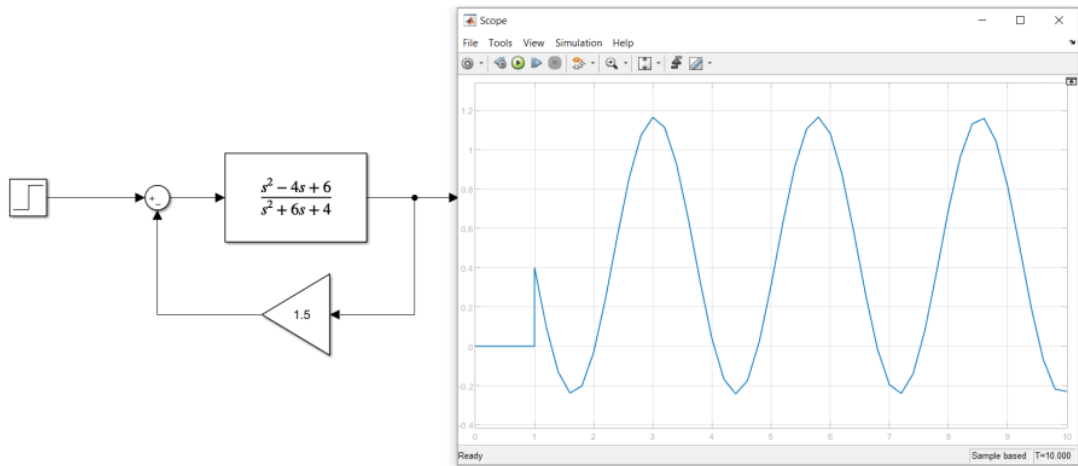


Figure 7 - Simulink graph K rating 1.5

With the Gain set to 2.8 in figure 8 below it shows that the system goes unstable and enters instability.

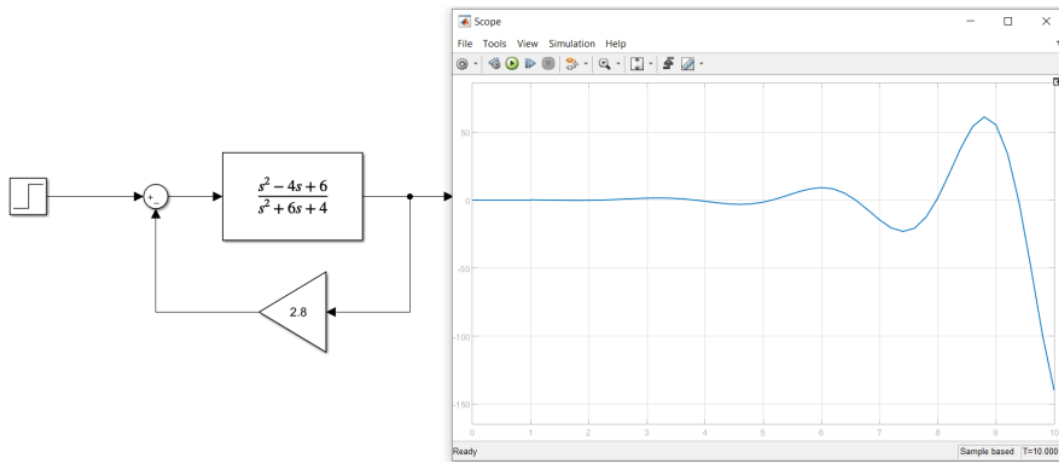


Figure 8 - Simulink graph K rating 2.8

The Root locus is used for control system modelling as show above to analyse and predict the control system is going to be stable or unstable and wither its over damped or under damped. this is carried out by the location of the poles and zeros on the argand diagram as shown in appendix 8.2.3. The argand diagram shows that the poles in this case are in the left hand plane on the real axis and the zeros on the right-hand plane as complex values. proving that the system will be stable. If the poles where in the right hand plane then the system will be become unstable.

This also shows wither the system will be under or over damped as figure 4 and appendix 8.2.3 shows this system will be overdamped as the calculated value for the damping factor is 1.5 and as shown in this section in figures 6,7 and 8 the three different values prove that if the system with 3 different added feedback values it proves by being equal to 1.5, above and below 1.5 shows Stability, Critical Stability and Instability.

4. Block Diagram Reduction method

Below in figure 9 is a block diagram containing 6 transfer blocks.

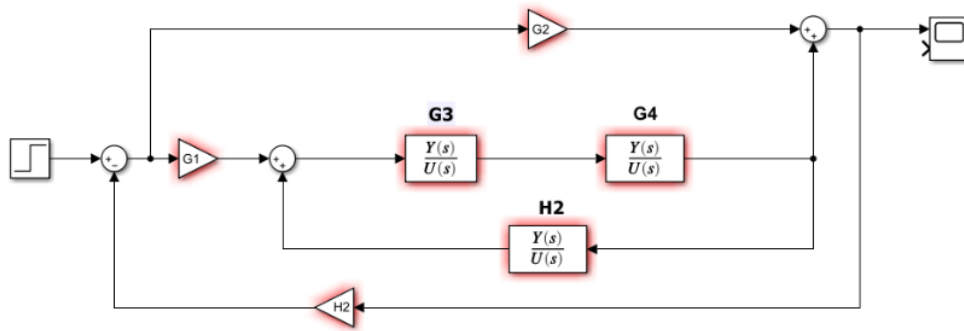


Figure 9 - Block diagram

The above block diagram has been reduced to a single transfer function as shown in Appendix 8.3 and an Expression derived for T as shown below in terms of G1,G2,G3,G4, H1 and H2.

$$T = \frac{G1 G3 G4 + G2 - G2 G3 G4 H1}{1 - G3 G4 H1 + H2 G1 G3 G4 + H2 G2 - H1 H2 G3 G4 G2}$$

Values for a,b,c and d have been chosen as shown below and entered into the transfer block equations for G1,G2,G3,G4,H1 and H2 as shown below.

$$a = 8$$

$$b = 6$$

$$c = 3$$

$$d = 14$$

$$G1 = a = 8$$

$$G4 = 1/s$$

$$G2 = b = 6$$

$$H1 = 1/(s+1)$$

$$G3 = c/(s+1) = 3/(s+1)$$

$$H2 = d = 14$$

Using the above transfer block equations the final expression was derived as shown in appendix 8.3 and shown below.

$$T = \frac{6s^3 + 12s^2 + 30s + 6}{85s^3 + 170s^2 + 421s + 81}$$

The System was analysed and the poles and zeros found as shown in appendix 8.3.1 and the argand diagram shown below in figure 10 and appendix 8.3.2 which shows by the response that the system will be stable as in the left hand plane.

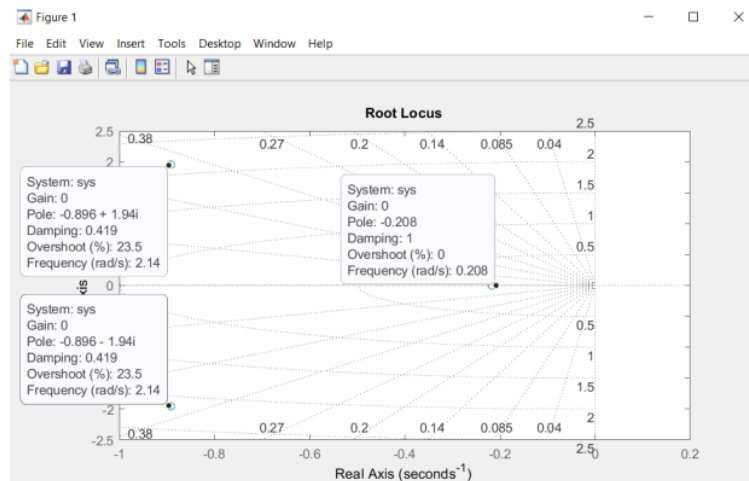


Figure 10 - Block diagram root locus argand diagram

The Transfer function was entered into matlab for comparison and as shown the below in figure 11 using the impulse function it was proven that the system would be stable.

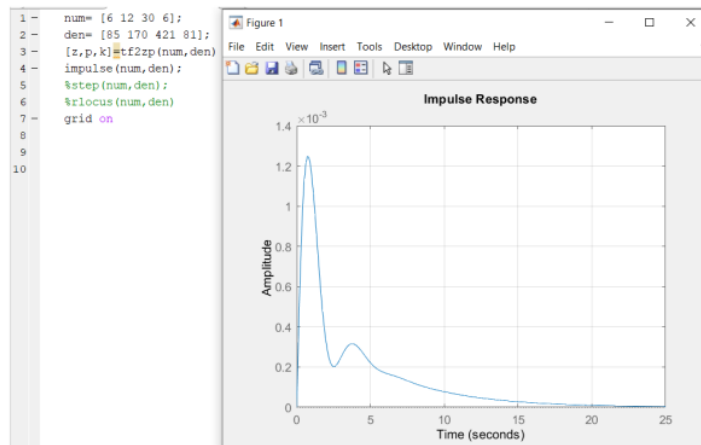


Figure 11 - Impulse function response

To prove the response further the transfer function was entered into matlab and a step command entered and a graph produced as shown below in figure 12 which also proves the system to be stable.

