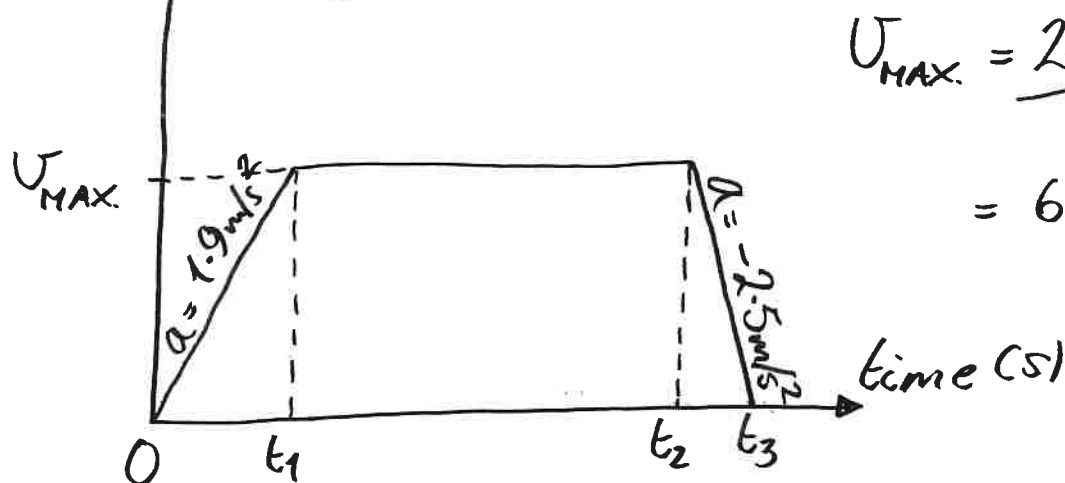


Q1(a) Velocity (m/s)



$$U_{MAX} = \frac{24 \times 1000}{3600} = 6.67 \text{ m/s}$$

(4 marks)

(b) Using, $V = u + a.t$, calculate t_1 and t_3 ;

$$t_1 = \frac{V - u}{a} = \frac{6.67 - 0}{1.9} = 3.51 \text{ s}$$

$$t_3 = \frac{0 - 6.67}{-2.5} = 2.67 \text{ s}$$

Now, distances covered; using $V^2 = u^2 + 2as$

$$\therefore S_1 = \frac{V^2 - u^2}{2a} = \frac{6.67^2 - 0^2}{2 \times 1.9} = 11.696 \text{ m}$$

$$\text{and, } S_3 = \frac{0^2 - 6.67^2}{2 \times (-2.5)} = 8.889 \text{ m}$$

So, distance covered for constant velocity section;

$$S_2 = 275 - (11.696 + 8.889) = 254.415 \text{ m}$$

Now, time for constant velocity section;

$$\text{Distance} = \text{Velocity} \times \text{time}$$

$$\therefore \text{Time} = \frac{254.415}{6.67} = 38.16 \text{ s}$$

So, fastest time to travel distance is;

$$\text{Time} = 3.51 + 2.67 + 38.16 = 44.34 \text{ s}$$

Assume: Smooth & level ground, no skidding, no stopping, etc. (6 marks).

Q1(c).

$$\text{Average velocity} = \frac{\text{Distance}}{\text{Time}} = \frac{275}{44.34} = \underline{6.2 \text{ m/s.}}$$

Kinetic energy, $K.E. = \frac{1}{2}mv^2$, where v is average velocity.

$$\therefore K.E. = \frac{1}{2}(450)(6.2)^2 = \underline{8654.79 \text{ J}}$$

(4 marks)

(d)

Applying principle of conservation of energy;
 $K.E. = P.E.$

$$\text{So, } \frac{1}{2}mv^2 = mgh \Rightarrow h = \frac{v_{\text{max}}^2}{2g} = 2.265 \text{ m}$$

