

University of Wales Trinity Saint David
School of Engineering - M.Sc. Automotive Engineering



COMPARATIVE STUDY OF PASSIVE AND ACTIVE ANTI-ROLL BAR SYSTEM FOR VEHICLE RIDE AND HANDLING USING MATLAB/SIMULINK

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ACFA7003 Master Project

DECLARATION

I, JAIMISHKUMAR VALLABHBHAI NAVADIYA, hereby declare that the work presented in this project COMPARATIVE STUDY OF PASSIVE AND ACTIVE ANTI-ROLL BAR SYSTEM FOR VEHICLE RIDE AND HANDLING USING MATLAB/SIMULINK, is entirely my own and has been conducted by the principles of academic integrity and ethical research.

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- 2. Any contributions or assistance received from others have been duly acknowledged in the acknowledgments section.*
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By signing this declaration, I affirm my commitment to upholding the highest standards of academic integrity and professionalism.

Signed: JV Navadiya

Date: 25/11/2023

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Abbreviation

ARB- Anti Roll Bar

AARB- Active Anti-Roll Bar

CG -Centre of Gravity

DLC- Double line change

FL- Front Left

FR- Front Right

RC- Roll Centre

RL- Rear Left

RR- Rear Right

SWA- Steering wheel angle

Abstract

This master's project aims to conduct a thorough comparative analysis of passive and active anti-roll bar systems to assess their influence on vehicle roll behaviour during steady-state cornering and double lane change manoeuvres. The anti-roll bar, also referred to as the sway bar, plays a crucial role in maintaining vehicle stability and controlling body roll during these manoeuvres.

Passive anti-roll bar systems have been extensively utilized in vehicles for many years due to their ability to provide a fixed level of stiffness, resulting in reduced body roll and improved vehicle stability. However, their lack of adaptability restricts their effectiveness in optimizing roll behaviour, as they cannot adjust to varying driving conditions or driver preferences.

In contrast, active anti-roll bar systems offer the flexibility to dynamically adjust the stiffness of the anti-roll bars based on real-time inputs such as vehicle speed, steering angle, and road conditions. This adaptability enables enhanced roll control and improved vehicle dynamics during steady-state cornering and double lane change manoeuvres.

To conduct the comparative analysis, this project will employ a designing of 7 DOF vehicle model using Simulink MathWorks software, a combination of simulations, laboratory tests, and real-world driving scenarios. The evaluation will focus on parameters such as roll angle, roll rate, lateral acceleration, and vehicle response to determine the effectiveness of each system in maintaining vehicle stability and control.

The findings of this study will provide valuable insights into the strengths and limitations of both passive and active anti-roll bar systems in steady-state cornering and double lane change manoeuvres.

Key Words: Active Anti-Roll bar system, Steady state cornering, Double Lane Change Manoeuvre, 7DOF vehicle model.

1. Introduction

An important consideration when assessing a vehicle's overall performance is its safety record (NHTSA Data). Individuals have suffered severe bodily and financial harm as a result of traffic accidents, which have increased significantly due to the complexity of traffic settings. Rollover incidents are second in terms of percentage of car accidents, behind collisions, according to the National Highway Traffic Safety Administration (NHTSA) (Shao et al. 2019). The percentage of road fatalities that are caused by rollover incidents is around 17.7%. As a result, improving the safety performance of automobiles is receiving more attention.

The anti-rollover performance of vehicles is a key indicator in the estimation of new vehicle performance in the United States. Rollover stability is a mandatory testing requirement in automotive safety, which falls under the broader categories of active and passive safety (Dong et al. 2019). Active safety measures centre on proactive steps taken by vehicles to prevent accidents, whereas passive safety measures can only prevent damage after an accident happens.

Vehicle rollovers can be classified into two categories: non-tripped rollovers and tripped rollovers.

Tripped rollover refers to a situation where a vehicle skids and one side of the tires loses contact with the road surface, either due to encountering slippery or muddy conditions or colliding with obstacles such as railings. This results in significant forces acting on the tires or body, leading to the vehicle rolling over.

Non-trip type rollover, also known as curvilinear motion rollover, typically occurs when a driver encounters unexpected situations at high speeds and urgently needs to avoid

pedestrians or obstacles by abruptly steering. This results in excessive lateral acceleration, causing one side of the wheels to lose vertical reaction force and leading to the vehicle rolling over. Non-trip rollovers are influenced by various factors such as vehicle parameters, road conditions, driving conditions, and external interferences, making them a subject of extensive research in vehicle rollover analysis. It is worth noting that non-trip rollovers, although less common than tripped rollovers, tend to occur more frequently in tall and heavy vehicles with a higher centre of gravity. Consequently, these rollovers can result in greater loss of life and property. Research and survey data from both domestic and international sources indicate that the proportion of non-tripped rollovers is twice as high as that of tripped rollovers (Palkovics et al. 2010). Therefore, active anti-rollover control systems that specifically target non-tripping rollovers are of highest importance in reducing

rollover accidents and save lives.



Figure 1 Risk factors affecting rollover phenomena.

A vehicle rollover control system normally operates in two stages. Using a variety of driving factors, including wheel lateral load transfer rate, rollover angle, folding angle, lateral acceleration, yaw rate, and folding angular velocity, the system determines the probability of a vehicle rollover in the first phase. The system decides whether to turn on the rollover prevention control system based on these measurements. In the second phase, if the

estimation result exceeds a predetermined threshold, the rollover control system is activated to modify the vehicle's driving conditions and prevent a rollover from occurring. The fundamental technology behind vehicle rollover control is centred around accurately estimating the probability of a rollover and implementing control measures to minimize this risk, thereby ensuring the safe operation of the vehicle.

2. Industrial Relevance

There is a lot of industrial significance to the study comparing active and passive anti-roll control for vehicle sprung mass. The results of this study may aid automakers in creating suspension systems for their cars that are more effective and efficient. Active anti-roll control can greatly improve a vehicle's handling and stability, which is an important consideration for both passenger and commercial vehicles. Furthermore, by contrasting active and passive systems, the study may offer manufacturers useful information about which kind of anti-roll control is best for their particular cars, taking into account variables like price, efficiency, and energy usage. All things considered, this research may have useful consequences for the auto industry, resulting in the construction of safer and more.

3. Literature Review

This study's major goal is to investigate new techniques for improving off-road vehicle handling capabilities without sacrificing ride comfort.

"Handling" describes how the car reacts to input from the driver via the steering wheel, creating a closed-loop control system. The steering wheel is used by the driver to establish the desired path for the car, and the car reacts to these inputs as well as outside influences like the state of the road. After then, the driver watches how the car responds and modifies their input as necessary.

A review of the literature will start with an analysis of traffic accident data, emphasising the need for handling improvements, in order to set the scene for this investigation. A review of handling theory and analysis, as well as an investigation of techniques for measuring ride comfort and handling in vehicles, will come next. The literature evaluation will also include a range of remedies that have been presented to address the handling issue identified in earlier research. Ultimately, the vehicle model will be designed, simulated, and presented in the conclusion using the findings of the preceding investigation.

3.1 Statistics about handling accidents and Roll-over

Stockholm, Sweden hosted the third Global Ministerial Conference on Road Safety in February 2020. All of the attendees of this meeting reaffirmed their unwavering commitment to reaching the target of a minimum 50% reduction in road accident-related mortality by 2030. According to the Global Status report on Road Safety 2018, road traffic continues to be a major problem for development, a public health concern, and a primary cause of death and injury worldwide, killing over 1.35 million people, with 90 percent of

these deaths occurring in developing nations. The transportation sector still requires greater security and perfection in order to solve this issue. The percentage of rollover fatalities for each type of vehicle from 2000 to 2020 in the United States is shown in Figure 2, which is based on statistics provided by the National Highway Traffic Safety Administration (NHTSA).

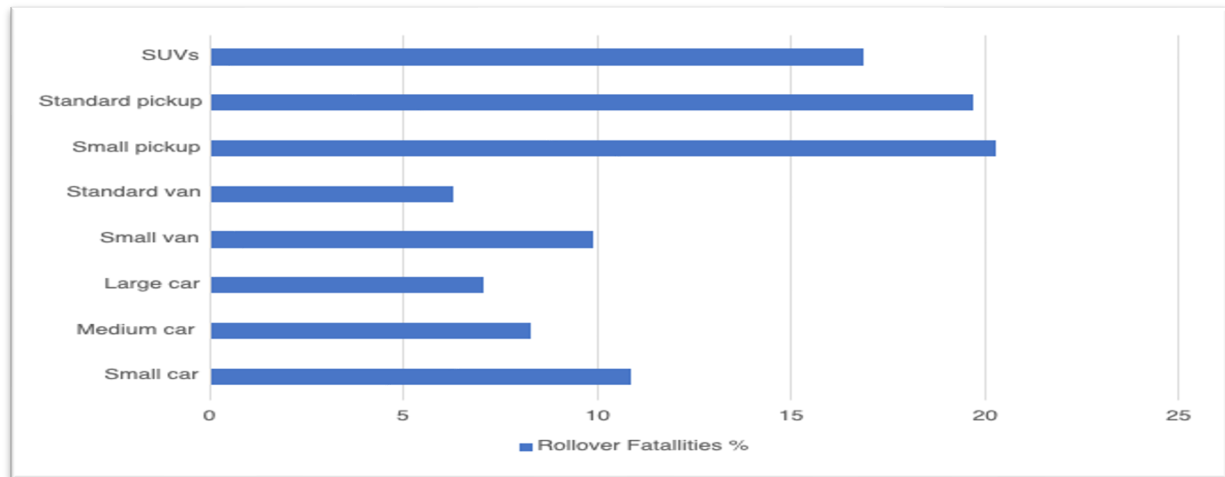


Figure 2: The percentage of rollover deaths per million registered vehicles in the US for various car types between 2000 and 2020. (NHTSA data)

According to Figure 2, compared to other vehicles, the vehicle with a greater centre of gravity had more fatal accidents.

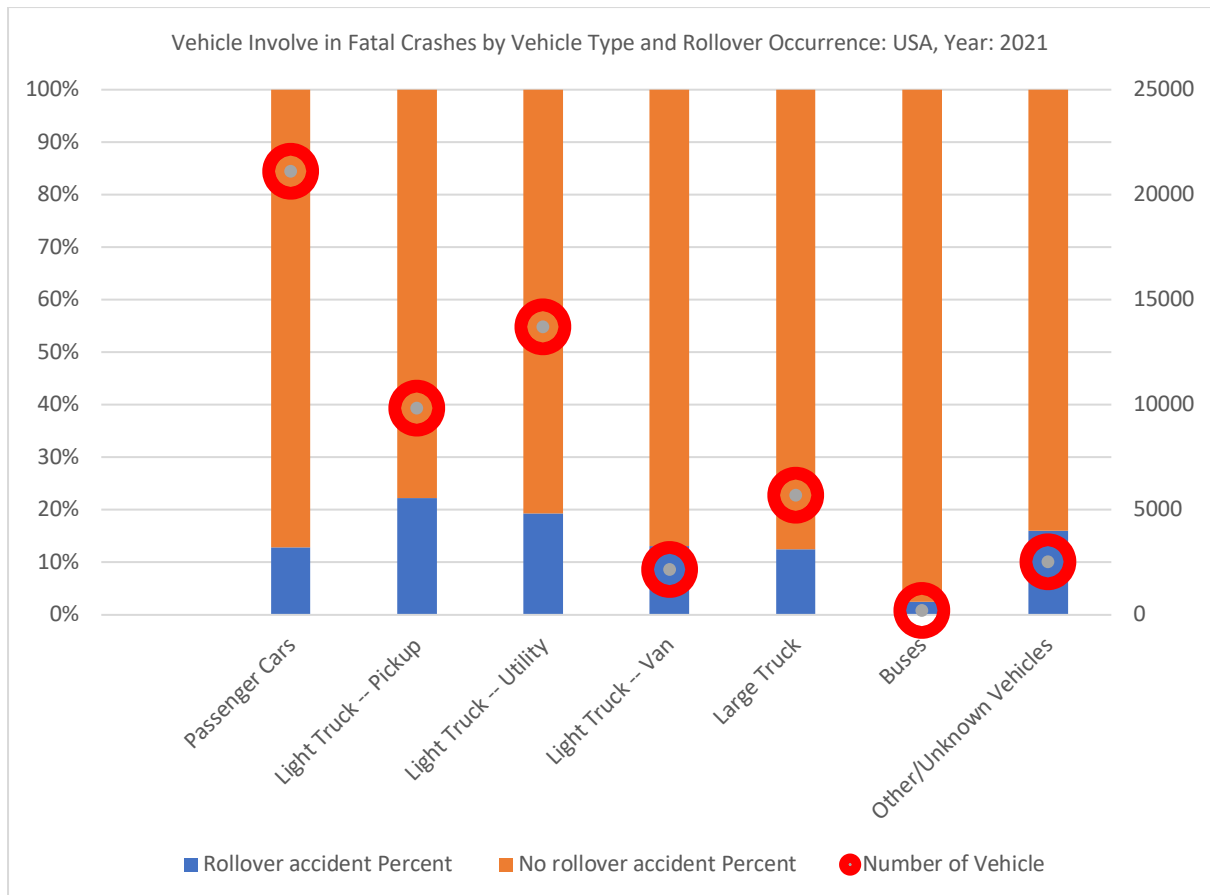


Figure 3: Vehicle Involve in Fatal Crashes by Vehicle Type and Rollover Occurrence: USA, Year: 2021 (NHTSA Rollover data)

Based on the provided statistics, it can be inferred that the safety concern arising from passenger cars rollovers needs to be addressed, considering the growing number of passenger cars.