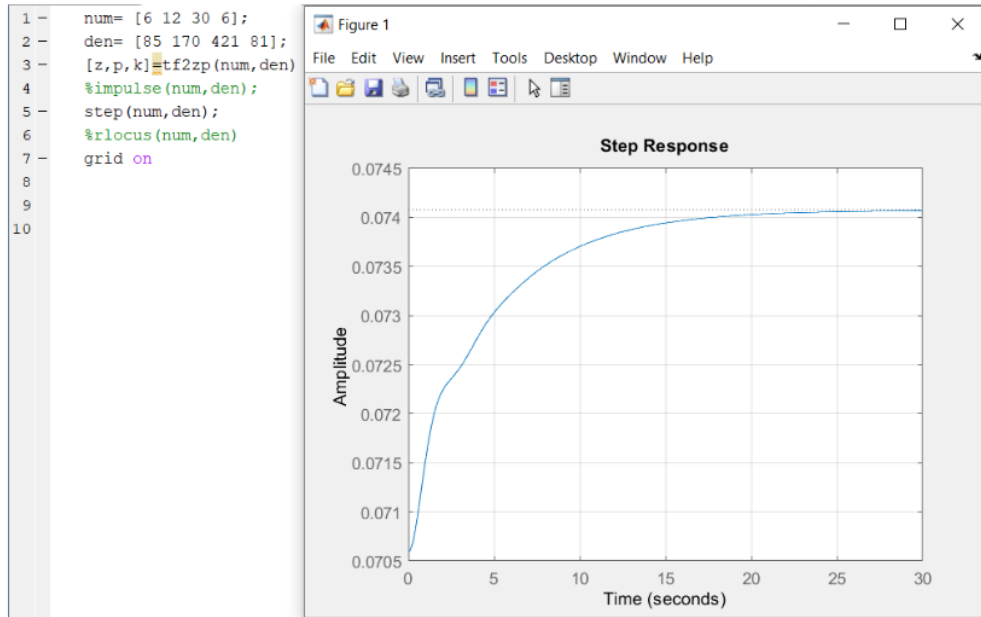


Figure 12 - Step command response

Using the block diagram with the values entered for G1, G2, G3, G4, H1 and H2 as shown below in figure 13, using Simulink the response was plotted as also shown in figure 14 below.

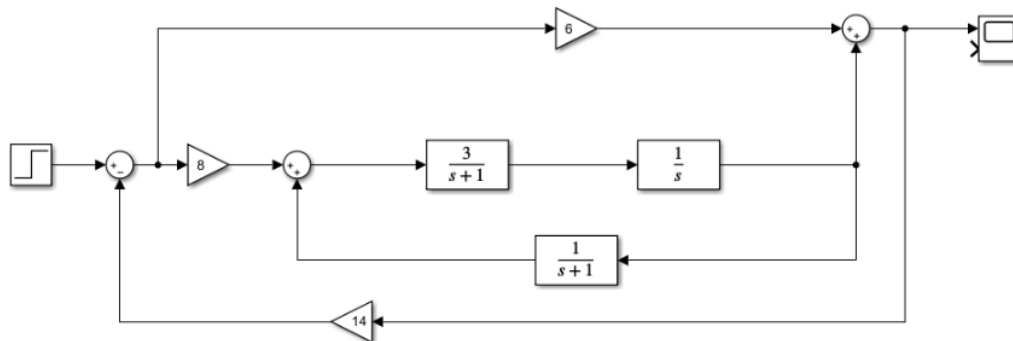


Figure 13 - Block diagram G1,G2,G3, H1 and H2 added values

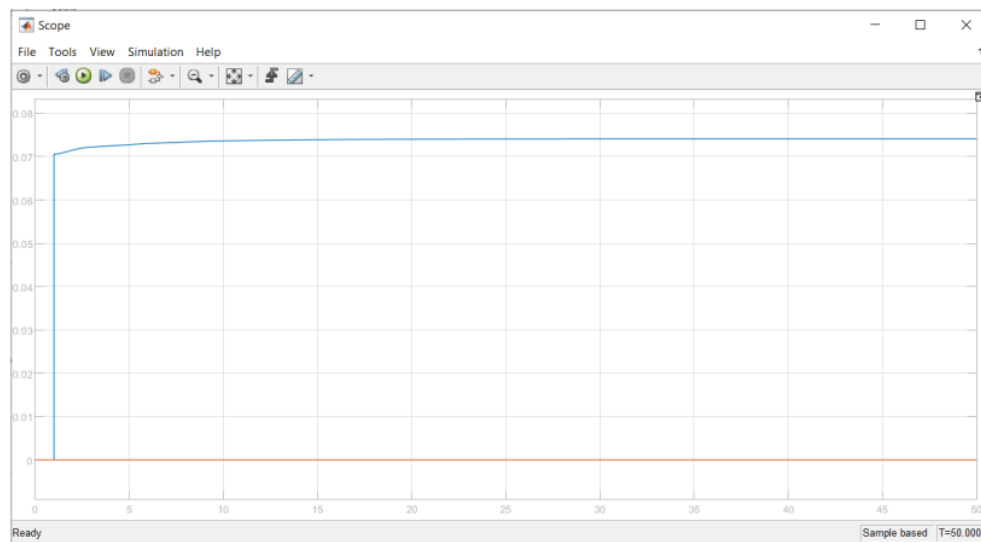


Figure 14 - Block diagram response

Using the overall transfer function entered into Simulink, alongside the non-reduced block diagram, using the same scope as shown below in figure 15 the block reduction calculation techniques were proven as shown in the scope graph shown below in figure 16.

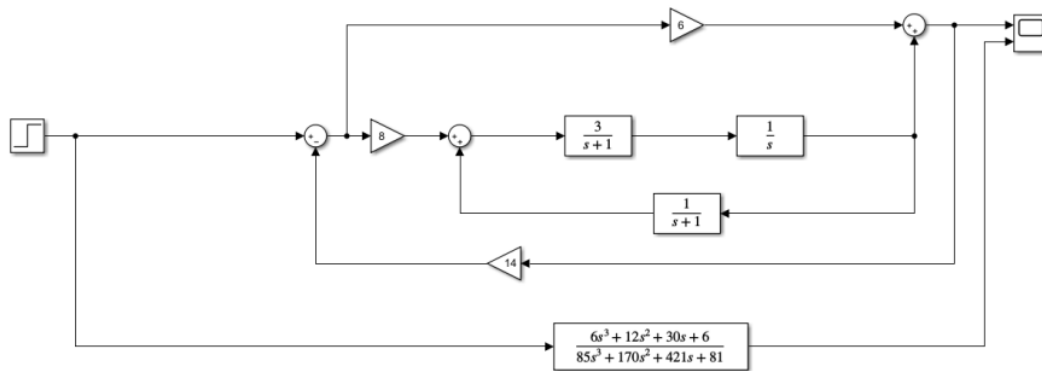


Figure 15 - Block diagram reduced method

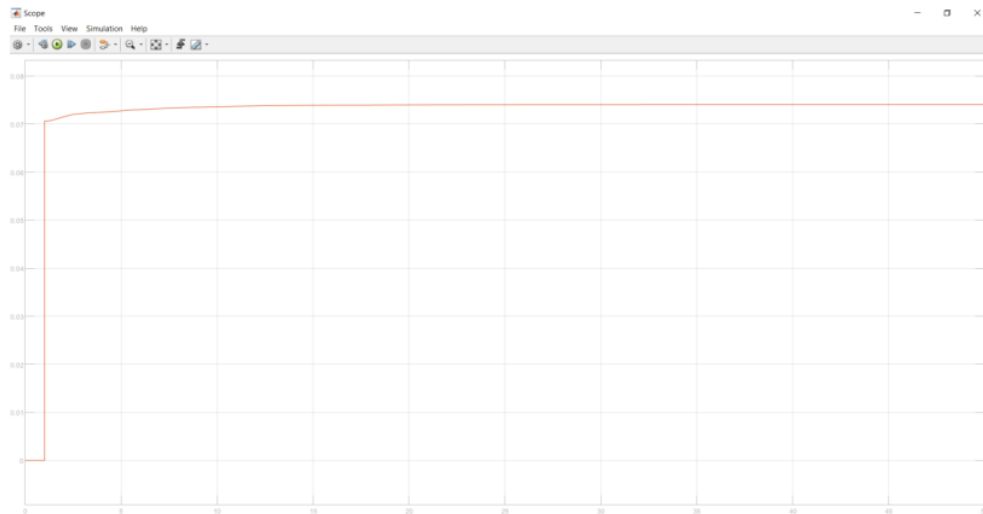


Figure 16 - Block diagram reduced method response

As shown in figures 15 and 16 above, the responses are identical proving that the block reduction technique is correct.