Now, 'h' is vertical height gained by trolley,
so distance travelled up incline;

$$x = \frac{h}{\sin \theta} \text{ where, } \sin \theta = \frac{1}{25} = 0.04$$

$$\therefore x = \frac{2.265}{0.04} = \underline{56.632 \text{ m.}}$$

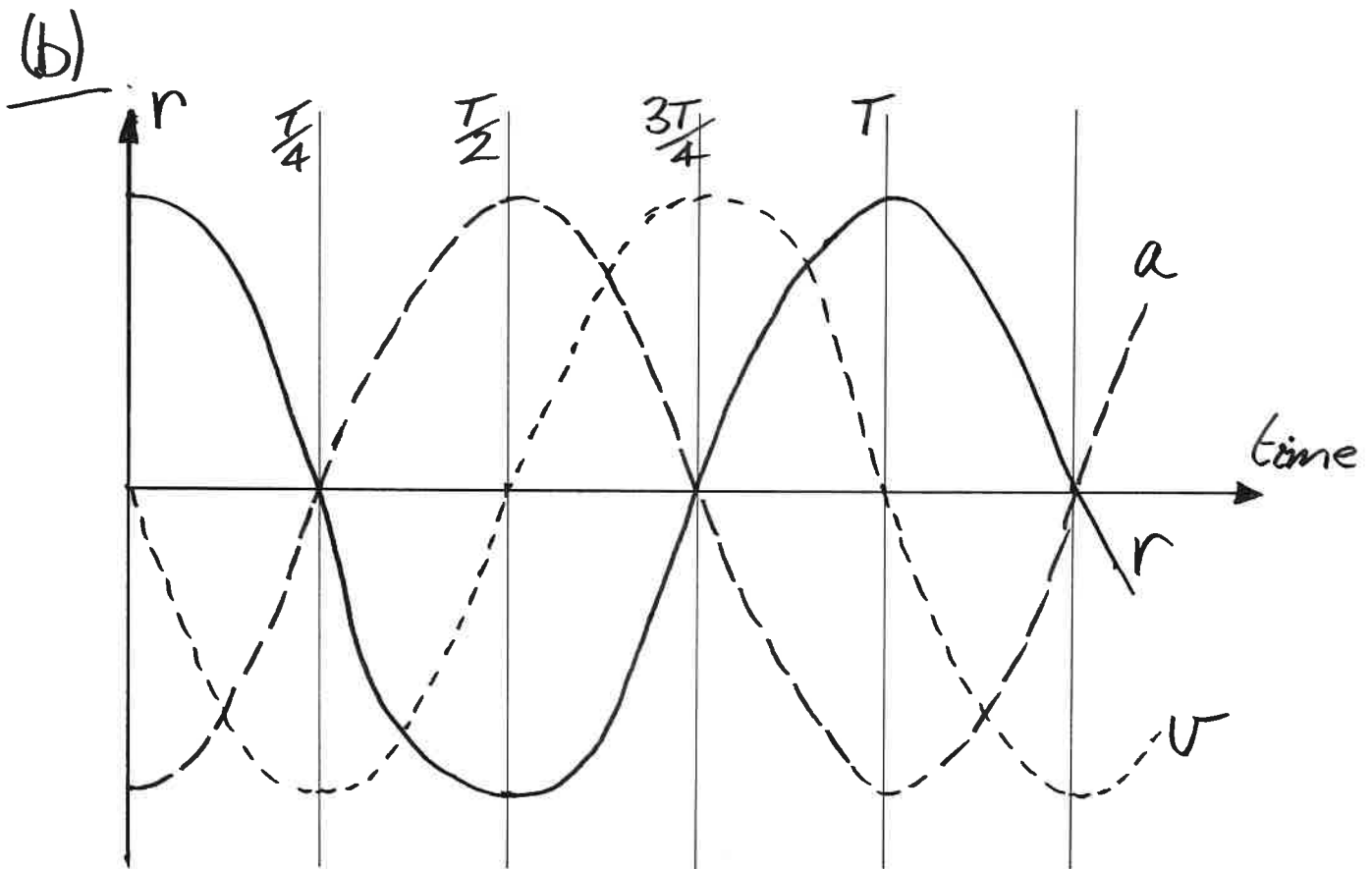
Assume: Incline is smooth, drag neglected,
no friction accounted for, etc.

(6 marks)

Q2(a)

- Acceleration of body is always directed towards a fixed point in its path, and
- magnitude of the acceleration of the body is proportional to its distance from the fixed point.