3.2 Previous Studies on Anti-roll bar system

Many studies aimed at reducing or eliminating the risk of rollovers have been conducted in the last several years. To reduce lateral acceleration, one often used strategy is to combine differential braking and electronic stability control (ESC) (Solmaz et al. 2007). This technique involves decreasing the desired yaw rate that vehicles should follow. However, this reduction in yaw rate reference can negatively impact the manoeuvrability of the vehicle,

potentially leading to other types of accidents, such as collisions with other vehicles (Yim 2011).

Additionally, researchers have explored the development of active suspension systems that employ a combination of mechanical, electrical, and hydraulic components to address vehicle weight transfer. These systems can generally be categorized into three classifications: active suspension (Weeks et al. 2000), semi-active suspension (Gao et al. 2006), and active anti-roll bar (AARB) systems (D.J.M. Sampson and Cebon 1999).

Many experiments conducted on active suspension systems (Gordon 1995; Hrovat 1997; Kawabe et al. 1998; Fischer and Isermann 2004) primarily utilize quarter-car models, which neglect the influence of sprung mass roll but significantly impact vehicle comfort. From a manufacturing perspective, cost plays a crucial role in production. In this regard, the active anti-roll bar (AARB) holds an advantage as it can effectively reduce body roll at a lower cost compared to active suspension systems (Zulkarnain et al. 2012).

The passive anti-roll bar is a suspension component that mitigates body roll acceleration and roll angle during cornering manoeuvres. It achieves this by counteracting any uneven vertical movement between the connected pair of wheels, thereby generating opposing moments against lateral acceleration. This reduction in roll motion enhances driving safety and stability (Zulkarnain et al. 2014). However, the passive anti-roll bar does have limitations. When driving in a straight line, vertical forces caused by road irregularities are transferred from one side to the other, compromising with the benefits of an independent suspension system and compromising ride comfort (Zulkarnain et al. 2012; Zulkarnain et al. 2014).

To minimise or eliminate body roll during cornering and other maneuverer, AARB systems have been developed. AARB system allowing independent bouncing of the wheels on both

side by decoupling the two sides of the anti-roll bar. This effectively reduces vehicle roll caused by bumps and increase ride comfort.(Gosselin-Brisson et al. 2009; Zulkarnain et al. 2012; Zulkarnain et al. 2014).

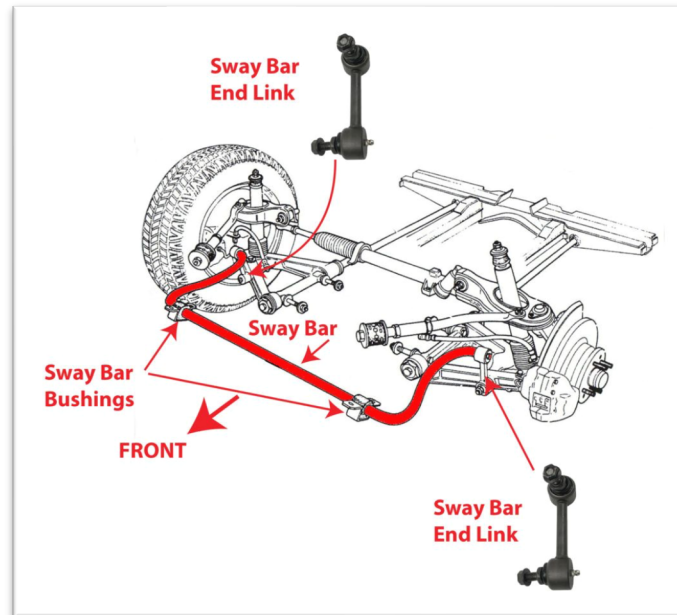


Figure 4: Schematic diagram of Passive Anti-roll bar system

4. Theory and Mathematical equations of vehicle dynamic

To better understanding and ease to design, author divided full car model into three parts.

Vehicle ride model

Vehicle handling model

Tyre model

4.1 Model assumption

The vehicle's sprung and un-sprung masses are represented by a conglomerate mass model.

The influence of aerodynamic drag force is disregarded, and the RC is assumed to coincide with the pitch centre, positioned just below the body's CG.

Rolling resistance resulting from the passive stabilizer bar and body flexibility is not considered.

The suspension between these masses is simulated using passive viscous dampers and spring components.

The vehicle body is assumed to be rigid.

Both the inner and outer front-wheel steer angles are equal.

The tire behaviour is modelled using a linear spring element without any damping.

Throughout the manoeuvring process, the vehicle maintains constant contact with the ground, ensuring that all four tires always remain in contact.

while the lateral and longitudinal behaviours are represented using the Pacejka Magic Formula model. The steering system is simplified as a constant ratio, and the impact of steering inertia is disregarded. So, this simulation model is not good for cases with excessive or frequent wheel lift-off.

4.2 Vehicle ride model

Vehicle ride model is represented by the equations of vertical acceleration and moment of inertia about roll axis and pitch axis. This model is consisting of 7 motion of the vehicle which are 4 unsprung mass (front left, front right, rear left and rear right) vertical motion, sprung mass vertical motion, roll rate about longitudinal axis and pitch rate about lateral axis. The equations were extracted from (Ahmad et al. 2009).

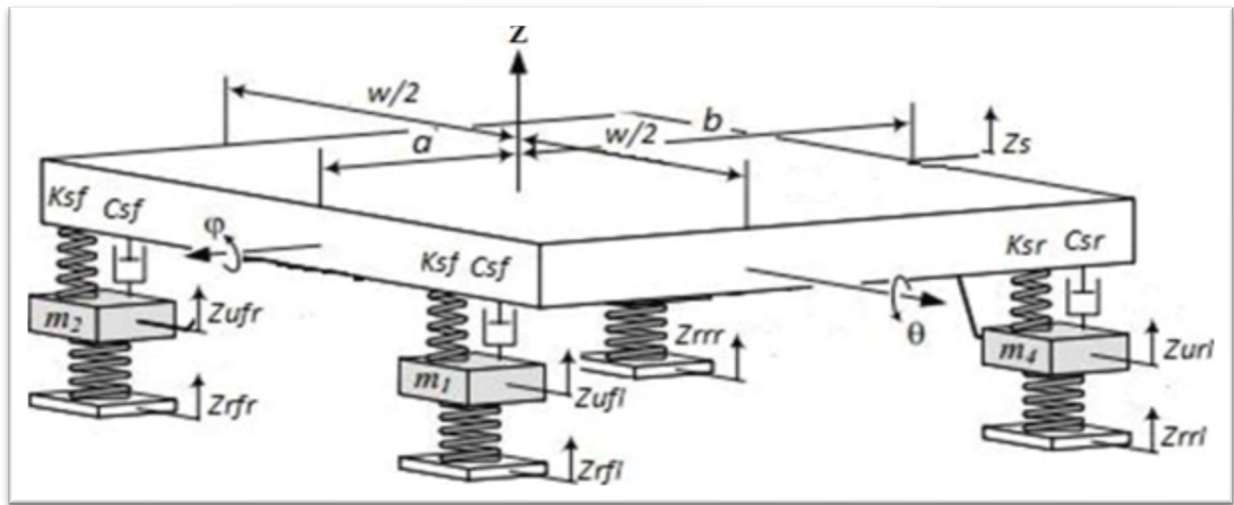


Figure 5 Vehicle 7DOF ride model

Considering Figure 4, Through the application of force balance analysis on the sprung mass, the resulting equation is as follows:

$$m_s \ddot{Z}_s = P_{fl} + P_{fr} + P_{rl} + P_{rr}$$

Equation 1 Sprung mass vertical displacement.

Where,

m_s = Sprung mass

$\ddot{Z}_s$ = Sprung mass acceleration in vertical direction

P_{fl} = Suspension force at front left wheel

P_{fr} = Suspension force at front right wheel

P_{rl} = Suspension force at rear left wheel

P_{fr} = Suspension force at rear right wheel

Suspension forces on each wheel can be define as

$$P_{fl} = K_{sf}(Z_{ufl} - Z_{sfl}) + C_{sf}(\dot{Z}_{ufl} - \dot{Z}_{sfl})$$

$$P_{fr} = K_{sf}(Z_{ufr} - Z_{sfr}) + C_{sf}(\dot{Z}_{ufr} - \dot{Z}_{sfr})$$

$$P_{rl} = K_{sr}(Z_{url} - Z_{srl}) + C_{sr}(\dot{Z}_{url} - \dot{Z}_{srl})$$

$$P_{rr} = K_{sr}(Z_{urr} - Z_{srr}) + C_{sr}(\dot{Z}_{urr} - \dot{Z}_{srr})$$

Equation 2 Force acting on each corner of sprung mass.

Where,

C_{sf} = the front suspension damping coefficient

C_{sr} = the rear suspension damping coefficient

K_{sf} = the front suspension spring stiffness

K_{sr} = the rear suspension spring stiffness

Z_{ufl} = the front left unsprung mass displacement

Z_{ufr} = the front right unsprung mass displacement

Z_{url} = the rear left unsprung mass displacement

Z_{urr} = the rear right unsprung mass displacement

$\dot{Z}_{ufl}$ = the front left unsprung mass velocity

$\dot{Z}_{ufr}$ = the front right unsprung mass velocity

$\dot{Z}_{url}$ = the rear left unsprung mass velocity

$\dot{Z}_{urr}$ = the rear right unsprung mass velocity

$Z_{sfl}, Z_{sfr}, Z_{srl}, Z_{srr}$ are spring mass position at each corner and can be formulated as

$$Z_{sfl} = Z_s - a\theta + 0.5w\phi$$

$$Z_{sfr} = Z_s - a\theta - 0.5w\phi$$

$$Z_{srl} = Z_s + b\theta + 0.5w\phi$$

$$Z_{srr} = Z_s + b\theta - 0.5w\phi$$

Equation 3 Sprung mass each corner displacement equation.