5. Frequency Response method

A control system with the transfer function shown below.

$$G(S) = \frac{K1 (s + p)}{(s + q)(s + r)}$$

Using my selected values for p, q and r as shown below, calculations were carried out as shown in appendix 8.4 and the transfer function derived as shown below

$$P = 2$$

$$q = 10$$

$$r = 100$$

$$K1 = 1800$$

$$T(s) = \frac{1800s + 3600}{s^2 + 110s + 1000}$$

Using the transfer function above entered into matlab a bode frequency response diagram is produced as shown below in figure 17.

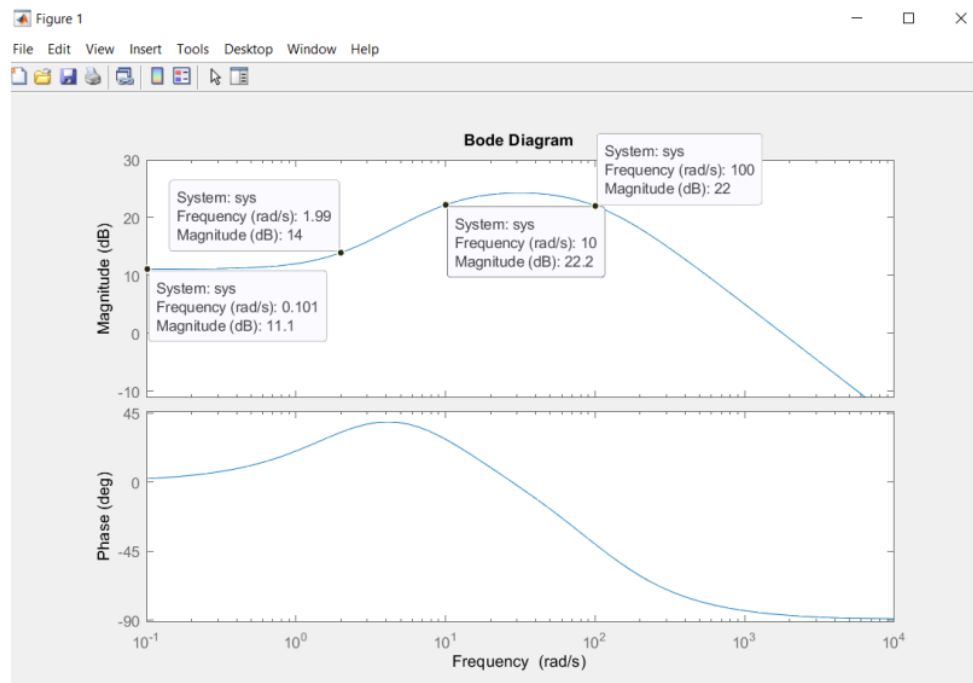


Figure 17 - Bode frequency response diagram

Below in figure 18 and appendix 8.4.1 shows the magnitude and phase graph of the magnitude and phase of the transfer function over the ranges showing the errors and measurements.

excellent analysis

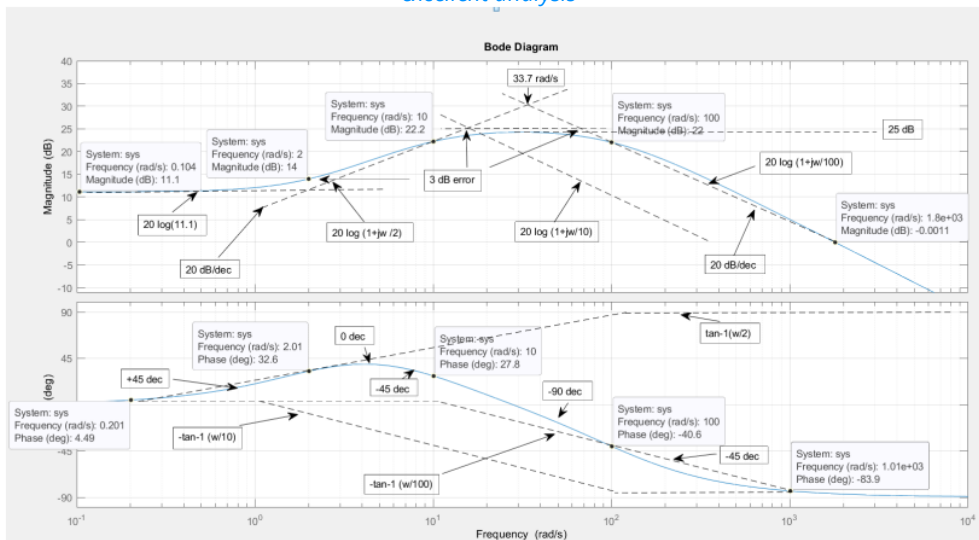


Figure 18 - Bode frequency magnitude and phase graph measurements

The phase margin method is used to determine the relative stability of the system as the gain margin method cannot be used, as the phase in the system does not reach 180 deg. Figure 16 below shows that at 1800 rad/s the gain hits 0 deg and the frequency phase shift at 1800 rad/s = -86.6 deg as shown below in figure 19.

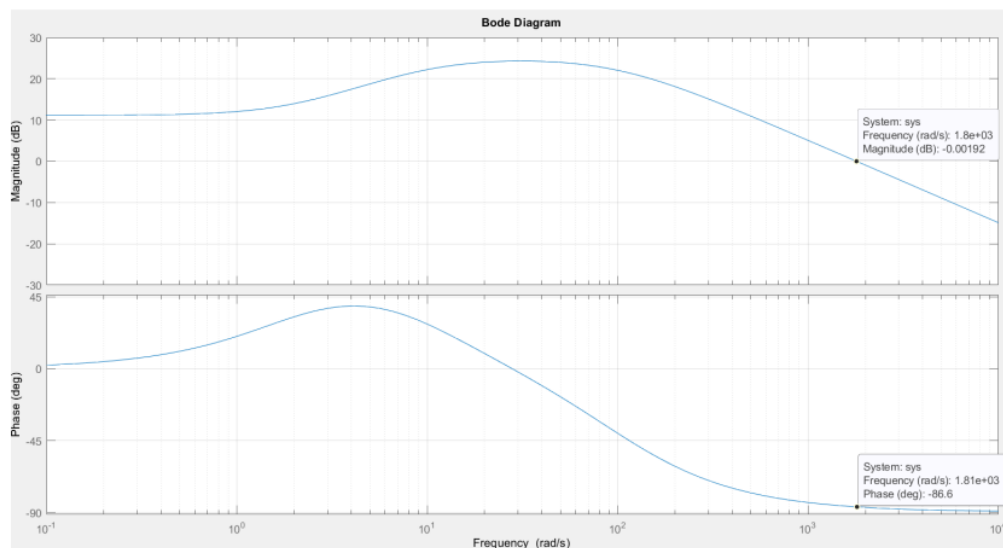


Figure 19 - Phase margin method graph

So as this shows above the distance between -86.6 and -180 deg = 93.4 deg which means that the phase margin = 93.4 deg before the critical point.

This shows that the system is stable because the minimum phase margin = 45 deg and with the system having a phase margin of 93.4 its within stability.

good conclusions

6. Digital Control systems

In figure 20 below shows the equivalent block diagram of a digital control system.

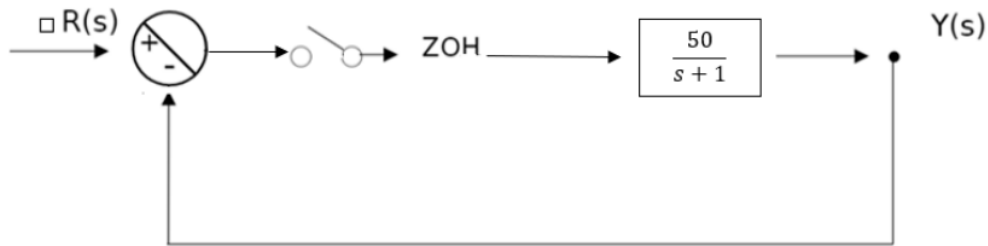


Figure 20 - Digital control system block diagram

The closed Loop Transfer function was derived as shown below and calculations shown in appendix 8.5.

$$T(Z) = \frac{50 (1 - e^{-t})}{z + 50 - 51e^{-t}}$$

Calculations were carried out as shown in appendix 8.5 to determine the sampling time and the minimum sampling frequency, this was proven using Simulink as shown in appendix 8.5 and shown in comparison table 4 below.

