

MATHEMATICS (TEST 2)

1(a) (i) $x^2 - 12x + 37 = 0$

$a = 1 \quad b = -12 \quad c = 37$

given $x = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(37)}}{2}$

$x = \frac{12 \pm \sqrt{-4}}{2}$

$\sqrt{-4} = 2j$

$\therefore$ roots are $x = \frac{12 \pm 2j}{2}$

i.e. $x = 6 + j$ and $x = 6 - j$

(ii) $4 + 5j$

1(b) (i) $\frac{-4 + 8j}{2} + \frac{2 + 6j}{2}$

i.e. $\frac{-2 + 4j + 1 + 3j}{1}$

(iii) $(-4 + 8j - 1 - 3j)(-4 + 8j + 1 + 3j)$
 $(-5 + 5j)(-3 + 11j)$
 $15 - 55j - 15j + 55j^2$
 $-40 - 70j$

(iii) $(-4 + 8j)(-4 + 8j) + (2 + 6j)(2 + 6j) + (-1 - 3j)(-1 - 3j)$
 $16 - 32j - 32j + 64j^2$
 $4 + 12j + 12j + 36j^2$
 $1 + 3j + 3j + 9j^2$
 adding gives $-58 - 34j$

(iv) $(2 + 6j) \times (-1 + 3j)$
 $(-1 + 3j)(-1 + 3j)$
 $-2 + 6j - 6j + 18j^2$
 $1 + 3j - 3j - 9j^2$

$\frac{-20}{10}$

i.e. $\frac{-2}{1}$

(3)

(V)

$$\frac{2(-4 + 8j) - 3(-1 - 3j)}{4(2 + 6j) + (-4 + 8j)}$$

reduces to $\frac{-5 + 25j}{4 + 32j}$

Multiply numerator & denominator by (their) conjugate of denominator

to give $\frac{780 + 260j}{1040}$

1
1

1

1
1

(6)

20

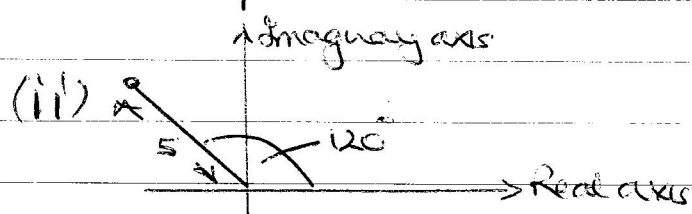
C (i) r is the modulus

the length of the line from the origin to the complex no

θ is the argument

the angle turned through in an anticlockwise direction from the positive real axis

(2)

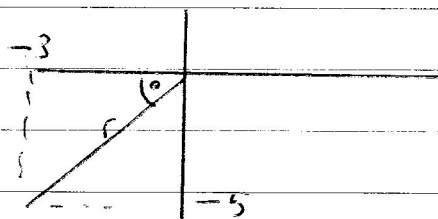


Axes labelled correctly

Correct position

(2)

(iii) $-3 - 5j$



$$r^2 = 3^2 + 5^2$$

$$r = \sqrt{34} \text{ (or } 5.83)$$

$$\tan \theta = \frac{5}{3}$$

$$\therefore \theta = 59^\circ$$

$$\theta = 180 + 59$$

$$\theta = 239^\circ$$

$\therefore$ In Polar form

$$5.83 (\cos 239^\circ + j \sin 239^\circ)$$

$$\text{or } 5.83 / 239^\circ$$

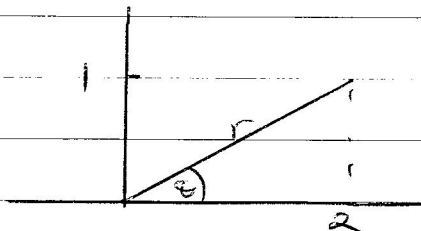
(3)

7

$$\begin{aligned}
 &1 \text{ (d)} \quad (2+j)(2+j)(2+j) \\
 &\quad (4+2j+2j+j^2)(2+j) \\
 &\quad (3+4j)(2+j) \\
 &\quad 2+11j
 \end{aligned}$$

1
(2)

Firstly converting $2+j$ into polar form



$$\begin{aligned}
 r^2 &= 2^2 + 1^2 \\
 r &= \sqrt{5} \quad (2.24)
 \end{aligned}$$

$$\tan \theta = \frac{1}{2}$$

$$\therefore \theta = 26.56^\circ$$

$$\text{giving } \sqrt{5} (\cos 26.56^\circ + j \sin 26.56^\circ)$$

Applying De Moivre's Theorem to give

$$\begin{aligned}
 &\sqrt{5}^3 (\cos(3 \times 26.56^\circ) + j \sin(3 \times 26.56^\circ)) \\
 &\quad (\text{i.e. } 2+11j)
 \end{aligned}$$

1
(5)

7

$$\begin{aligned}
 2 \text{ (a)} \quad 2x + 4 &= -7 \\
 2x &= -7 - 4 \\
 2x &= -11 \\
 \therefore x &= -5.5
 \end{aligned}$$