Where,

a = distance from CG to front axle

b = the distance from CG to rear axle

w = wheelbase

θ = pitch angle of sprung mass

ϕ = roll angle of sprung mass

By using value of equation 2 and equation 3. The Equation of sprung mass vertical acceleration can be written as

$$\begin{aligned} m_s \ddot{Z}_s = & -2(K_{sf} + K_{sr})Z_s - 2(C_{sf} + C_{sr})\dot{Z}_s + 2(aK_{sf} - bK_{sr})\theta + 2(aC_{sf} - bC_{sr})\dot{\theta} \\ & + K_{sf}(Z_{ufl} + Z_{ufr}) + C_{sf}(\dot{Z}_{ufl} + \dot{Z}_{ufr}) + K_{sr}(Z_{url} + Z_{urr}) + C_{sr}(\dot{Z}_{url} + \dot{Z}_{urr}) \end{aligned}$$

Equation 4 Sprung mass vertical acceleration equation.

The equation of pitch acceleration ($\ddot{\theta}$) and roll acceleration ($\ddot{\phi}$) can be derived from the moment equilibrium, and can be written as

$$\begin{aligned} I_{yy} \ddot{\theta} = & 2(aK_{sf} - bK_{sr})Z_s + 2(aC_{sf} - bC_{sr})\dot{Z}_s - 2(a^2K_{sf} + b^2K_{sr})\theta - 2(a^2C_{sf} - b^2C_{sr})\dot{\theta} \\ & - aK_{sf}(Z_{ufl} + Z_{ufr}) - aC_{sf}(\dot{Z}_{ufl} + \dot{Z}_{ufr}) + bK_{sr}(Z_{url} + Z_{urr}) \\ & + bC_{sr}(\dot{Z}_{url} + \dot{Z}_{urr}) - m_s h_{cgM} a_x \end{aligned}$$

Equation 5 Pitch angle equation.

$$\begin{aligned} I_{xx} \ddot{\phi} = & -0.5w^2(K_{sf} + K_{sr})\phi - 0.5w^2(C_{sf} + C_{sr})\dot{\phi} + 0.5wK_{sf}(Z_{ufl} - Z_{ufr}) \\ & + 0.5wC_{sf}(\dot{Z}_{ufl} - \dot{Z}_{ufr}) + 0.5wK_{sr}(Z_{url} - Z_{urr}) + 0.5wC_{sr}(\dot{Z}_{url} - \dot{Z}_{urr}) \\ & + m_s h_{cgM} a_y - \frac{w}{2}F_{fl} + \frac{w}{2}F_{fr} - \frac{w}{2}F_{rl} + \frac{w}{2}F_{rr} \end{aligned}$$

Equation 6 Roll angle equation.

Where,

a_x = longitudinal acceleration of vehicle

a_y = lateral acceleration of vehicle

F_{fl} = Front left anti-roll force

F_{fr} = Front right anti-roll force

F_{rl} = Rear left anti-roll force

F_{rr} = Rear right anti-roll force

I_{xx} = moment of inertia of vehicle about pitch axis

I_{yy} = moment of inertia of vehicle about roll axis

h_{cgM} = centre of gravity height

The vertical Acceleration of Sprung mass is calculated by the following equations.

$$m_{uf}\ddot{Z}_{ufl} = (K_{sf})Z_s + (C_{sf})\dot{Z}_s - (aK_{sf})\theta - (aC_{sf})\dot{\theta} + 0.5wK_{sf}\phi + 0.5wC_{sf}\dot{\phi} \\ - (K_{sf} + K_t)Z_{ufl} - C_{sf}\dot{Z}_{ufl} + K_tZ_{rfl} - F_{fl}$$

Equation 7 Front left unsprung mass vertical acceleration.

$$m_{uf}\ddot{Z}_{ufr} = (K_{sf})Z_s + (C_{sf})\dot{Z}_s - (aK_{sf})\theta - (aC_{sf})\dot{\theta} + 0.5wK_{sf}\phi + 0.5wC_{sf}\dot{\phi} \\ - (K_{sf} + K_t)Z_{ufr} - C_{sf}\dot{Z}_{ufr} + K_tZ_{rfr} - F_{fr}$$

Equation 8 Front right unsprung mass vertical acceleration.

$$m_{ur}\ddot{Z}_{url} = (K_{sr})Z_s + (C_{sr})\dot{Z}_s - (aK_{sr})\theta - (aC_{sr})\dot{\theta} + 0.5wK_{sr}\phi + 0.5wC_{sr}\dot{\phi} \\ - (K_{sr} + K_t)Z_{url} - C_{sr}\dot{Z}_{url} + K_tZ_{rrl} - F_{rl}$$

Equation 9 Rear left unsprung mass vertical acceleration.

$$m_{ur}\ddot{Z}_{urr} = (K_{sr})Z_s + (C_{sr})\dot{Z}_s - (aK_{sr})\theta - (aC_{sr})\dot{\theta} + 0.5wK_{sr}\phi + 0.5wC_{sr}\dot{\phi} \\ - (K_{sr} + K_t)Z_{urr} - C_{sr}\dot{Z}_{urr} + K_tZ_{rrr} - F_{rr}$$

Equation 10 Rear right unsprung mass vertical acceleration.

Where,

K_t = tire stiffness

m_{uf} = front unsprung mass

m_{ur} = rear unsprung mass

$\ddot{Z}_{ufl}, \ddot{Z}_{ufr}, \ddot{Z}_{url}, \ddot{Z}_{urr}$ are the front left and right and rear left and right unsprung mass vertical acceleration respectively.

4.3 Vehicle Handling Model

Dynamic handling and steady state handling are the two other subcategories of vehicle handling. Steady state cornering refers to a scenario in vehicle control where the vehicle is travelling on a circle with a stable radius and steering angle. A severe double-lane-change (DLC) manoeuvre is handled dynamically or kinetically. An essential test for determining a vehicle's road holding capability is the critical double-lane-change (DLC) manoeuvre, which was defined by the International Organization for Standardization (ISO) in 1975.

Evaluating the dynamic behaviour of the vehicle starts with the DLC test, which is described by ISO as "a dynamic process consisting of driving a vehicle from its original lane to another lane parallel to the initial lane as fast as possible, and perhaps returning to the initial lane."

Table 1 provides particular track dimensions, while Figure 6 illustrates the visual arrangement of the operation.

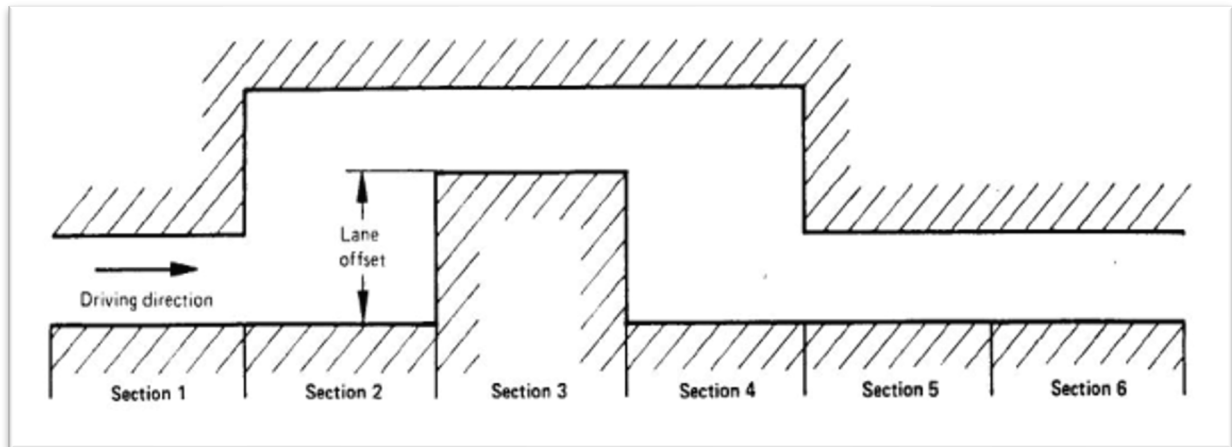


Figure 6: Double Lane change track and designation of track (International Organisation for Standardisation, 1975)

Section	Length	Width
1	15	$1.1 \times \text{Vehicle width} + 0.25\text{m}$
2	30	Not applicable
3	25	$1.2 \times \text{vehicle width} + 0.25\text{m}$
4	25	Not applicable
5	15	$1.3 \times \text{vehicle width} + 0.25\text{m}$
6	15	$1.3 \times \text{vehicle width} + 0.25\text{m}$
Lane Offset	3.5	

Table 1: Double Lane change track dimension (International Organisation for Standardization, 1975)

As per the guidelines established by the British Standards Institution in 1987 concerning the assessment of human exposure to whole-body mechanical vibration, the prescribed approach involves the utilization of the weighting function W_b . This weighting function is specifically designed for instances where an individual is seated and subjected to vibration in the vertical direction. The graphical representation of this weighting function can be observed in Figure 7.

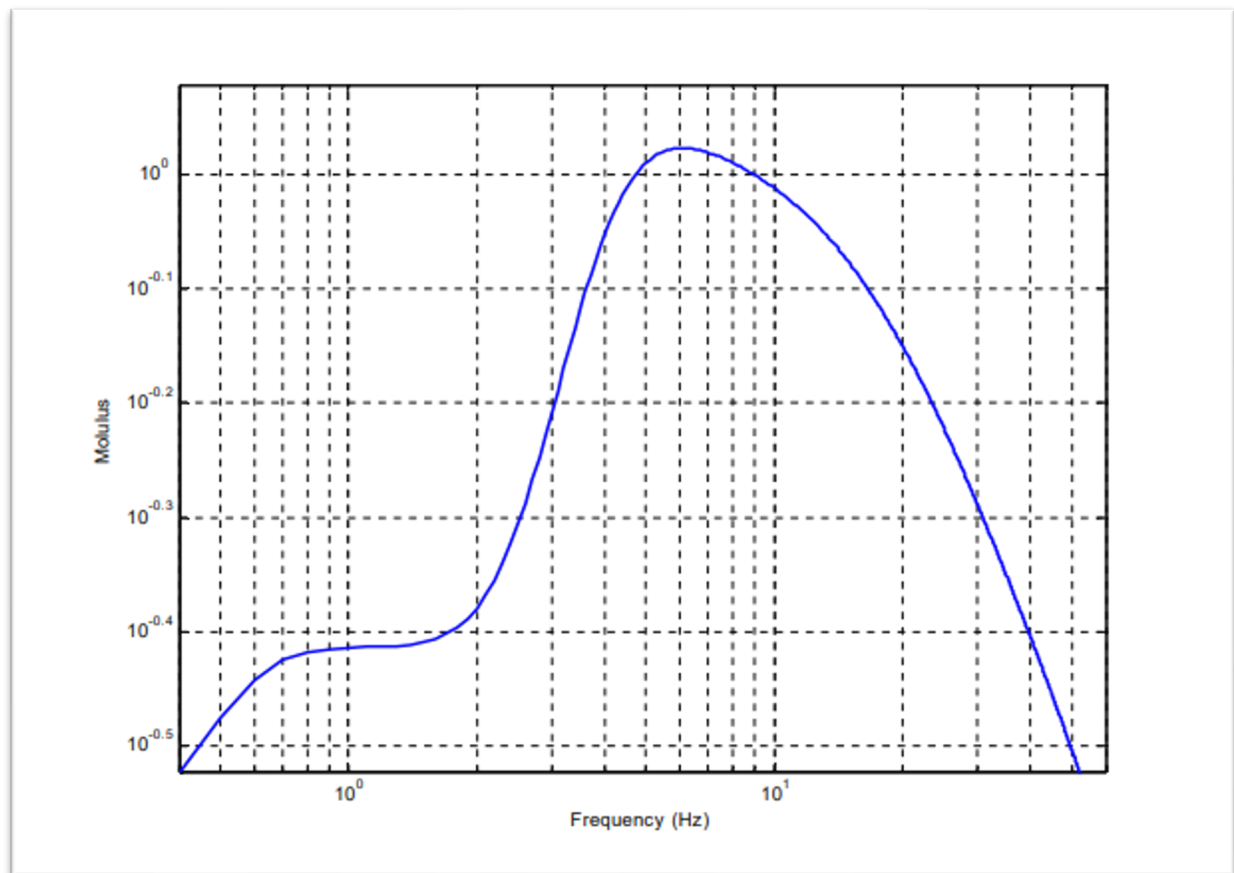


Figure 7 function for weighting W_b is utilised to measure vibration in a vertical manner on a seated individual.