Calculated and simulated comparison table		
	Calculated	Simulated
Sampling time (ms)	40	40
Minimum sampling Frequency (Hz)	25	25

Table 4 - Digital control system comparison table

As the table 4 shows above the calculated and simulated values are identical which proves the calculated values are correct showing that the minimum sampling frequency is 25HZ and stability in the Z-plane.

At 40ms the system is critically stable as shown in appendix 8.5.1 and by altering the sampling time above and below the sampling time it can be seen the system becomes stable or unstable. As shown in appendix 8.5.2 the system becomes stable at a sampling time of 0.15, and becomes unstable by altering the sampling time to 65ms as shown in appendix 8.5.3.

These are mainly a description or summary of what was done confirming the answers were correct. However, it has few conclusions or recommendation that show what you have learnt from the assignment.

7. Conclusion and Recommendations

Transient response of a second order system was calculated and simulated with all results shown in table 2, which shows that both the simulated and calculated values are identical with very minor rounding errors proving that the transient response of a control system can be determined throughout calculations and simulation and provide accurate predictions without causing any disruption to the operational process within industry and allowing the control engineer to ensure the response is correct.

An open loop control system was modelled and represented by a differential equation and calculations were carried out and a poles and zeros determined showing the response to be stable and overdamped with the calculated damping factor being 1.5. As shown in table 3 the calculated and simulated results for the poles and zeros are identical verifying the calculations are correct. Feedback added to the system and was proven that above the calculated value of 1.5 would make the system unstable, below the value of 1.5 would make the system stable and equal to 1.5 the system would be critically stable. Also showing the benefits of the root locus method for control system analysis and predicting the responses.

A control system was shown in a block diagram with 6 transfer blocks. The block reduction method was used to reduce the block diagram to one transfer function. The non-reduced block diagram and the reduced transfer function were plotted using Simulink and it was proven that the calculated block reduction technique was correct showing the exact same responses from them both. The system was analysed and using the calculated transfer function it was also proven using matlab root locus, Impulse function and step input code that the system will be stable.

Frequency response method proved that the response of a control system can be determined by using the bode frequency response method and the derived transfer function can be used to determine the stability of the system.

The digital control system was represented using an equivalent block diagram where calculations were carried out to determine the minimum sampling frequency, which as shown in table 4 the calculated and simulated values are identical and proven using Simulink by showing the responses above, below and equal to the minimum sampling frequency proving wither they are critical stability, stable or instable at these points as proven.

Overall throughout the sections it has shown that a Control system can be calculated, Simulated and tested before implementation or maintenance and be accurate throughout, due to no discrepancies between all results. This will allow production to carry on with no interruptions and ensure the system is 100% accurate before being implemented on the plant.

8. Appendix

8.1. Transient Response of a second order system Calculations

$$K = 18$$

$$a = 4$$

Single Transfer function :-

$$\begin{aligned} T(s) &= \frac{G(s)}{1 + G(s) H(s)} \\ &= \frac{\frac{18}{s(s+4)}}{1 + \frac{18}{s(s+4)} \times 1} \\ &= \frac{18}{s(s+4) + 18} \\ &= \frac{18}{s^2 + 4s + 18} \end{aligned}$$

Natural Frequency :-

$$\begin{aligned} \omega_n^2 &= 18 \\ \omega_n &= 4.24 \\ K\omega_n^2 &= 18 \\ K &= 1 \end{aligned}$$

$$\frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Damping factor :-

$$2\zeta\omega_n = 4$$

$$\zeta = \frac{4}{2\omega_n}$$

$$\zeta = \frac{4}{2 \times 4.24}$$

$$\zeta = \frac{4}{8.48}$$

$$\zeta = 0.47$$

Damped frequency :-

$$\begin{aligned}\omega &= \omega_n \sqrt{1 - \zeta^2} \\ \omega &= 4.24 \sqrt{1 - 0.47^2} \\ \omega &= 3.85 \text{ rad/s}\end{aligned}$$

Rise time from 0 to 100% of final value :-

$$\begin{aligned}t_R &= \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)}{\omega} \\ t_R &= \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - 0.47^2}}{0.47}\right)}{3.85} \\ t_R &= 0.535 \text{ seconds}\end{aligned}$$

Peak time :-

$$\begin{aligned}t_p &= \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \\ t_p &= \frac{\pi}{3.67} \\ t_p &= 0.86 \text{ seconds}\end{aligned}$$

Percentage overshoot :-

$$\begin{aligned}100e^{-\left(\frac{\zeta\pi}{\sqrt{1 - \zeta^2}}\right)} \\ 100e^{-\left(\frac{0.47 \times \pi}{\sqrt{1 - 0.47^2}}\right)} \\ = 18.77\%\end{aligned}$$

Settling time to within +2% :-

$$\begin{aligned}ts = 2\% &= \frac{4}{\zeta\omega_n} \\ &= \frac{4}{0.47 \times 4.24} \\ &= 2.00 \text{ Seconds}\end{aligned}$$

Number of oscillations during settling time :-

$$\begin{aligned}n &= \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} \\ n &= \frac{2}{\pi} \sqrt{\frac{1}{0.47^2} - 1} \\ n &= 1.196\end{aligned}$$

8.2. Control System Modelling Calculations

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = \frac{d^2x}{dt^2} - p \frac{dx}{dt} + qx$$

Laplace Transform dy :-

$$\mathcal{L} \left\{ \frac{d^2y}{dt^2} + a \frac{dy}{dt} + by \right\} = s^2 Y(s) + as Y(s) + bY(s)$$

Laplace Transform dx :-

$$\mathcal{L} \left\{ \frac{d^2x}{dt^2} - p \frac{dx}{dt} + qx \right\} = s^2 X(s) - ps X(s) + qX(s)$$

Open loop Transfer Function :-

$$\begin{aligned} \frac{Y(s)}{X(s)} &= \frac{s^2 - ps + q}{s^2 + as + b} \\ G(s) &= \frac{s^2 - ps + q}{s^2 + as + b} \end{aligned}$$

Chosen Values :-

$$\begin{aligned} a &= 6 \\ b &= 4 \\ p &= 4 \\ q &= 6 \end{aligned}$$

$$G(s) = \frac{s^2 - 4s + 6}{s^2 + 6s + 4}$$

Zero's :-

$$a = 1$$

$$b = -4$$

$$c = 6$$

$$\begin{aligned} s &= \frac{-b \pm \sqrt{(b)^2 - 4(ac)}}{2(a)} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 6}}{2(1)} \\ &= \frac{4 \pm \sqrt{16 - 24}}{2} \\ &= \frac{4 \pm \sqrt{-8}}{2} \\ &= \frac{4 \pm j\sqrt{8}}{2} \\ &= 2 \pm j1.4142 \end{aligned}$$

Poles :-

$$a = 1$$

$$b = 6$$

$$c = 4$$

$$\begin{aligned} s &= \frac{-b \pm \sqrt{(b)^2 - 4(ac)}}{2(a)} \\ &= \frac{-6 \pm \sqrt{(6)^2 - 4 \times 1 \times 4}}{2(1)} \\ &= \frac{-6 \pm \sqrt{36 - 16}}{2} \\ &= \frac{-6 \pm \sqrt{20}}{2} \\ &= -5.236, -0.764 \end{aligned}$$

Natural frequency and Damping factor :-

$$s^2 + 6s + 4$$

$$= s^2 + 2\zeta\omega_n s + \omega_n^2$$

Therefore :-

$$\omega_n^2 = 4$$

$$\omega_n = \sqrt{4}$$

$$2\zeta\omega_n = 6$$

$$\zeta = \frac{6}{2 \times \sqrt{4}}$$

$$\zeta = 1.5$$

With Negative Feedback $H(s) = K$

$$T(s) = \frac{G(s)}{1 + H(s) \times G(s)}$$

$$= \frac{\frac{(s^2 - 4s + 6)}{(s^2 + 6s + 4)}}{1 + K \frac{(s^2 - 4s + 6)}{(s^2 + 6s + 4)}}$$

$$= \frac{s^2 - 4s + 6}{s^2 + 6s + 4 + Ks^2 - 4Ks + 6K}$$

$$= \frac{s^2 - 4s + 6}{(1 + K)s^2 + (6 - 4K)s + (4 + 6K)}$$

$$T(s) = \frac{1}{1 + K} \times \frac{(s^2 - 4s + 6)}{s^2 + \frac{(6 - 4K)s}{(1 + K)} + \frac{(4 + 6K)}{(1 + K)}}$$