1

1

$$y^2 = 49$$

$$\therefore y = +7 \text{ and } y = -7$$

1
1

$$\frac{2z-1}{5} = \frac{z}{3}$$

$$3(2z-1) = 5z$$

$$6z-3 = 5z$$

$$z = 3$$

1

1

6

$$\begin{aligned}
 2(6) \quad & x(5x-8) - 3(25+12) - 2(-10-3x) = 0 \\
 & 5x^2 - 8x - 75 - 36 + 20 + 6x = 0 \\
 & \underline{5x^2 - 2x - 91 = 0}
 \end{aligned}$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(-91)}}{10}$$

$$x = \frac{2 \pm \sqrt{1824}}{10}$$

gives $x = 4.5$ and $x = -4.1$

$$2C \quad (i) \quad P \& \quad \text{Impossible} \quad 1$$

$$(ii) \quad P^2 \quad \text{Impossible} \quad 1$$

$$(iii) \quad Q^2 = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix} \quad 2$$

$$\begin{aligned}
 (iv) \quad R^2 &= \begin{pmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 5 & -1 & -3 \end{pmatrix} \begin{pmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 5 & -1 & -3 \end{pmatrix} \\
 &= \begin{pmatrix} 34 & -11 & -6 \\ 0 & 9 & 0 \\ -5 & -5 & 39 \end{pmatrix} \quad 3
 \end{aligned}$$

$$(v) \quad PR \quad \text{Impossible} \quad 1$$

$$\begin{aligned}
 (vi) \quad RP &= \begin{pmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 5 & -1 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ -3 \\ -5 \end{pmatrix} \\
 &= \begin{pmatrix} -29 \\ -9 \\ 13 \end{pmatrix} \quad 2
 \end{aligned}$$

$$2(c) \quad (vii) \quad P^T = \begin{pmatrix} -1 & -3 & -5 \end{pmatrix} \quad 1$$

$$(cont) \quad (viii) \quad Q^T = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix} \quad 1$$

$$(ix) \quad P^T R^T$$

$$R^T = \begin{pmatrix} 2 & 0 & 5 \\ -1 & 3 & -1 \\ 6 & 0 & -3 \end{pmatrix} \quad 1$$

$$\therefore P^T R^T = \begin{pmatrix} -29 & -9 & 13 \end{pmatrix} \quad 2$$

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2(d) Equations in Matrix form

$$\underbrace{\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & -3 \\ -3 & 4 & 7 \end{pmatrix}}_A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ -7 \end{pmatrix} \quad 1$$

$$|A| = 1(7+12) - 2(21-9) + 3(12+3) \\ = \underline{40} \quad 2$$

$$|A_1| = -5(7+12) - 2(28-21) + 3(16+7) \\ = \underline{-40} \quad 2$$

$$|A_2| = 1(28-21) + 5(21-9) + 3(-21+12) \\ = \underline{40} \quad 2$$

$$|A_3| = 1(-7-16) - 2(-21+12) - 5(12+3) \\ = \underline{-80} \quad 2$$

$$\therefore x = -1 \quad y = 1 \quad z = -2 \quad 1$$

10

$3(a) (i) \left[4\frac{x^4}{4} - 6\frac{x^3}{3} + \frac{8x^2}{2} - x \right]_0^2$ $= \underline{\underline{14}}$	1111
$(ii) \left[\frac{e^{2x}}{2} - e^x + 3x \right]_0^1$ $= \underline{\underline{4.47}}$	(5) 111
$(iii) 4\left(-\frac{1}{2}\cos 2x\right) - 3\left(\frac{1}{4}\sin 4x\right) + c$	(4) 111 (3) 12

3(b) $\int x \sin 4x \, dx$

Let $u = x$
 $\therefore \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = \sin 4x$

$v = -\frac{1}{4}\cos 4x$

Applying Parts formula.

$$\int x \sin 4x \, dx = \frac{-x \cos 4x}{4} - \int \frac{-1 \cos 4x}{4} \, dx$$

$$= \frac{-x \cos 4x}{4} + \frac{1 \sin 4x}{16} + C$$

3(c) $u = 5x^2 - 4$

$\therefore \frac{du}{dx} = 10x$

$\therefore dx = \frac{du}{10x}$

$\therefore \int \frac{10x}{u} \cdot \frac{du}{10x}$

$\int \frac{1}{u} \, du$

$\ln u$

$$3(c) \quad \left[\ln(5x^2 - 4) \right]_1^4$$

(continued)

$$\left[\ln 16 - \ln 1 \right]$$

$$= \ln 16 \quad \text{hence shown.}$$

$$3(d) \quad \frac{A}{(x+1)} + \frac{B}{(x-4)} = \frac{5x-5}{(x+1)(x-4)}$$

$$A(x-4) + B(x+1) = 5x-5$$

$$\text{Put } x=4 \quad 5B=15$$

$$\underline{B=3}$$

$$\text{Put } x=-1 \quad -5A=-10$$

$$\underline{A=2}$$

$$\int_5^6 \left\{ \frac{2}{(x+1)} + \frac{3}{(x-4)} \right\} dx$$

$$\left[2 \ln(x+1) + 3 \ln(x-4) \right]_5^6$$

$$= 2.39 \quad (\text{to 2 decimal places})$$

$$3(e) \quad \int \frac{x^2+1}{x^3+3x} dx$$

differentiating denominator gives $3x^2+3$

for the integral multiply numerator by 3 and $\frac{1}{3}$

$$\text{i.e. } \frac{1}{3} \int 3 \frac{(x^2+1)}{x^3+3x} dx$$

$$\text{gives } \frac{1}{3} \ln(x^3+3x) + c$$