Applying the weighting function W_b to the measured vertical acceleration a test subject experiences is the filtering procedure. Next, the Fast Fourier Transform of this recorded vertical acceleration is computed to convert it into the frequency domain (FFT). The weighting function is then applied to the FFT result, and the result is subsequently converted back to the time domain. The last step is to find the weighted vertical acceleration's Root Mean Square (RMS). The test ride comfort experienced is directly reflected in this RMS number. Table 2 contains recommendations for acceptable RMS values as defined by the British Standards Institution in 1987. These standards are provided for reference and interpretation.

Weighted RMS values	Rating
$< 0.315 \text{ m/s}^2$	Not uncomfortable
$0.315 - 0.63 \text{ m/s}^2$	A little uncomfortable

0.5 – 1.0 m/s ²	Fairly uncomfortable
0.8 – 1.6 m/s ²	Uncomfortable
1.25 – 2.5 m/s ²	Very uncomfortable
> 2.0 m/s ²	Extremely uncomfortable

Table 2 Comfort recommendations based on weighted RMS (British Standards institution, 1987).

4.4 Bicycle model

4.4.1 Low speed cornering

It's thought that when a car turns slowly (as in a parking lot), its tyres don't need to exert any lateral force, hence there isn't any slip angle created. In Figure 8, you can see that the car in this example is circling a stationary point.

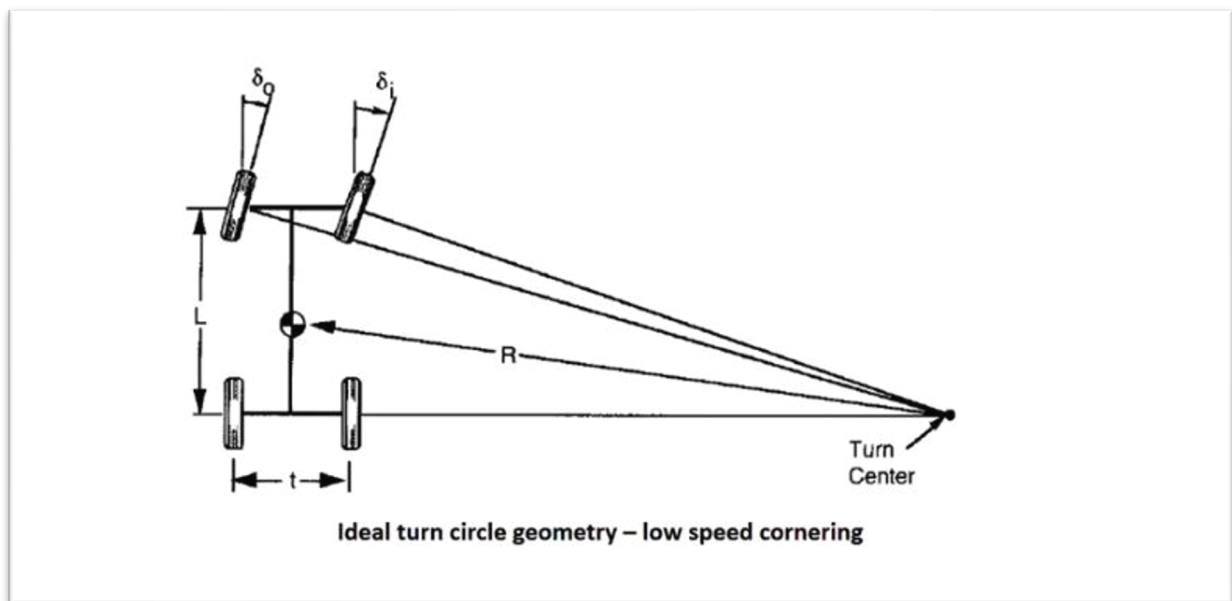


Figure 8 Low speed cornering

The term "turn centre" refers to the location that the car turns around. Every wheel and the turn centre's lines are perpendicular to each other. This is because there isn't any slide when the manoeuvre is being performed. The steering angle, represented by δ_o for the outside wheel and δ_i for the inside wheel, is the angle formed by the direction the wheel is pointing and the direction the vehicle is heading (Gillespie 1992).

For small angle assumption, steering angle

$$\delta_o = \frac{L}{R + \frac{t}{2}}$$
$$\delta_i = \frac{L}{R - \frac{t}{2}}$$

δ_o = steering angle of the outer wheel

δ_i = steering angle of the inner wheel

R = turning radius

t = trackwidth

The term "Ackerman steering" or "Ackerman geometry" refers to this relationship. At low speeds, this is the optimal steering angle for the vehicle's "bicycle model." The left and right wheels are treated as one wheel in the "bicycle model," with each wheel producing twice as much vertical force and twice as much lateral force at the same slip angle. This indicates that the two-dimensional model does not account for body roll or lateral load transfer (Gillespie 1992).

4.4.2 High speed cornering

The vehicle's lateral acceleration is crucial to handling characteristics during high-speed cornering and cannot be overlooked. In order to offset the lateral acceleration, the tyres must therefore generate lateral forces. This implies that each wheel will produce a slip angle.

The velocity and angle are cited in figure 8. The centre of gravity of the model is at (x,y). velocity of the vehicle is V. Steering angle is δ_f . Angle made by the velocity vector to the line of CG is β . ψ is the current heading angle made by the axis of vehicle model. l_f and l_r is the distance of center of front and rear wheel from center of gravity point. Radius of cornering (distance between O to the point of centre of gravity) is R.

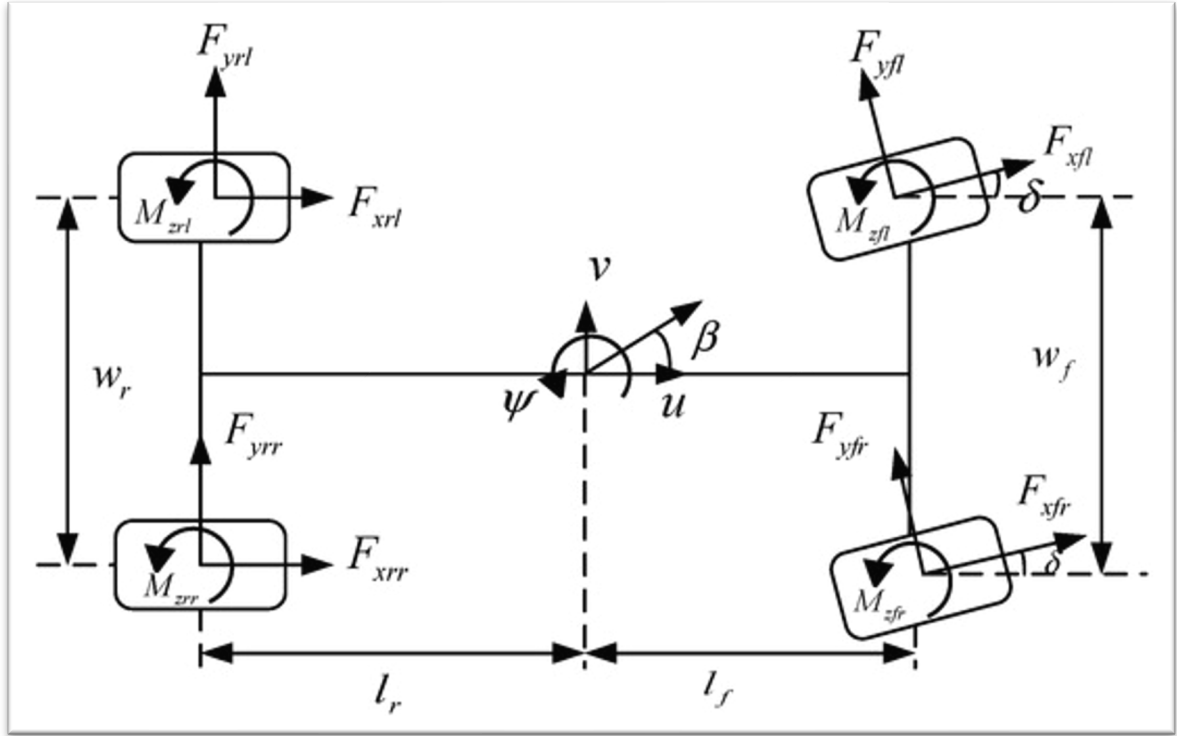


Figure 9 Dynamic Bicycle Model with linear motion.

The notation v and u substitute by V_x and V_y respectively.

From the geometry of model, yaw velocity is given by,

$$\omega = \frac{v}{R}$$

Equation 11 Yaw velocity of vehicle

And linear velocity components in longitudinal v_x and lateral v_y direction are given by following equations respectively.

$$V_x = V \cos(\psi + \beta)$$

$$V_y = V \sin(\psi + \beta)$$

Equation 12 Linear velocity component of bicycle model

From geometry of figure 8, $\triangle AOB$,

$$\tan \delta = \frac{l_f + l_r}{R_1}$$

From geometry of figure 8, ΔOAV ,

$$\tan\beta = \frac{l_r}{R_1} = \frac{l_r \tan\delta}{l_f + l_r}$$

$$\cos\beta = \frac{R_1}{R} = \frac{l_f + l_r}{\tan\delta} \times \frac{1}{R}$$

$$R = \frac{l_f + l_r}{\tan\delta} \times \frac{1}{\cos\beta} = \frac{l_f + l_r}{\tan\delta \cos\beta}$$

So, by substituting the value of R in Equation 11,

$$\omega = \frac{V}{\frac{l_f + l_r}{\tan\delta \cos\beta}} = \frac{V \tan\beta \cos\delta}{l_f + l_r}$$

Equation 13 Yaw rate in term of side slip angle and steering angle

Where,

$$\beta = \tan^{-1}\left(\frac{l_r \tan\delta}{l_f + l_r}\right)$$

These formulas aid in establishing the motion and force model for the vehicle. Some assumptions about this model, such as treating the car body like a two-wheeled bicycle. This model's centre of gravity is located at point C. The motion is limited to the X and Y axes.

Vehicle has a completely rigid body. The model's longitudinal velocity is constant.

Where,

m_s = Vehicle sprung mass

I_{zz} = Vehicle moment of inertia about Z axis

l_f = Distance from CG to front axle

l_r = Distance from CG to rear axle

$C_{\alpha r}$ = Cornering stiffness of rear tire

$C_{\alpha f}$ = Cornering stiffness of front tire

$\ddot{\theta}$ = angular acceleration of the vehicle

I_{zz} = moment of inertia in lateral direction

F_{xfl} = Longitudinal force acting on the front left wheel

F_{xfr} = Longitudinal force acting on the front right wheel

F_{xrl} = Longitudinal force acting on rear left wheel

F_{xrr} = Longitudinal force acting on rear right wheel

F_{yfl} = lateral force acting on the front left wheel

F_{yfr} = lateral force acting on the front right wheel

F_{yrl} = lateral force acting on rear left wheel

F_{yrr} = lateral force acting on rear right wheel

f_{rfl} = Friction force on front left wheel

f_{rfr} = Friction force on front right wheel

f_{rrl} = Friction force on rear left wheel

f_{rrr} = Friction force on rear right wheel

M_{tfl} = Self aligning torque on front left wheel

M_{tfr} = Self aligning torque on front right wheel

M_{trl} = Self aligning torque on rear left wheel

M_{trr} = Self aligning torque on rear right wheel

a_y = Lateral acceleration of vehicle

V = Vehicle velocity

V_x = vehicle longitudinal velocity

V_y = vehicle lateral velocity

ω = vehicle angular velocity (Yaw rate)

V_{xf} = Front wheel longitudinal velocity

V_{yf} = Front wheel lateral velocity

V_{xr} = Rear wheel longitudinal velocity

V_{yr} = Rear wheel lateral velocity

α_f = Front wheel slip angle

α_r = Rear wheel slip angle

m = Total vehicle mass

The longitudinal acceleration of the vehicle's CG a_y is composed of two terms: $\dot{V}_x$ is the result of acceleration along the x-axis, while the rotation of the vehicle around the z-axis produces centripetal acceleration. Lateral acceleration of the body is derived from the following equation.

$$ma_y = \sum F_y = F_{yr} + F_{yf} \cos \delta$$

If steering angle δ is very small than $\cos \delta = 1$.

$$ma_y = F_{yr} + F_{yf}$$

a_y has two components because one from lateral velocity component and other one is from centripetal force.