Critical damping value of K

Poles :-

$$s^2 + \frac{(6-4k)s}{(1+k)} + \frac{(4+6k)}{(1+k)} = 0$$

$$\frac{-b}{2a} = 0$$

$$a = 1$$

$$b = \frac{(6-4k)}{(1+k)}$$

$$c = \frac{(4+6k)}{(1+k)}$$

$$\begin{aligned} s &= \frac{-b \pm \sqrt{(b)^2 - 4(ac)}}{2(a)} \\ &= \frac{-b \pm j\sqrt{4ac - b^2}}{2(a)} \\ &= \frac{-\left(\frac{6-4k}{1+k}\right)}{\frac{2 \times 1}{-6+4k}} = 0 \\ &= \frac{1+k}{-6+4k} = 0 \\ &= -6+4k = 0 \end{aligned}$$

Therefore Critical damping :-

$$4k = 6$$

$$k = \frac{6}{4}$$

$$k = 1.5$$

You mean critical stability

8.2.1 Matlab command, poles and zeros :-

```
1 - clc;
2 - hold off
3
4 - s= tf('s');
5 - system1 = (s^2 - 4*s + 6)/(s^2 + 6*s + 4)
6 - pole(system1)
7 - zero(system1)
8
9 - step(system1);
10 %rlocus(system1);
11 - grid on
```

Command Window

```
s^2 - 4 s + 6
-----
s^2 + 6 s + 4

Continuous-time transfer function.

ans =

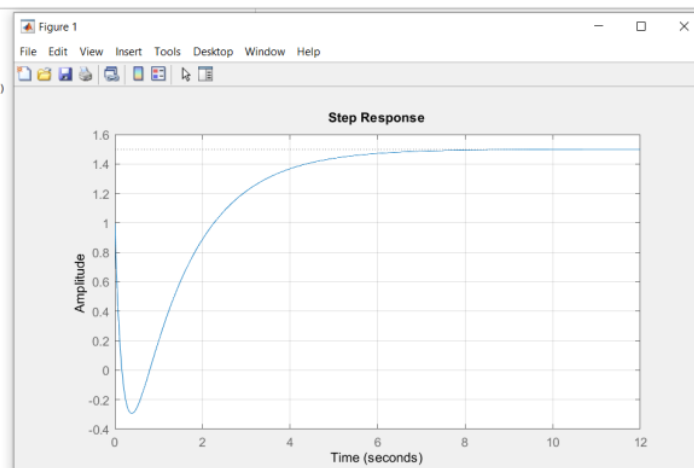
    -5.2361
    -0.7639

ans =

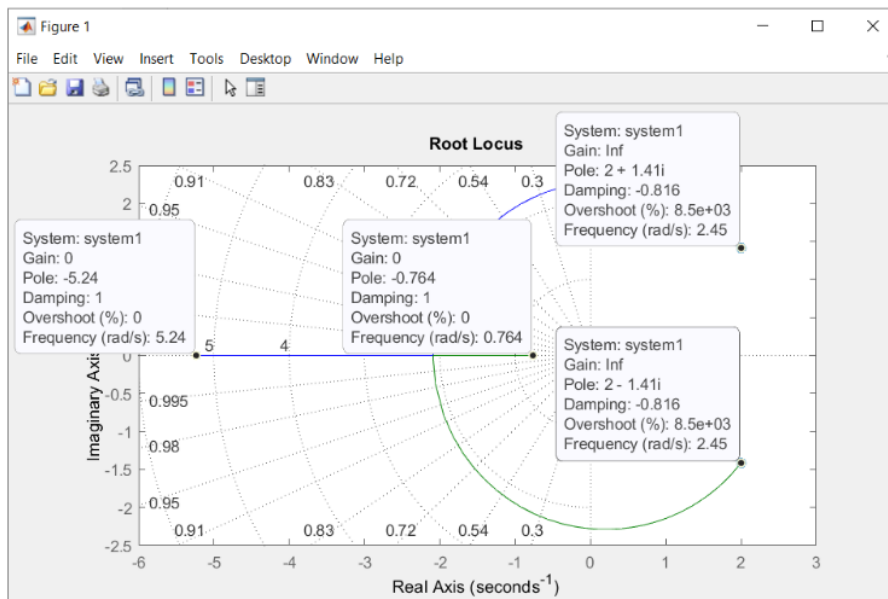
    2.0000 + 1.4142i
    2.0000 - 1.4142i
```

8.2.2 Matlab step input :-

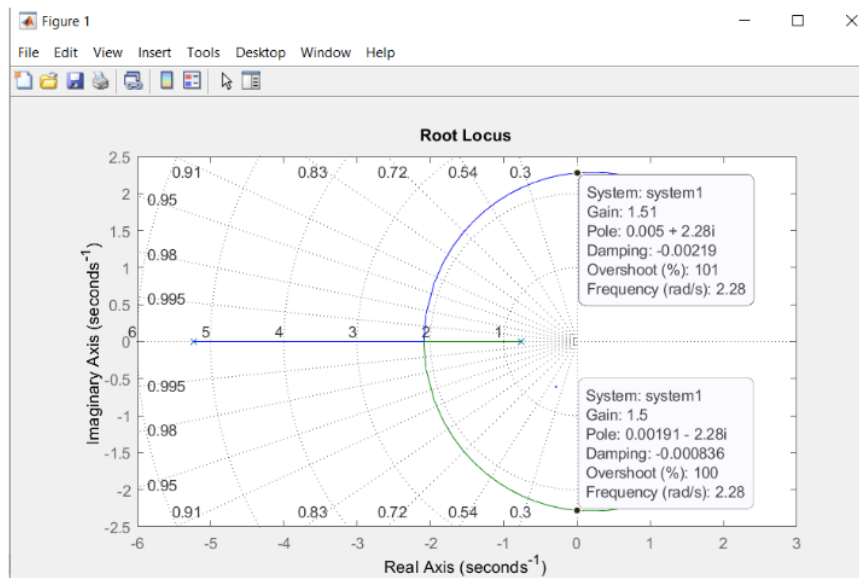
```
1 - clc;
2 - hold off
3
4 - s= tf('s');
5 - system1 = (s^2 - 4*s + 6)/(s^2 + 6*s + 4)
6 - pole(system1)
7 - zero(system1)
8
9 - step(system1);
10 %rlocus(system1);
11 - grid on
12
```



8.2.3 Matlab Root Locus :-

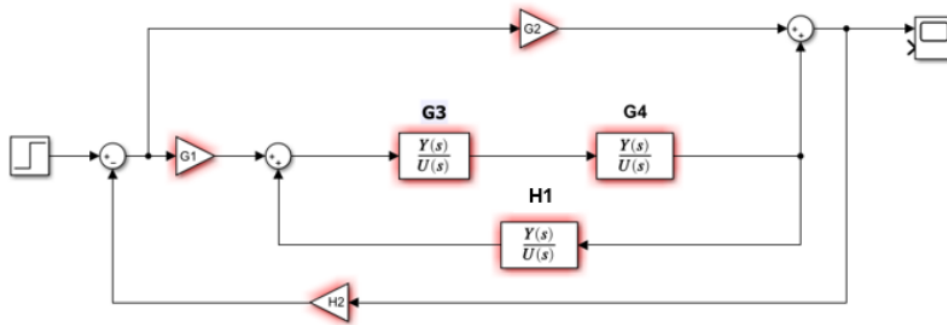


8.2.4 Root locus Critical Stability, 1.5 :-



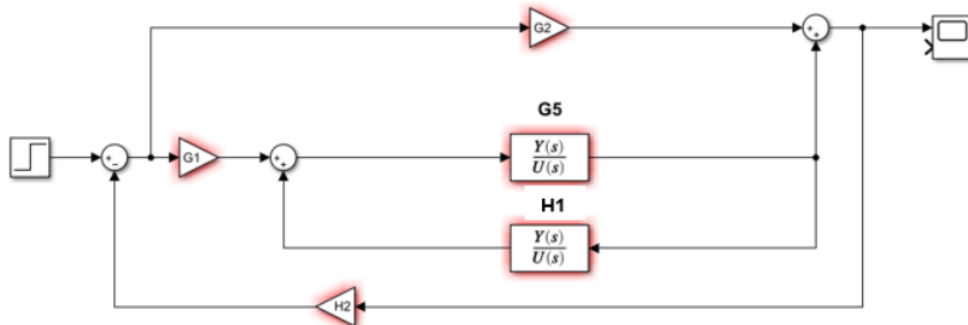
8.3. Block Diagram Reduction Method Calculations