(2 marks)



Acceleration, $a = \omega^2 r$

Velocity, $v = \omega r$

Displacement, $= r$

(4 marks)

Q2(c).

(i) Determine amplitude of oscillation from;

$$v = \omega \sqrt{(r^2 - x^2)}$$

$$\text{So, } 15 = \omega \sqrt{(r^2 - 0.0064)} \quad \text{--- ①}$$

$$3 = \omega \sqrt{(r^2 - 0.0256)} \quad \text{--- ②}$$

Divide ① by ② to eliminate ' ω ', and square both sides;

$$25 = \frac{r^2 - 0.0064}{r^2 - 0.0256}$$

$$25r^2 - 0.64 = r^2 - 0.0064$$

$$\therefore r = \sqrt{\frac{0.6336}{24}} = \underline{0.1625\text{m}} \quad (r = 162.5\text{mm}) \quad (8\text{ marks})$$

To calculate frequency, find angular velocity ω ;

Using Equation ② above, $3 = \omega \sqrt{0.02641 - 0.0256}$

$$\therefore \omega = 105.07 \text{ rad/s.}$$

$$\text{So, frequency of motion, } f = \frac{\omega}{2\pi}$$

$$\therefore \underline{f = 16.88 \text{ Hz}} \quad (3\text{ marks})$$

(d)

Due to very high accelerations/decelerations of pistons, high forces would be experienced for even moderate masses, hence high material stresses experienced. Aluminium alloy adopted to attempt to reduce high stresses in these pistons. (3 marks)

Q3(a).

Knowing that fundamental equation for torque is;

$$T = I\alpha \quad \text{hence, necessary to find both moment of inertia (I), and angular acceleration (\alpha).}$$

Assuming flywheel is solid disc construction, then; $I = \frac{1}{2} Mr^2$, where $M = \text{Volume} \times \text{Density}$

Assuming solid steel shaft passing through p.d. axis;

$$\text{Volume of flywheel} = \pi \cdot r^2 \cdot t$$

$$\therefore \text{Mass, } M = \pi \cdot (0.325)^2 \cdot (0.055) \times 7800$$

$$M = 142.36 \text{ kg}$$

$$\text{Now, } I = 0.5 (142.36) (0.325)^2 = 7.518 \text{ kg m}^2$$

For constant linear acceleration of flywheel;

$$\omega_2 = \omega_1 + \alpha t \Rightarrow \alpha = \frac{\omega_2 - \omega_1}{t}$$

$$\text{but, } \omega_2 = \frac{2\pi \times 250 \text{ rpm}}{60} = 26.18 \text{ rad/s}$$

$$\therefore \alpha = \frac{26.18 - 0}{40} = 0.6545 \text{ rad/s}^2$$

Hence, accelerating torque, $T = 7.518 \times 0.6545$

$$\underline{T = 4.921 \text{ Nm}}$$

Assume also: no drag on rotating flywheel, no loss of transmitted accelerating torque, etc.
(8 marks).

Q3(b)

'New' available torque, $T = 4.921 - 0.75$
 $= 4.171 \text{ Nm}$

Knowing $T = I \cdot \alpha$, $\therefore \alpha = \frac{T}{I}$

So, $\alpha = \frac{4.171}{7.518} = ~~0.555~~ 0.555 \text{ rad/s}^2$
(4 marks)

(c)(i)

Retardation caused by frictional torque;

So, $T = I \cdot \alpha \Rightarrow \alpha = \frac{0.75}{7.518} = 0.0998 \text{ rad/s}^2$
(2 marks)

(ii) Using, $\omega_2 = \omega_1 + \alpha t \therefore t = \frac{\omega_2 - \omega_1}{\alpha}$

So, time to come to rest, $t = \frac{0 - 26.18}{-0.0998} = 262.435$
(3 marks)

(iii) Using, $\omega_2^2 = \omega_1^2 + 2\alpha\theta \therefore \theta = \frac{\omega_2^2 - \omega_1^2}{2\alpha}$

So, $\theta = \frac{0^2 - 26.18^2}{(2 \times -0.0998)} = 3435.19 \text{ rad}$

Hence, number of revolutions, $n = 546.73 \text{ revs.}$
(3 marks)

Q4(a)

Contact between surfaces only at a few points due to surface roughness. These small areas bear load between surfaces, and in metals, plastic deformation occurs at points of contact due to high pressures, hence causing cold-welding of the metals.