Centripetal force $\frac{mv^2}{R}$. So centripetal acceleration is due to centripetal force is define as

$$\frac{v^2}{R} = \frac{vR\omega}{R} = V\omega.$$

And other component is due to centrifugal force which is $\dot{V}_y$.

$$\dot{V}_y = a_y + \omega V_x$$

Where a_y can be calculated from force equilibrium along Y- axis,

$$ma_y = (F_{xfl} - f_{rfl})\sin \delta + F_{yfl}\cos \delta + (F_{xfr} - f_{rfr})\sin \delta + F_{yfr}\cos \delta + F_{yrl} + F_{yrr}$$

Equation 14 lateral acceleration equation

Similarly, longitudinal acceleration can be defined as

$$\dot{V}_x = a_x - \omega V_y$$

Where a_x can be obtained by force equilibrium along X- axis,

$$ma_x = (F_{xfl} - f_{rfl})\cos\delta - F_{yfl}\sin\delta + (F_{xfr} - f_{rfr})\cos\delta + F_{yfr}\sin\delta + F_{xrl} + F_{xrr} - f_{rrl} - f_{rrr}$$

Equation 15 longitudinal acceleration equation

Vehicle yaw acceleration can be calculated from the force and moment equilibrium along Z- axis.

$$I_{zz}\ddot{\psi} = \frac{w_f}{2}(F_{xfl} - f_{rfl})\cos\delta + \frac{w_f}{2}(F_{xfr} - f_{rfr})\cos\delta - \frac{w_r}{2}F_{xrl} + \frac{w_r}{2}F_{xrr} + \frac{w_f}{2}F_{yfl}\sin\delta - \frac{w_f}{2}F_{yfr}\sin\delta - l_r F_{yrl} - l_r F_{yrr} + l_f F_{yfl}\cos\delta + l_f F_{yfr}\cos\delta + l_f(F_{xfl} - f_{rfl})\sin\delta + l_f(F_{xfr} - f_{rfr})\sin\delta + \frac{w_r}{2}f_{rrl} - \frac{w_r}{2}f_{rrr} - M_{tfl} - M_{tfr} - M_{trl} - M_{trr}$$

Equation 16 Yaw acceleration equation

By integrating $\dot{V}_x$ and $\dot{V}_y$, the longitudinal and lateral velocities, or V_x and V_y can be derived.

These velocities are essential for determining the tyre forces by computing the longitudinal slip ratio and side slip angle.

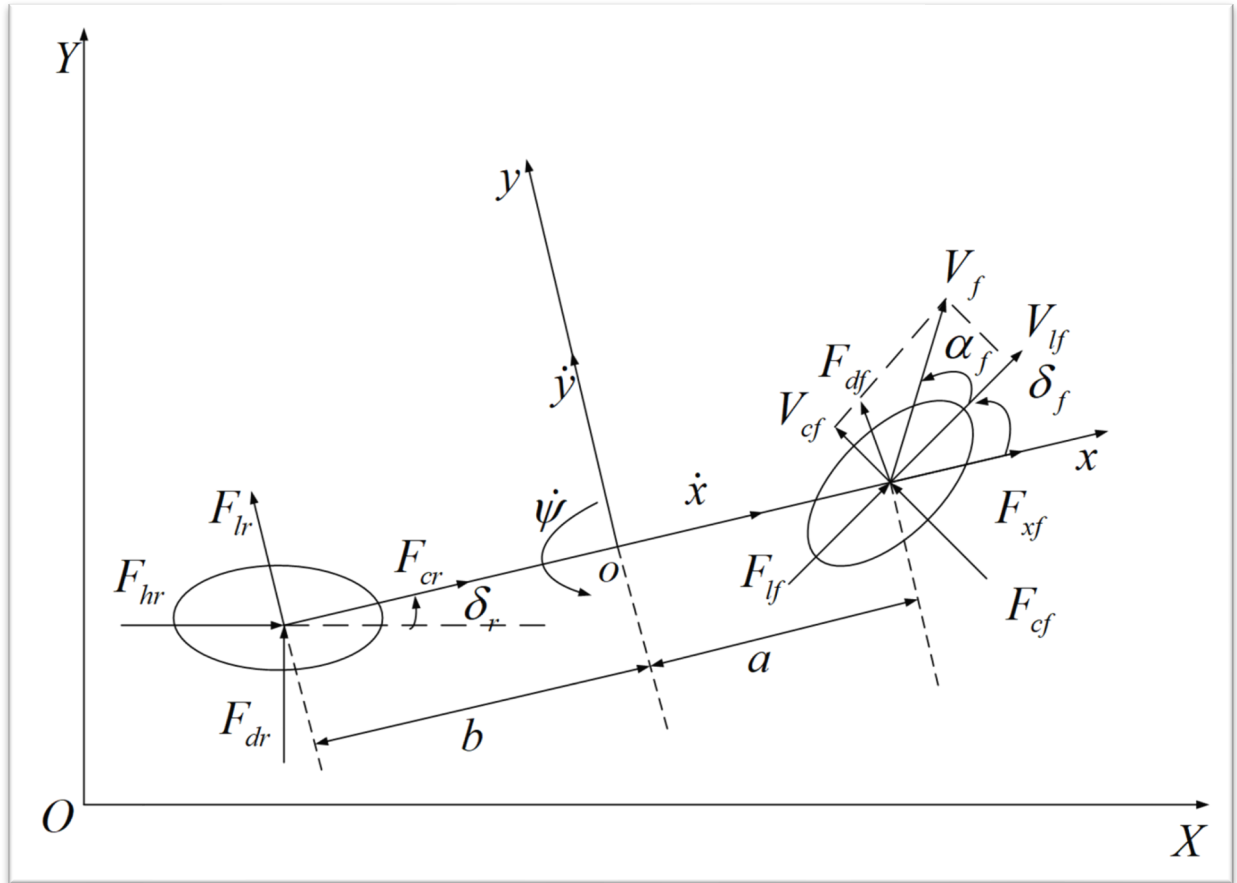


Figure 10 Global coordinate system (XY) and vehicle coordinate system (xy).

The notation V_{lf} and V_{cf} substitute by $V_{A,\varepsilon}$ and $V_{B,\eta}$.

From vehicle coordinate system,

$$\alpha_r \approx \tan \alpha_r = \frac{\overline{V_{B,y}}}{\overline{V_{B,x}}}$$

And

$$\alpha_f \approx \tan \alpha_f = \frac{\overline{V_{A,\eta}}}{\overline{V_{A,\varepsilon}}}$$

From kinematics,

$$V_{B,x} = V_x$$

$$V_{B,y} = V_y - \omega l_r$$

$$V_{A,x} = V_x$$

$$V_{A,y} = V_y + \omega l_f$$

coordinate transformation relation between vehicle coordinate system to Global coordinate system $(X, Y) \rightarrow (\epsilon, \eta)$.

$$V_{A,\epsilon} = V_{A,x}\cos\delta - V_{A,y}\sin\delta = V_x + (V_y + \omega l_f)\delta$$

$$V_{A,\eta} = -V_{A,x}\sin\delta + V_{A,y}\cos\delta = -V_x\delta + (V_y + \omega l_f)$$

By substituting these values in the equation of slip angle.

$$\alpha_f \approx \tan\alpha_f = \frac{\overline{V_{A,\eta}}}{\overline{V_{A,\epsilon}}} = \frac{-V_x\delta + (V_y + \omega l_f)}{V_x + (V_y + \omega l_f)\delta}$$

$$\alpha_r \approx \tan\alpha_r = \frac{\overline{V_{B,y}}}{\overline{V_{B,x}}} = \frac{V_y - \omega l_r}{V_x}$$

By assuming, $V_x \gg (V_y + \omega l_f)\delta$, so dominating part $V_x + (V_y + \omega l_f)\delta = V_x$.

Therefore,

$$\alpha_f = \tan^{-1}\left(\frac{V_y + \omega l_f}{V_x}\right) - \delta$$

Equation 17 Front wheel slip angle equation

And

$$\alpha_r = \tan^{-1}\left(\frac{V_y - \omega l_r}{V_x}\right)$$

Equation 18 Rear wheel slip angle equation

4.5 Pacejka Tyre model

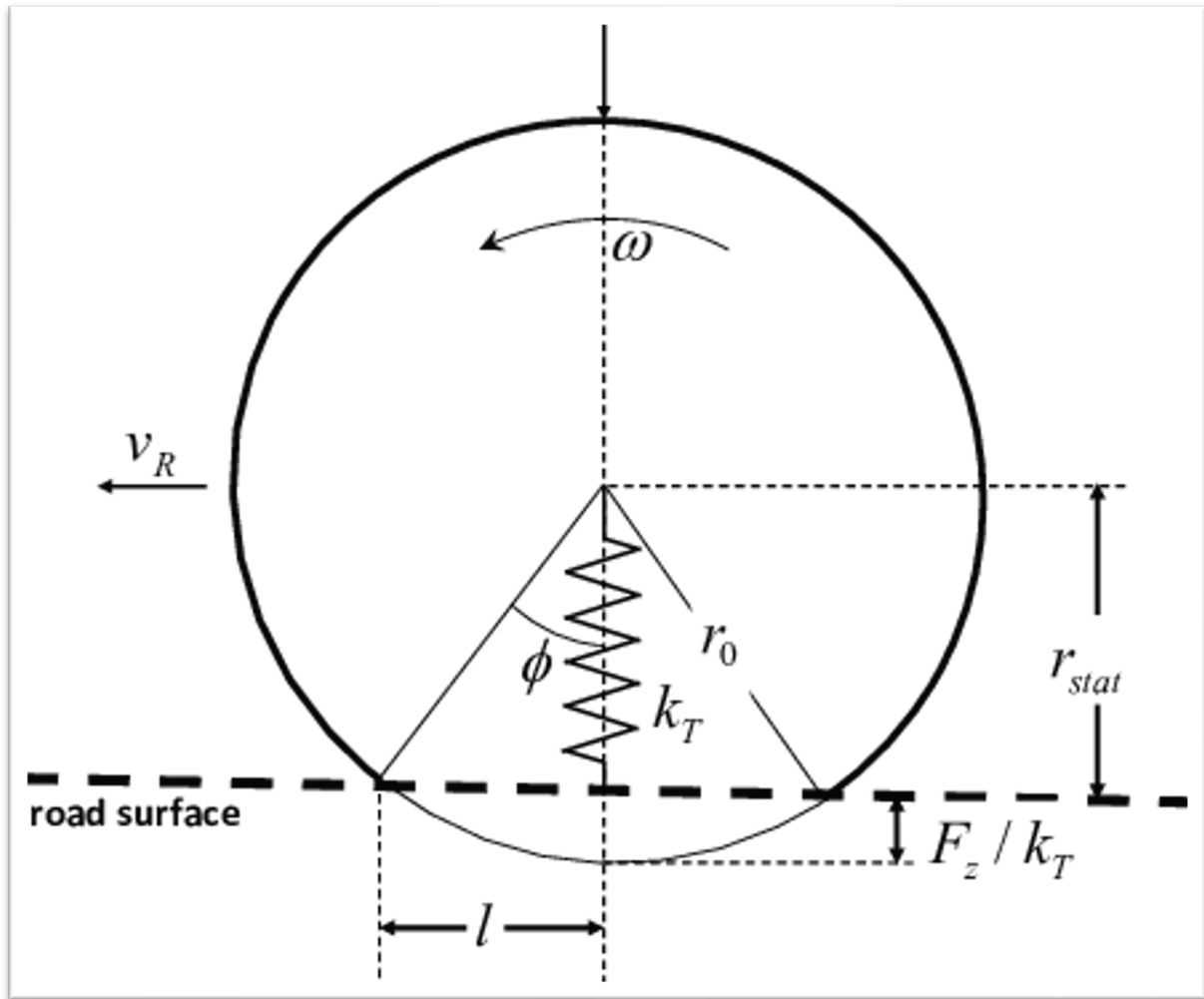


Figure 11 Tyre spinning motion.