Block Diagram :-



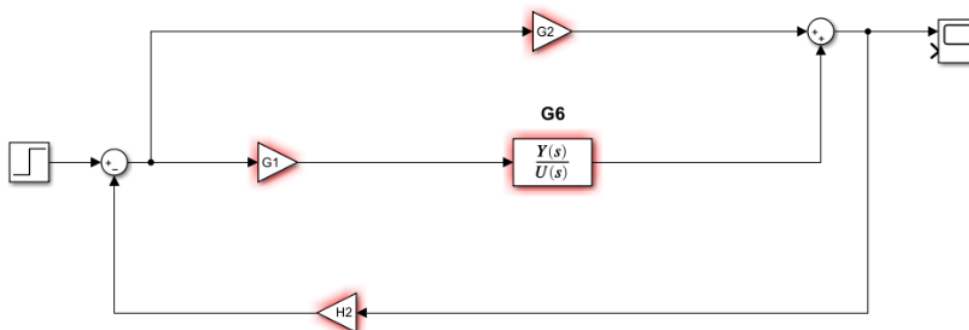
Step 1 :- G3 and G4 combined to form G5

Where $G5 = G3 \times G4$ because they are in Series



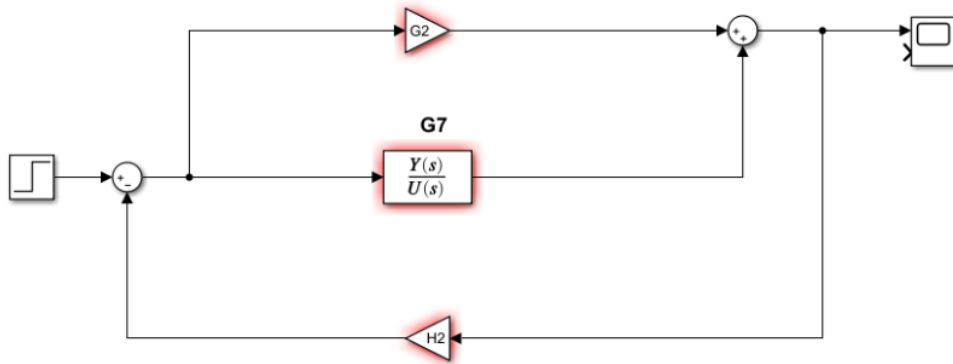
Step 2 :- G5 and H1 combined to form G6

Where $G6 = G5 / 1 - G5 H1$ as Positive feedback



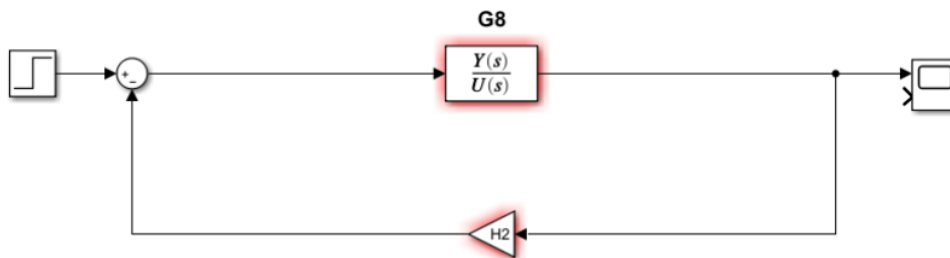
Step 3 :- G6 and G1 combine to form G7

Where $G7 = G1 \times G6$ as in series



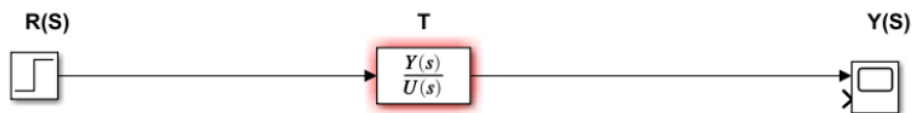
Step 4 :- G7 and G2 combine to form G8

Where $G8 = G7 + G2$ as parallel



Finally step 5 :- G8 and H2 are combined to form T

Where $T = G8 / 1 + H2 G8$ due to Negative Feedback



Expression in terms of G1,G2,G3,G4,H1 and H2 derived in terms of T

Where T equals :-

$$\frac{G8}{1 + G8 H2}$$

Substitute for G8 :-

$$T = \frac{(G7 + G2)}{1 + H2 (G7 G2)}$$

Substitute for G7 :-

$$\frac{G1 G6 + G2}{1 + H2(G1 G6 + G2)}$$

Substitute for G6 :-

$$\begin{aligned} & \frac{G1 \frac{G5}{(1 - G5 H1)} + G2}{1 + H2 \left[\frac{G1 G5}{1 - G5 H1} + G2 \right]} \\ &= \frac{G1 G5 + G2 (1 - G5 H1)}{1 - G5 H1 + H2 G1 G5 + H2 G2 (1 - G5 H1)} \end{aligned}$$

Substitute for G5 :-

$$\frac{G1 G3 G4 + G2 - G2 G3 G4 H1}{1 - G3 G4 H1 + H2 G1 G3 G4 + H2 G2 - H1 H2 G3 G4 G2}$$

T equals :-

$$T = \frac{G1 G3 G4 + G2 - G2 G3 G4 H1}{1 - G3 G4 H1 + H2 G1 G3 G4 + H2 G2 - H1 H2 G3 G4 G2}$$

Using the values for G1, G2, G3, G4, H1 and H2 shown below an expression was derived in terms of the complex frequency variables 's' as shown below :-

$$G1 = a = 8$$

$$G4 = 1/s$$

$$G2 = b = 6$$

$$H1 = 1/(s+1)$$

$$G3 = c/(s+1) = 3/(s+1)$$

$$H2 = d = 14$$

$$T = \frac{G1 G3 G4 + G2 - G2 G3 G4 H1}{1 - G3 G4 H1 + H2 G1 G3 G4 + H2 G2 - H1 H2 G3 G4 G2}$$

$$T = \frac{8 \times \frac{3}{(s+1)} \times \frac{1}{s} + 6 - 6 \times \frac{3}{(s+1)} \times \frac{1}{s} \times \frac{1}{(s+1)}}{1 - \frac{3}{(s+1)} \times \frac{1}{s} \times \frac{1}{(s+1)} + 14 \times 8 \times \frac{3}{(s+1)} \times \frac{1}{s} + 14 \times 6 - \frac{1}{(s+1)} \times 14 \times \frac{3}{(s+1)} \times \frac{1}{s} \times 6}$$

$$T = \frac{\frac{24}{s(s+1)} + 6 \frac{-18}{s(s+1)^2}}{1 - \frac{3}{s(s+1)^2} + \frac{336}{s(s+1)} + 84 - \frac{252}{s(s+1)^2}}$$

$$T = \frac{24(s+1) + 6s(s+1)^2 - 18}{s(s+1)^2 - 3 + 336(s+1) + 84s(s+1)^2 - 252}$$

$$T = \frac{24s + 24 + 6s^3 + 6s^2 + 6s^2 + 6s - 18}{s^3 + s^2 + s^2 + s - 3 + 336s + 336 + 84s^3 + 84s^2 + 84s^2 + 84s - 252}$$

$$T = \frac{6s^3 + 12s^2 + 30s + 6}{85s^3 + 170s^2 + 421s + 81}$$

8.3.1 Matlab command window, poles and zeros

```
1 - num= [6 12 30 6];  
2 - den= [85 170 421 81];  
3 - [z,p,k]=tf2zp(num,den)  
4 - rlocus(num,den)  
5 - grid on  
6  
7  
8
```