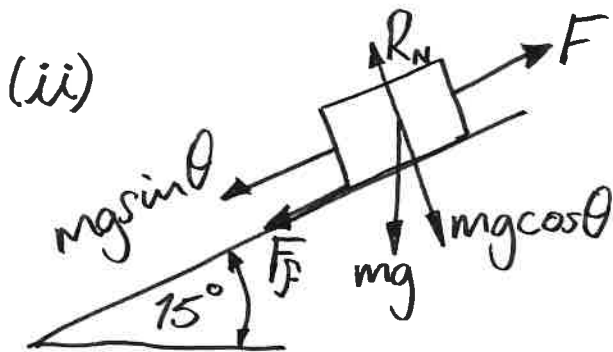
(3 marks)

(b)(i)

$$\text{Friction force, } F = \mu_s R_N \Rightarrow \mu_s = \frac{F}{R_N} = \frac{F}{mg}$$

$$\text{so, } \mu_s = \frac{1.15 \times 10^3}{350 \times 9.81} = \underline{0.335}.$$

(4 marks)



Resolving forces perpendicular to the plane, indicates zero net force, i.e.

$$R_N = mg \cos \theta = 0.$$

Resolving forces parallel to inclined plane;

$$F = mg \sin \theta + F_f \quad \text{where, } F_f = \mu_s R_N$$

(friction force)

$$= mg \sin \theta + \mu_s R_N \quad \text{where, } R_N = mg \cos \theta$$

$$\therefore F = mg \sin \theta + \mu_s mg \cos \theta$$

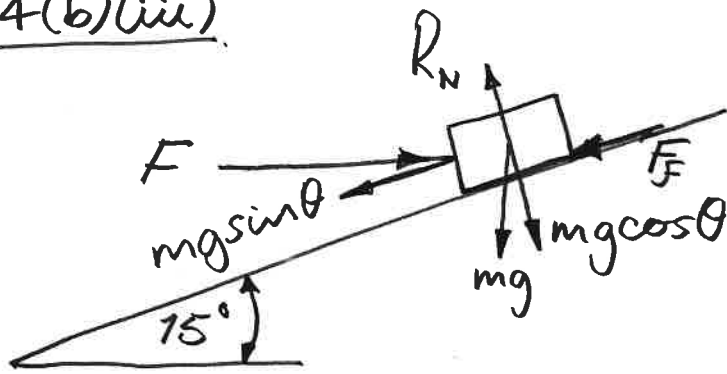
$$= (350 \times 9.81 \times \sin 15^\circ) + (0.335 \times 350 \times 9.81 \times \cos 15^\circ)$$

$$\text{So, } F = 888.66 + 1111.03 \text{ N}$$

$$F = \underline{1999.69 \text{ N}} \quad (\text{i.e. approx. } 2 \text{ kN})$$

(6 marks)

Q4(b)(iii).



Resolve F into perpendicular ($F \sin \theta$) and parallel ($F \cos \theta$) components.

For components of forces perpendicular to plane;

$$F \sin \theta + mg \cos \theta = R_N \text{ ————— ①}$$

For components of forces parallel to plane;

$$F \cos \theta = mg \sin \theta + F_f \text{ where, } F_f = \mu_s R_N$$

$$\text{So, } F \cos \theta = mg \sin \theta + \mu_s R_N \text{ ————— ②}$$

Substituting equation ① into ② for R_N ;

$$F \cos \theta = mg \sin \theta + \mu_s (F \sin \theta + mg \cos \theta)$$

$$\therefore F = \frac{mg (\sin \theta + \mu_s \cos \theta)}{(\cos \theta - \mu_s \sin \theta)}$$

$$= (350 \times 9.81) \cdot \frac{(\sin 15^\circ + 0.335 \cos 15^\circ)}{(\cos 15^\circ - 0.335 \sin 15^\circ)}$$

$$\text{So, } F = \underline{2274.38 \text{ N}}$$

(7 marks).