The torque about wheel axle on each wheel can be defined by the following equation.

$$I_w \dot{\omega}_{fl} = T_{af} - T_{bf} + (F_{xfl} - f_{rfl})e - F_{zfl} L$$

$$I_w \dot{\omega}_{fr} = T_{af} - T_{bf} + (F_{xfr} - f_{rfr})e - F_{zfr} L$$

$$I_w \dot{\omega}_{rl} = T_{ar} - T_{br} + (F_{xrl} - f_{rrl})e - F_{zrl} L$$

$$I_w \dot{\omega}_{rr} = T_{ar} - T_{br} + (F_{xrr} - f_{rrr})e - F_{zrr} L$$

Equation 19 Torque at each wheel equation.

I_w = Moment of inertia of wheel

$\dot{\omega}_{ij}$ = wheel spin acceleration (i = front and rear, j = left and right)

T_{ai} = Driving torque (i = front and rear)

T_{bi} = Braking torque (i = front and rear)

$$e = F_z / k_T = r_o (1 - \cos\phi)$$

$$L = r_o (\sin\phi)$$

r_o = tire radius

Tire friction force can be calculated by following equations:

$$f_{rfl} = \sigma F_{zfl}$$

$$f_{rfr} = \sigma F_{zfr}$$

$$f_{rrl} = \sigma F_{zrl}$$

$$f_{rrr} = \sigma F_{zrr}$$

Equation 20 Friction force on each tire.

F_{zfl} = Front left wheel normal force

F_{zfr} = Front right wheel normal force

F_{zrl} = Rear left wheel normal force

F_{zrr} = Rear right wheel normal force

where tyre speed is the primary determinant of the rolling resistance coefficient, σ (v). The value of σ vs V tyre speed is displayed in Table 1. The tyres are 235/70 R15, the normal size specified by the vehicle's handbook (2001) and utilised in the simulation in Section 4; the tyre pressure is approximately 35 Psi.

Rolling speed v (m/s)	Resistance coefficient(σ)
0	0
1.126	0.0129
6.944	0.0129
13.888	0.01376
21.02	0.014178
27.777	0.015205
34.722	0.016438

41.666	0.0182
48.423	0.021369
55.555	0.024657

Table 3 Rolling resistance coefficient value on different speed.(Wiegand 2016)

Normal forces on each wheel can be calculated by the following equations:

$$F_{zfl} = \frac{mgl_r}{2(l_f + l_r)} - m_s a_x \frac{H_{cg}}{2(l_f + l_r)} - \frac{K_{\phi f}}{K_{\phi f} + K_{\phi r}} \left(\frac{m_s a_y H_{cg}}{w_f} \right) + F_{fl}$$

Equation 21 Normal force on front left wheel

$$F_{zfr} = \frac{mgl_r}{2(l_f + l_r)} - m_s a_x \frac{H_{cg}}{2(l_f + l_r)} + \frac{K_{\phi f}}{K_{\phi f} + K_{\phi r}} \left(\frac{m_s a_y H_{cg}}{w_f} \right) + F_{fr}$$

Equation 22 Normal force on front right wheel

$$F_{zrl} = \frac{mgl_f}{2(l_f + l_r)} + m_s a_x \frac{H_{cg}}{2(l_f + l_r)} - \frac{K_{\phi r}}{K_{\phi f} + K_{\phi r}} \left(\frac{m_s a_y H_{cg}}{w_r} \right) + F_{rl}$$

Equation 23 Normal force on rear left wheel

$$F_{zrr} = \frac{mgl_f}{2(l_f + l_r)} + m_s a_x \frac{H_{cg}}{2(l_f + l_r)} + \frac{K_{\phi r}}{K_{\phi f} + K_{\phi r}} \left(\frac{m_s a_y H_{cg}}{w_r} \right) + F_{rr}$$

Equation 24 Normal force on rear right wheel

$K_{\phi f}$ = Front roll stiffness

$K_{\phi r}$ = Rear roll stiffness

g = Gravitational acceleration (9.81 m/s/s)

4.6 Wheel rate and Chassis roll stiffness.

The effective leverage, or the distance between the springs and the track, determines the wheel rate, which is different from the spring rate. Therefore, as illustrated in Figure (12), double-wishbone suspension wheel rate is given by the equation:

$$K_{wheel} = K_s IR^2 \cos \phi + K_{ARB} IR^2 \cos \phi \text{ N/mm}$$

Equation 25 Wheel rate

K_s = Spring rate

IR = Installation ratio

K_{ARB} = Anti roll bar rate

ρ = Angle of spring damper from vertical axis

ϱ = Angle of drop link form vertical axis

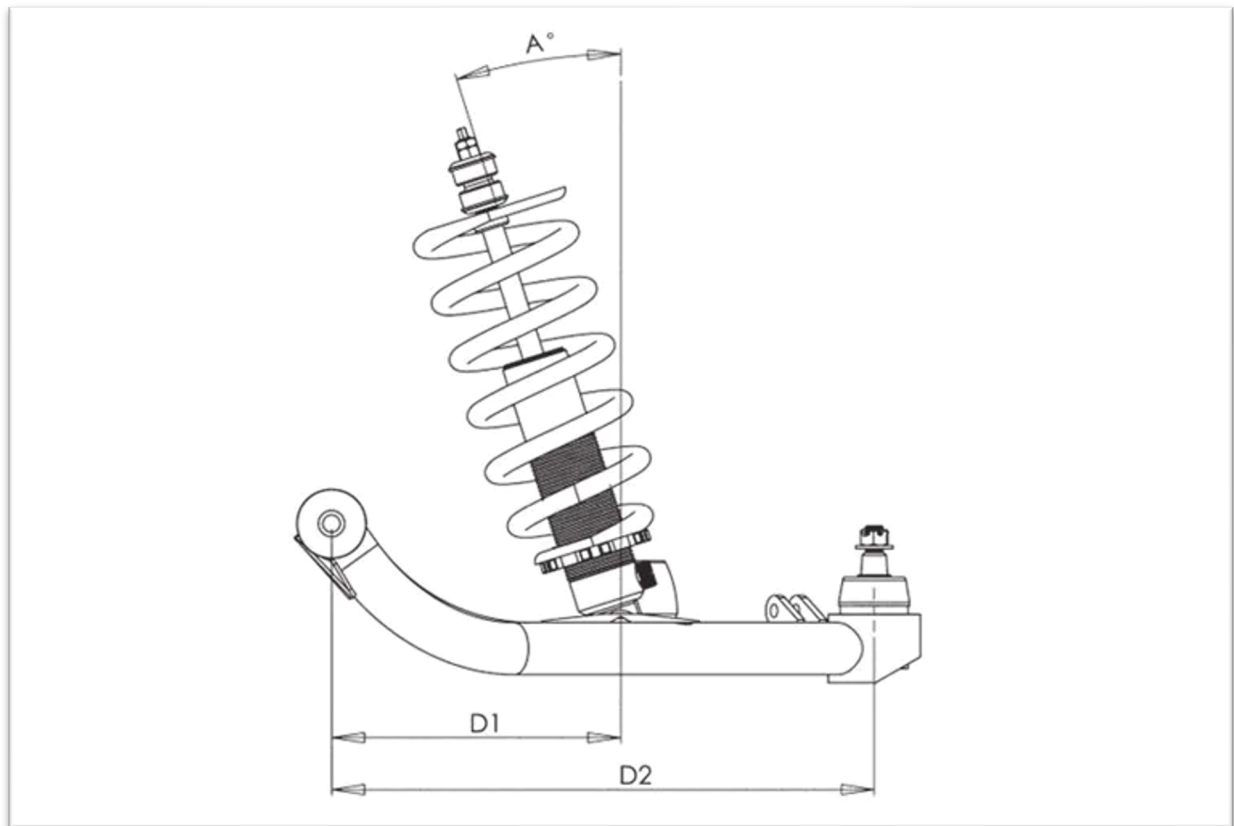


Figure 12 Schematic double wishbone suspension (SuspensionSecret)

To calculate the chassis roll stiffness per degree of roll, substituting the track width of the vehicle and the value of the combined wheel rate in roll into the equation 26.

$$K_{chassis} = \frac{K_{wheel} w^2}{\left(\frac{360}{\pi}\right)}$$

Equation 26 Vehicle roll stiffness equation.

$K_{chassis}$ = Chassis roll stiffness

w = Track width

4.7 Understeer and Oversteer

The definition of Ackermann steer angle $\bar{\delta}$ is the angle formed by the line connecting O to the mid-point of the rear wheel and O to the mid-point of the front wheel, as illustrated in figures (13) and (14).

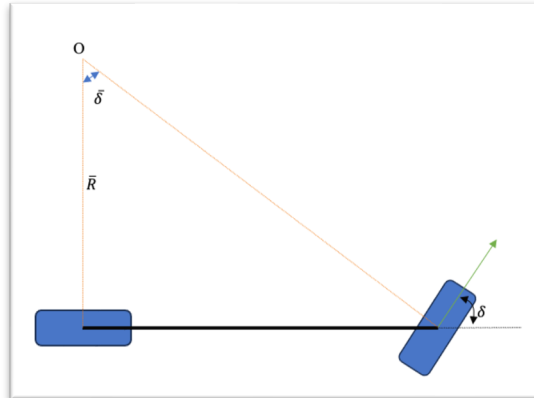


Figure 13 Kinematic bicycle model

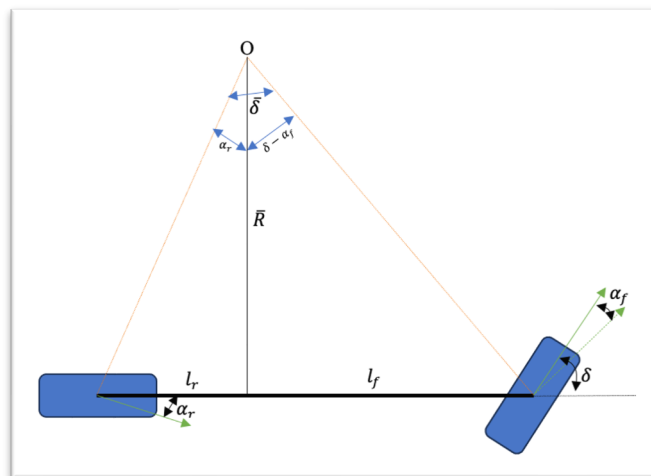


Figure 14 Dynamic bicycle model

Tangential Radius:

Let $\bar{C}$ represent the location where the slip angle equals O in the kinematic bicycle model.

The distance between the instantaneous centre turns O and $\bar{C}$ is known as the tangential radius $\bar{R}$.

In the kinematic model, $\bar{\delta}$ is always equal to δ , as illustrated in figure (13). However, from figure (14), $\bar{\delta} = \delta - \alpha_f + \alpha_r$.

For kinematic model figure (13),

$$\tan \bar{\delta} = \frac{l}{\bar{R}}$$

$$\bar{\delta}_{kinematic} = \tan^{-1} \frac{l}{\bar{R}}$$

For dynamic model figure (15),

$$\tan \alpha_r = \frac{l_r}{\bar{R}}$$

And

$$\tan(\delta - \alpha_f) = \frac{l_f}{\bar{R}}$$

But $\bar{\delta} = \delta - \alpha_f + \alpha_r$

$$\bar{\delta}_{dyn} = \tan^{-1} \frac{l_r}{\bar{R}} + \tan^{-1} \frac{l_f}{\bar{R}}$$

A dynamic bicycle model is said to be neutral steering during steady state cornering, if its steering angle $\bar{\delta}_{dyn}$ equals so that of a kinematic bicycle model $\bar{\delta}_{kinematic}$ when both are driving on a circle with same tangential radius $\bar{R}$.