Command Window

z =

-0.8916 + 1.9541i
-0.8916 - 1.9541i
-0.2168 + 0.0000i

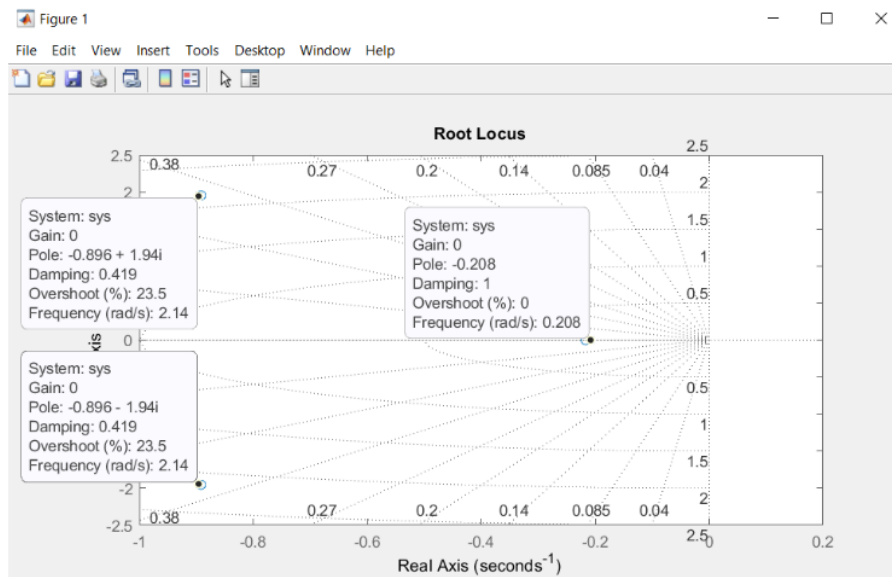
p =

-0.8960 + 1.9435i
-0.8960 - 1.9435i
-0.2081 + 0.0000i

k =

0.0706

8.3.2 Matlab Root locus



8.4. Frequency Response Method Calculations

$$P = 2$$

$$q = 10$$

$$r = 100$$

$$K1 = 1800$$

$$G(s) = \frac{K1 (S + P)}{(S + q) (s + r)}$$

Replace S with $j\omega$:-

$$G(j\omega) = \frac{K1 (j\omega + P)}{(j\omega + q) (j\omega + r)}$$

In standard form, $G(j\omega)$ becomes :-

$$G(j\omega) = \frac{K1 P (1 + j\omega/p)}{q(1 + j\omega/q) r (1 + j\omega/r)}$$

$$G(j\omega) = \frac{K1 P}{q r} \times \frac{(1 + \frac{j\omega}{p})}{(1 + \frac{j\omega}{q}) (1 + \frac{j\omega}{r})}$$

The overall gain is :-

$$K = \frac{K1 p}{q r}$$

$$K = \frac{1800 \times 2}{10 \times 100}$$

$$K = \frac{3600}{1000}$$

$$K = 3.6$$

Therefore the Log of K :-

$$20 \log_{10}(3.6) = 11.12$$

Transfer function :-

$$T(s) = \frac{1800(s + 2)}{(s + 10)(s + 100)}$$

$$T(s) = \frac{1800s + 3600}{s^2 + 110s + 1000}$$

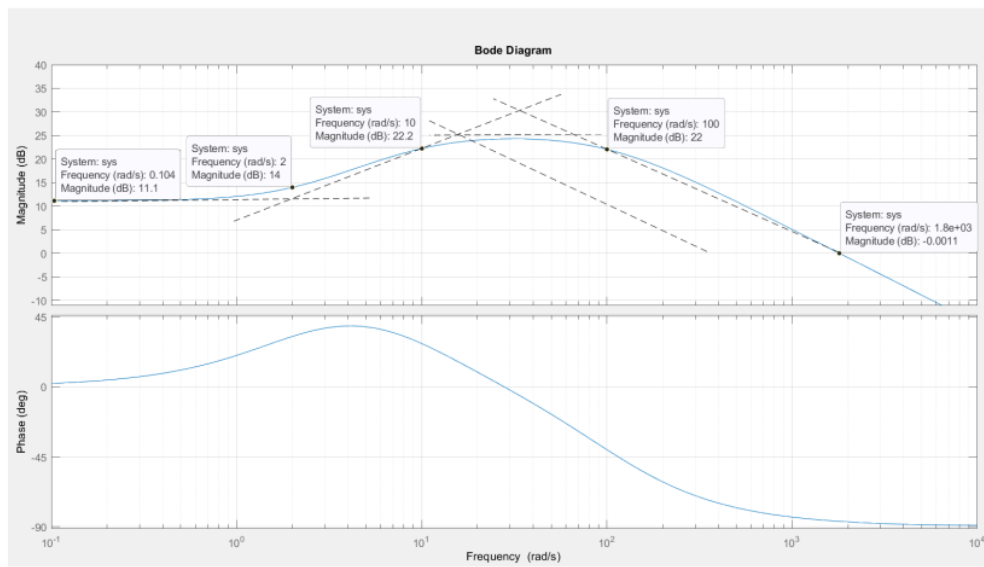
Matlab code :-

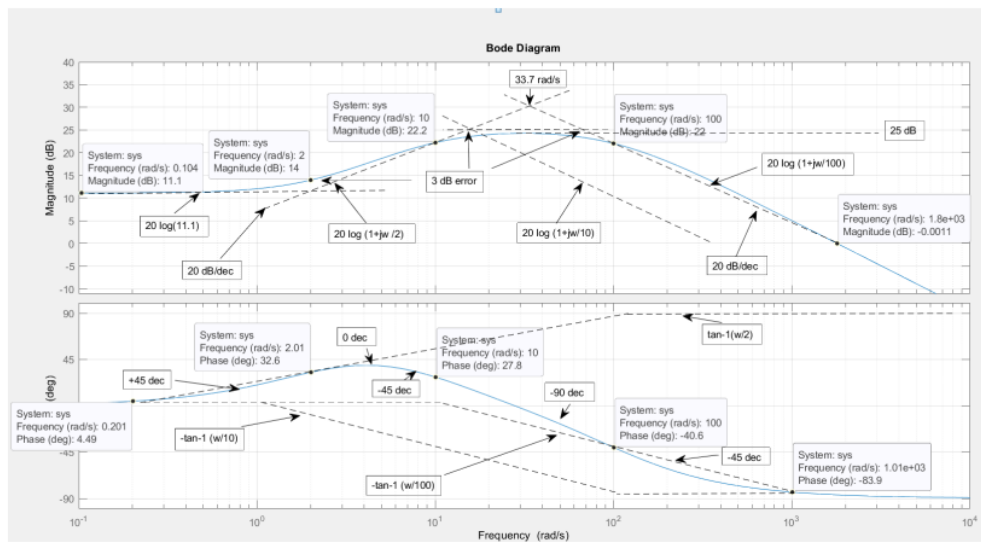
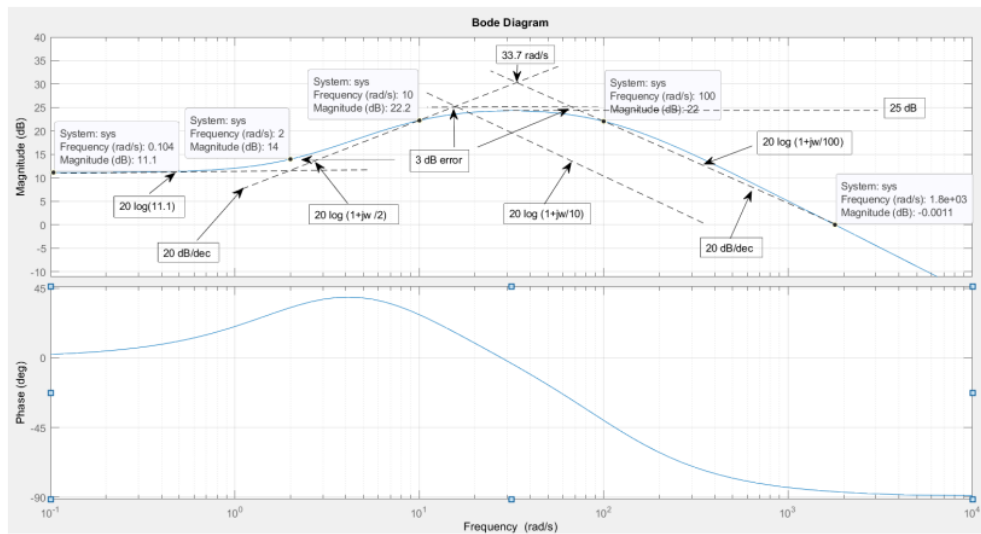
```
Num [1800 3600]
```

```
Den [1 110 1000]
```

```
Bode (num, den)
```

8.4.1 Bode frequency response magnitude and phase graphs/measurements





8.5. Digital Control Systems Calculations

$$\frac{50}{(s+1)}$$

ZOH Laplace transform :-

$$\frac{1 - e^{-st}}{s}$$

Open loop transfer function :-

$$G(s) = \frac{1 - e^{-st}}{s} \times \frac{50}{(s+1)}$$

Z transforms :-

$$\begin{aligned} G(z) &= Z\{G(s)\} \\ &= (1 - z^{-1})50 z \left\{ \frac{1}{s(s+1)} \right\} \end{aligned}$$

Partial fractions method :-

$$\begin{aligned} \frac{1}{s(s+1)} &= \frac{A}{s} + \frac{B}{s+1} \\ \frac{1}{s(s+1)} &= \frac{A(s+1) + B(s)}{s(s+1)} \end{aligned}$$

$$A(s+1) + B(s) = 1$$

When $s = 0$:-

$$\begin{aligned} s &= 0 \\ A(0+1) + B(0) &= 1 \\ A &= 1 \end{aligned}$$

When $s = -1$:-

$$\begin{aligned} s &= -1 \\ A(-1+1) + B(-1) &= 1 \\ -B &= 1 \\ B &= -1 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{s(s+1)} \\
 &= \frac{1}{s} - \frac{1}{(s+1)} \\
 & Z \left\{ \frac{1}{s(s+1)} \right\} \\
 &= Z \left\{ \frac{1}{s} - \frac{1}{(s+1)} \right\} \\
 &= 1 \left\{ \frac{z}{z-1} - \frac{z}{z-e^{-1t}} \right\}
 \end{aligned}$$

Therefore :-

$$\begin{aligned}
 G(Z) &= (1-z^{-1}) \times 50 \times 1z \left\{ \frac{z-e^{-1t}-z+1}{(z-1)(z-e^{-1t})} \right\} \\
 &= \frac{50(1-e^{-1t})}{z-e^{-t}}
 \end{aligned}$$

Using the closed loop transfer Function :-

$$\begin{aligned}
 T(z) &= \frac{G(z)}{1+G(z)} \\
 &= \frac{\frac{50(1-e^{-t})}{(z-e^{-t})}}{1+\frac{50(1-e^{-t})}{z-e^{-t}}} \\
 &= \frac{50(1-e^{-t})}{z-e^{-t}+50(1-e^{-t})} \\
 &= \frac{50(1-e^{-t})}{z+50-51e^{-t}}
 \end{aligned}$$

Poles found :-

$$Z + 50 - 51e^{-t} = 0$$

Therefore :-

$$Z = 51e^{-t} - 50$$

Stability :-

$$-1 < Z < +1$$

For $Z = +1$

$$51e^{-t} - 50 = 1$$

$$e^{-t} = \frac{51}{51} = 1$$

log base of e :-

$$\begin{aligned} -T &= \ln(1) \\ -T &= 0 \\ T &= 0 \end{aligned}$$

For $Z = -1$

$$51e^{-t} - 50 = -1$$

$$e^{-t} = \frac{50 - 1}{51} = 0.96078$$

Log e :-

$$\ln(0.96078) = -0.04$$

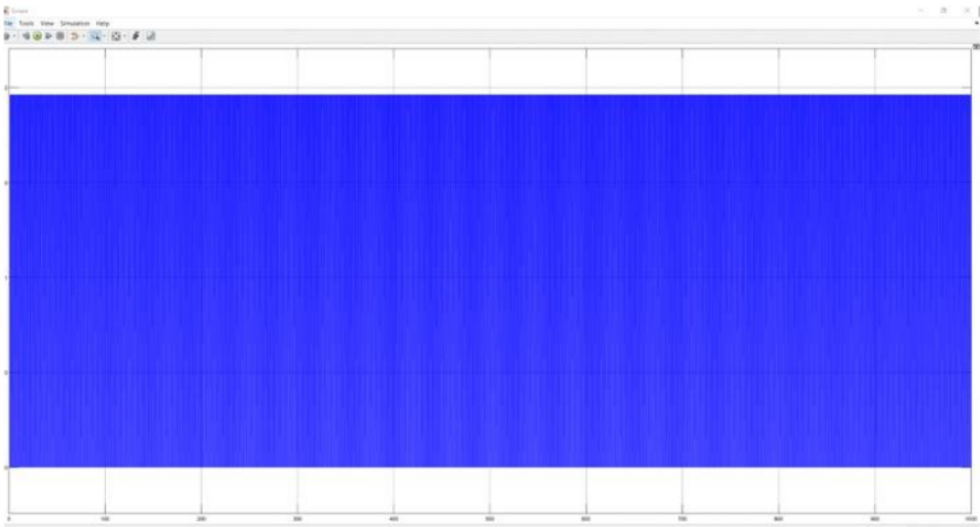
$$T = 0.04$$

$$T = 40 \text{ ms}$$

Sampling frequency :-

$$\begin{aligned} &\frac{1}{0.04} \\ &= 25 \text{ Hz} \end{aligned}$$

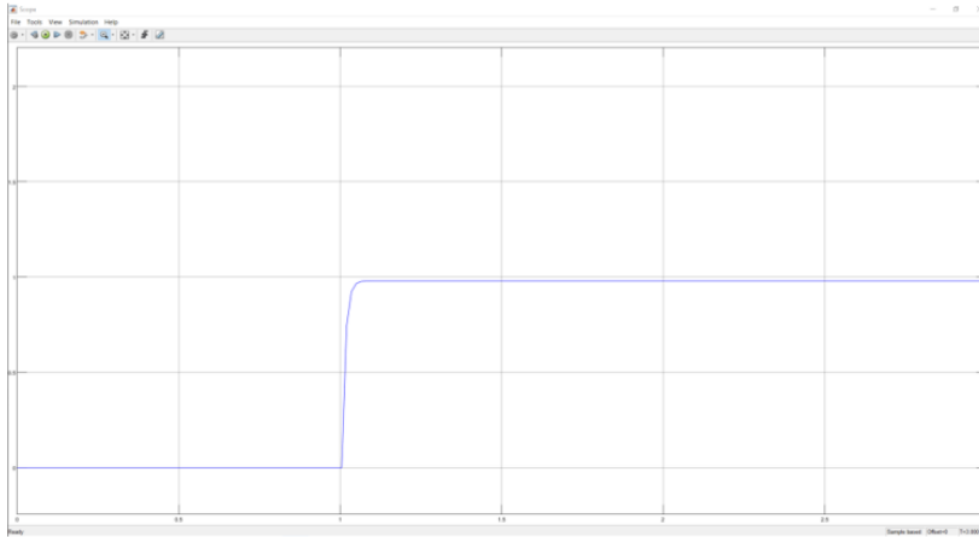
8.5.1 Simulink model, sampling at calculated frequency showing critical stability



Cursor Measurements			
Settings			
Measurements			
	Time	Value	
1	0.040	0.000e+00	
2	0.000e+00	0.000e+00	
ΔT	40.000 ms	ΔY	0.000e+00
	$1 / \Delta T$	25.000 Hz	
	$\Delta Y / \Delta T$	0.000 (/s)	

8.5.2 Simulink model, sampling at above the calculated frequency showing stability

Reduced to 0.15 to show stability



8.5.2 Simulink model, sampling at above the calculated frequency showing unstable

Increased to 65ms to show instability



1809812

GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

Instructor

67/100

PAGE 1

PAGE 2

PAGE 3

PAGE 4

Text Comment. section 5

PAGE 5

PAGE 6

PAGE 7

Text Comment. Your Z's are poles and your P's are zeros ??????

PAGE 8

Text Comment. Your Z's are poles and your P's are zeros ??????

PAGE 9

PAGE 10

Text Comment. what about critical damping?

PAGE 11

PAGE 12

PAGE 13

PAGE 14

PAGE 15

PAGE 16

PAGE 17

PAGE 18

Text Comment. excellent analysis

Text Comment. good conclusions

PAGE 19

PAGE 20

Text Comment. These are mainly a description or summary of what was done confirming the answers were correct. However, it has few conclusions or recommendation that show what you have learnt from the assignment.

PAGE 21

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PAGE 26

Text Comment. You mean critical stability

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PAGE 40

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GRADING FORM: NEW CSG-RUBRIC

LIAM GEORGE

67

1. ENGAGEMENT WITH LITERATURE



N/A

2. KNOWLEDGE AND UNDERSTANDING



Knowledge is reasonably extensive. Exhibits very competent critical understanding of the well-established concepts and principles of the subject and the main methods of enquiry. Breadth and depth of knowledge. 13/20

13

3. COGNITIVE AND INTELLECTUAL



Sound critical evaluation and analysis of concepts. Is selective in the range of evidence used and synthesises rather than describes. Ability to devise arguments that show awareness of different stances, and use evidence convincingly, to support appropriate and valid judgements. (19/30)

19

4. PRACTICAL APPLICATION



An advanced application of a range of established methods, materials, tools and/or techniques. The context of the application is well considered, and insightful. Consistent, accurate and logical application of theory to practice, making well-developed links between the two. Can identify problems and propose excellent, creative solutions outside the context in which they were first studied. (21/30)

21

5. TRANSFERABLE SKILLS



Can communicate professionally confidently and consistently in a suitable medium for the audience. Work is coherent, very fluent and is presented proficiently. Can work autonomously with initiative. Where relevant can work professionally within a team, showing leadership skills as appropriate, and meeting obligations. Demonstrates excellent skills for employment requiring the exercise of personal responsibility and decision-making and an appetite for further development. (14/20)

14