Comparison of Ackermann angles for the same radius $\bar{R}$

$$\bar{\delta}_{kinematic} = \tan^{-1} \frac{l}{\bar{R}} = \frac{l}{\bar{R}}$$

And

$$\bar{\delta}_{dyn} = \tan^{-1} \frac{l_r}{\bar{R}} + \tan^{-1} \frac{l_f}{\bar{R}} = \frac{l_r}{\bar{R}} + \frac{l_f}{\bar{R}} = \frac{l}{\bar{R}}$$

So from the above equations, $\bar{\delta}_{kinematic} = \bar{\delta}_{dyn}$ if both model drive on a circle with same $\bar{R}$. Substituting the steering angle $\bar{\delta}_{kinematic} = \delta_{kin}$ and $\bar{\delta}_{dyn} = \delta_{dyn} - \alpha_f + \alpha_r$.

$$\delta_{dyn} = \delta_{kin} + \alpha_f - \alpha_r$$

Equation 27 Relation between kinematic steering angle and dynamic steering angle

So, the dynamic bicycle model is neutrally steering if $\alpha_f = \alpha_r$.

If $\alpha_f < \alpha_r$ oversteering condition.

If $\alpha_f > \alpha_r$ understeering condition.

4.8 Understeer gradient

Understeer or oversteer behaviour of the vehicle changes with the vehicle lateral acceleration. If the lateral acceleration is small (which is by low speed and/or large turning radius), the slip angle generally small and hence the vehicle is almost neutrally steer.

Understeer gradient quantifies the rate of changes of a vehicle's understeer (or oversteer, if the gradient is negative) with its lateral acceleration.

Assumption:

Stationary conditions during steady state cornering

Small steering angle

Steady state cornering $\delta \cong \delta_{dyn} = \delta_{kin} + \alpha_f - \alpha_r$

Linear tyre model where $F_f = C_{\alpha f} \alpha_f$ and $F_r = C_{\alpha r} \alpha_r$.

The cornering stiffness coefficient $C_{\alpha f}$ and $C_{\alpha r}$ depends linearly on vertical tire load.

$$\sum F_{i,z} = 0$$

$$F_{zf} + F_{zr} = mg$$

$$\sum_{COG} M_{i,z} = 0$$

$$F_{zf} l_f - F_{zr} l_r = 0$$

From above equations, $F_{zf} = \frac{l_r}{l} mg$ and $F_{zr} = \frac{l_f}{l} mg$. where $\frac{l_r}{l} = W_f$ is fraction of weight in front tire and $\frac{l_f}{l} = W_r$ is fraction of weight on rear tire.

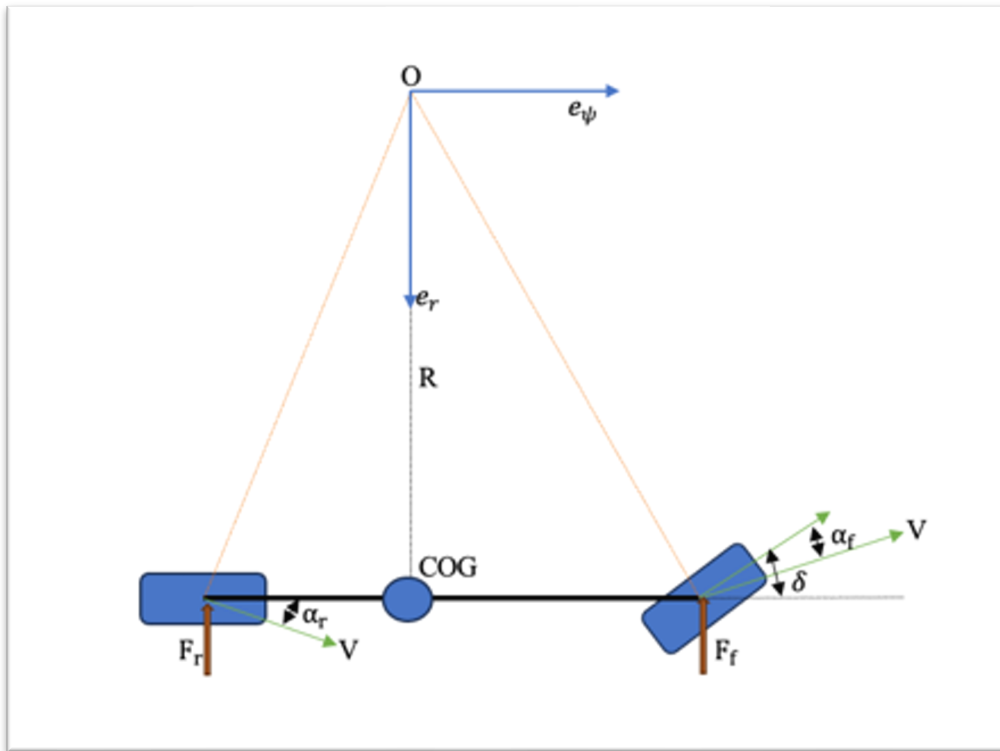


Figure 15 Dynamic model

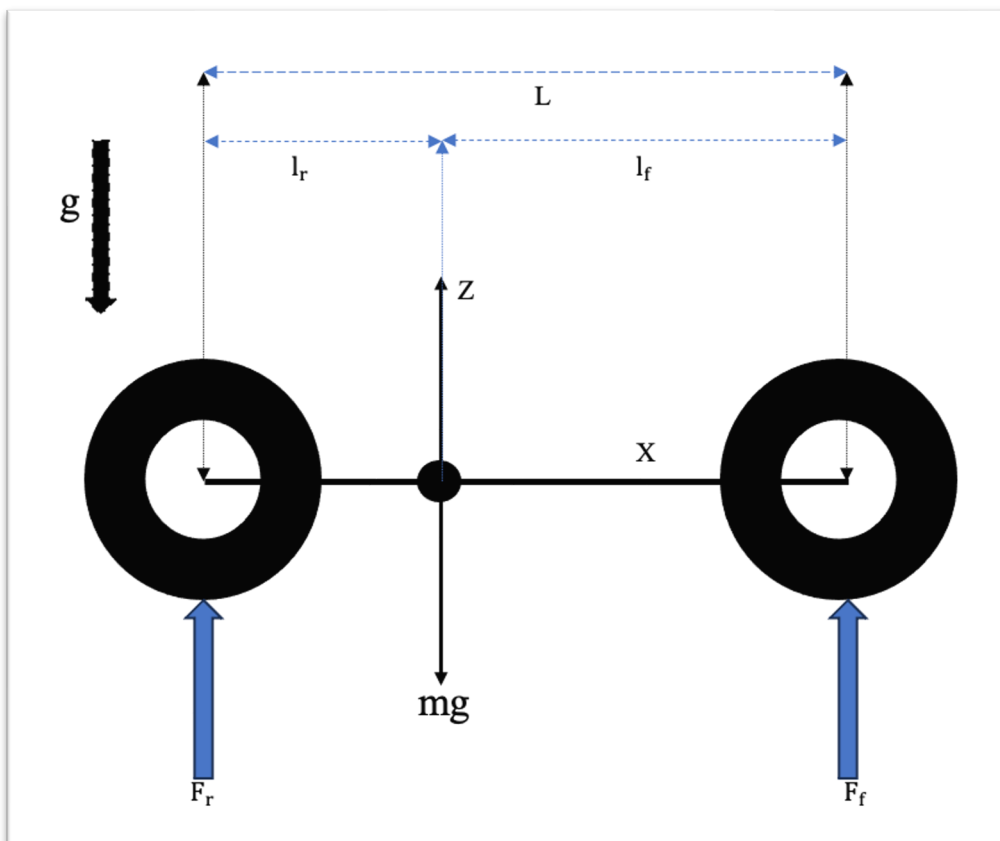


Figure 16 Force acting on dynamic vehicle model

Considering figure (15), balance of forces in radial direction e_r .

$$ma_r = -F_{zf} - F_{zr} \quad i1$$

$$a_r = \text{centripetal acceleration} = -\frac{V_{\text{long}}^2}{R}$$

Balancing of moment in Z-direction

$$I_z \ddot{\psi} = F_{zf} l_f - F_{zr} l_r \quad i2$$

$\ddot{\psi} = 0$ for stationary rotation.

Solving for equation i1 and i2.

$$F_f = \frac{l_r}{l} \frac{V_{\text{long}}^2}{R}$$

And

$$F_r = \frac{l_f}{l} \frac{V_{\text{long}}^2}{R}$$

Substituting into linear tyre model force equation for front and rear slip angle:

$$\alpha_f = \frac{1}{C_{\alpha f}} F_f = \frac{W_f}{C_{\alpha f}} \frac{V_{\text{long}}^2}{R}$$

And

$$\alpha_r = \frac{1}{C_{\alpha r}} F_r = \frac{W_r}{C_{\alpha r}} \frac{V_{\text{long}}^2}{R}$$

By knowing Steady state cornering condition;

$$\delta_{\text{dyn}} = \delta_{\text{kin}} + \alpha_f - \alpha_r = \delta_{\text{kin}} + \left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right] \frac{V_{\text{long}}^2}{R}$$

The equation effectively illustrates how various parameters affect the necessary steering

angle. The steering input required, in terms of both magnitude and direction, is determined

by the expression $\left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right]$. It is composed of two terms, each representing the wheel load

to cornering stiffness ratio. The understeer gradient is the term for this (K) and $\frac{V_{\text{long}}^2}{R}$ is

lateral acceleration.. There are three options for handling a vehicle in a steady state: neutral steer, understeer, and oversteer. Now, each of these will be covered:

- Neutral steer

If $\left[\frac{W_f}{C_{\alpha f}} = \frac{W_r}{C_{\alpha r}} \right]$ that means $\alpha_f = \alpha_r$

This is known as neutral steer. Regardless of the vehicle speed or lateral acceleration, the steering wheel angle in a constant radius test remains constant and is equivalent to the Ackerman steering angle.

- Understeer

If $\left[\frac{W_f}{C_{\alpha f}} > \frac{W_r}{C_{\alpha r}} \right]$ that means $\alpha_f > \alpha_r$.

It's known as understeer. A constant radius test would require the steering angle to increase as the speed increased. The lateral acceleration, or the square of the speed, is directly proportional to the steering angle growth. A greater steering angle puts more lateral force on the front tyres in an effort to keep the total moments around the CG point at zero. The tendency of the vehicle's front to stray outward is countered by increasing the steering angle.

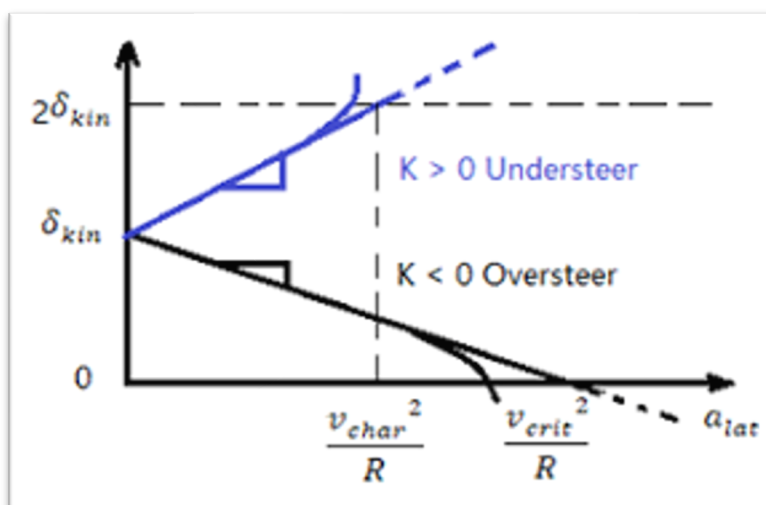


Figure 17 Understeer gradient characteristic with respect to vehicle speed and input steering angle.

- Oversteer

If $\left[\frac{W_f}{C_{\alpha f}} < \frac{W_r}{C_{\alpha r}} \right]$ that means $\alpha_f < \alpha_r$.

This term is called oversteer. In a constant radius test, the steering angle would need to decrease as the speed increased. The steering angle decreases according to the lateral acceleration and, hence, the square of the speed. A decrease in steering angle decreases the lateral force given to the front tyres in order to maintain zero sum of moments around the CG point. The car's propensity to drift out of the back is mitigated by lowering the steering angle. If the speed increases more, counter steering might be necessary to keep the vehicle on the intended course (Gillespie 1992).

4.9 Roll moment distribution.

The vehicle is impacted by lateral acceleration produced during high speed cornering. Newton's second law states that this causes a lateral force (F_y) to be applied to the CG point. The lateral force imposes a moment around the roll centre because the height of CG (H_{cg}) is often greater than the height of the roll centre (h_{rc}) on a conventional vehicle. As a result, upon rotating, the body leans to the outer.

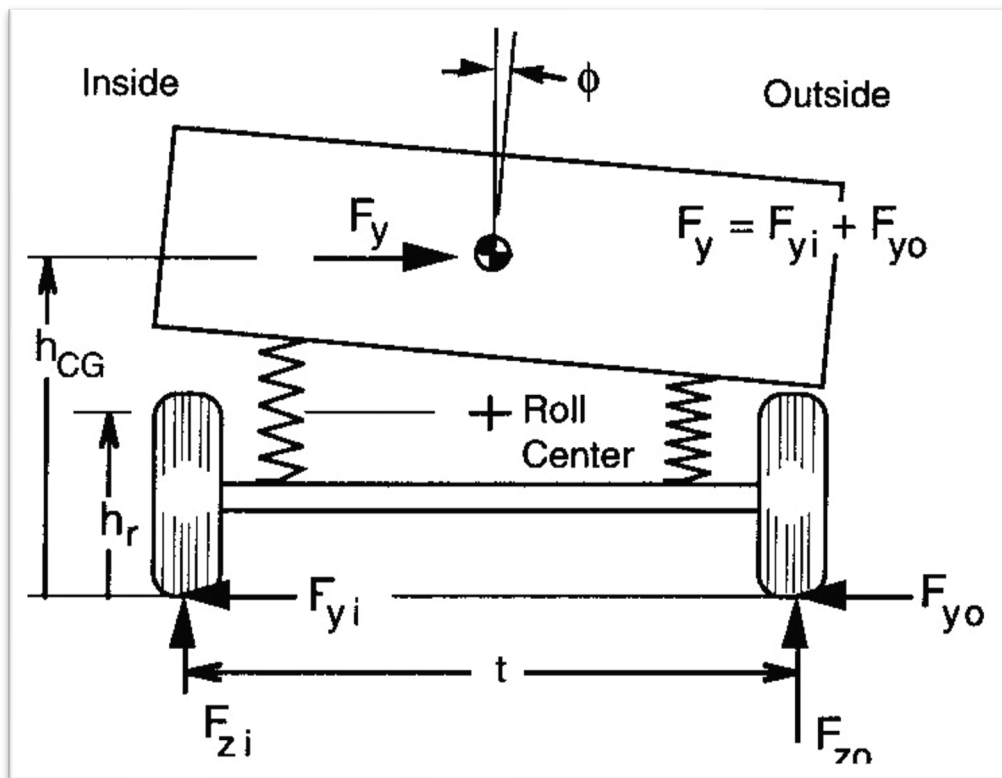


Figure 18 Force acting on vehicle during cornering (Tsirogiannis 2015)

As a result, there is an equal increase in vertical loads on the outside wheels and a decrease on the inside wheels. Because of the non-linear relationship between the lateral force and the vertical load, the vehicle's ability to create lateral force is reduced when the vertical load between its inside and outside wheels changes. Reduced lateral force that the tyres can produce is achieved by increasing the lateral load transfer (e.g., by employing stiff springs or an anti-roll bar).

This explains, among other things, why the CG point is moved inside a Winston Cup race car to create an asymmetrical design. When cornering on the oval track at a particular speed, this causes the vertical stress on the tyres to be equal. Because of this, the tyres are able to produce more lateral force, which speeds up the car's cornering acceleration (Haubenreich and Law 2000).

The vehicle's oversteer or understeer characteristics may also be influenced by this lateral load transfer. The lateral force that the front tyres of the car may produce will decrease if

the front wheels of the car transmit more lateral load than the back wheels. By increasing the steering angle, the front tire's slip angle will need to be raised to make up for this. As a result, understeering will occur more frequently. Similarly, a car will oversteer if its rear wheels transfer more lateral force than its front wheels.

On a road contact point, the total of the moments around the point of rotation during rollover is analysed to determine whether a vehicle will slide before rolling. The car revolves around the outside wheel's contact point as it rolls over. Assuming steady state conditions and rigid suspension, the sum of the moments around this position during cornering, with reference to Figure 18, is given by:

$$\sum M_o = 0$$

$$Mg \frac{t}{2} - Ma_y H_{cg} = 0$$

From above equation, $\frac{a_y}{g} = \frac{t}{2H_{cg}}$. The rollover threshold, denoted by $\frac{a_y}{g}$, is the value of the lateral acceleration at which the vertical force on the inside wheel (F_{zi}) is zero. The following relationship needs to hold in order to guarantee that the car will always slide rather than roll:

$$\frac{a_y}{g} < \frac{t}{2H_{cg}}$$

But, $F_{zmax} = Ma_y = \mu Mg$, where μ is coefficient of friction

From above equation,

$$\mu < \frac{t}{2H_{cg}}$$

This equation describes this relationship: the car will slide before rolling if the track width divided by two times the CG height is greater than the friction coefficient of the road. The

static stability factor (SSF), which is found on the right side of this equation, is expressed as(Sayers 1999)

$$SSF = \frac{t}{2H_{cg}}$$

Equation 28 Static Stability Factor equation

This theoretical examination of the lateral dynamics of a simplified linear vehicle model leads to the conclusion that increasing a vehicle's lateral acceleration capacity is necessary to improve handling performance. However, if the lateral acceleration is considered the optimising variable, Equation μ will rise, causing the car to roll before sliding. This is an unfavourable circumstance because it jeopardises the vehicle's safety. This is true for the majority of cars, including SUVs, that have high CGs.

A study on the parameters that should be utilised to measure and optimise a vehicle's handling was carried out. Test results clearly indicated that roll angle is a useful variable for handling quantification. It can also be used to optimise suspension settings when a specific road and manoeuvre are used(Uys et al. 2006).

This means that in order to measure and improve the vehicle's handling, the roll angle will be employed. The vehicle's roll stiffness increases as the roll angle is reduced during a handling manoeuvre, increasing the vertical load transfer on the tyres. This must be kept in mind. Equation $\mu < \frac{t}{2H_{cg}}$ will have less μ because of the tyres' reduced ability to produce lateral force as a result. Put otherwise, maximising the vehicle's roll angle off-road increases safety and decreases the likelihood that the car would roll over.

One of the following three techniques can be used to optimise the roll angle during a handling manoeuvre:

1. Reduce the vehicle's centre of gravity. The roll centre of the vehicle and the CG of the vehicle are closer when the CG point is lowered. As a result of the vehicle's mass, the shortened distance reduces the moment about the roll centre during lateral acceleration. Because of the reduced moment, the vehicle's roll angle will also be reduced, which will lessen the vertical load transfer on the tyres. The car will be more likely to slide before rolling because of the lower CG height's increased SSF. Semi-active, slow-active, or active suspension systems can all achieve a lower CG point.
2. Stiffen and/or dampen the suspension more. The car will handle better in dynamic situations by raising suspension damping, and it will corner more steadily in steady condition when the suspension stiffness is increased. This is because reducing the suspension's damping and/or stiffness causes the car's body roll to decrease. This decrease in body roll raises the tyres' vertical load transfer while lowering the total side forces that are available at each axle. Systems for passive, semi-active, or active suspension can improve the stiffness and/or damping of the suspension.
3. It is possible to add another system to make the car more roll stiff. This will reduce the total available side forces at each axle and increase the vertical load transfer on the tyres. Examples of these systems include semi-active and active anti-roll bars, as well as semi-active and active suspension.

5. Selection of solution method

The literature contained a good deal of suggestions for enhancing vehicle handling. The following is a list of these solutions:

- Passive suspension system
- Four state Semi active suspension system (4S4)
- Active suspension system
- Passive anti roll bar
- Semi active anti roll bar system
- Active anti roll bar system
- Tilting vehicles

Two of these approaches will be covered in this study. The suggested solutions will be weighed in accordance with a set of criteria in order to identify the best option. These standards are:

1. Can a sport car use this solution?
2. Is this solution going to work well in a challenging off-road setting?
3. In the literature, how is this solution improved upon?
4. Is it possible for this method to enhance the car in other ways as well?
5. How much energy does this solution use?
6. Is it possible to include or add this solution to the present vehicle?

Every criterion question is given a weight based on how significant a role it plays in selecting the best answer. After then, a score out of 10 is assigned to each solution for each question. After multiplying the supplied mark by the weighting, each solution's weighted marks are added together. Table 3 displays this technique of weighing. The following are the solutions derived from the literature:

1. The passive suspension system (Wilde et al. 2005)
2. 4S4 explained by (Els et al. 2007)
3. The semi-active suspension system with variable damping (Ahmadian and Simon 2004; Kim et al. 2005; Yoon et al. 2006)
4. The active anti-roll bar, as explained by the following authors: (DARLING and HICKSON 1998; Everett et al. 2000; Danesin et al. 2003; Kim and Park 2004; Cimba et al. 2006; Sorniotti 2006)
5. Active roll control using fully active suspension, as described (Hubert and Kumar 2005)
6. The tilting vehicles (Gohl et al. 2004).

Neither the tilting vehicle concept nor the 4S4 will be considered throughout the decision-making process. This is due to the fact that the 4S4 has already undergone extensive investigation, and the fact that it cannot switch to the handling mode when manoeuvring on a bumpy road is sufficient justification for dismissing it as a viable option. An SUV or other large vehicle cannot use the tilting vehicle concept.

Method	Question	1	2	3	4	5	6	Total	Rank
	Weighting	0.2	0.1	0.2	0.2	0.1	0.2		
Passive suspension		10	9	4	2	10	7	6.5	4
Semi-active suspension		10	9	5	4	8	8	7.1	3
Active suspension		10	8	8	7	3	8	7.7	2
Active anti roll bar		10	9	7.5	5	6	9	7.8	1

Table 4 Method selection weighting table

For the first question, all of the answers received 10 points because they are all workable and completely compatible with a standard SUV. In response to question 2, every solution—aside from the active suspension—was given nine points since, despite their extreme resilience, systems can still malfunction. Because there are more dependent sensors in the system that may give inconsistent readings or possibly fail, the active suspension received

eight points. Points were given at question three in direct proportion to the average percentage improvement for each solution discovered in the literature. The answer to Question 4 was determined by looking at how the system affects other components of the dynamics of the vehicle and how flexible it is to enhance other areas as well. The passive suspension system received a score of two points due to its limited adjustability and capacity to address only two unique conditions. Although there is a limited degree of improvement, the semi-active damping system can be beneficial in many different situations. A restricted range of situations can be improved by the active anti-roll bar; nevertheless, the improvement is greater because of the larger energy input compared to the semi-active dampening system. Out of all the systems, the active suspension has the most potential to enhance several elements. Points were given for the quantity of energy required in question 5 based on the literature, and points were given for the number of changes needed in question 6 to integrate the system with the current 4S4.

It's clear from Table 4 that the active anti-roll bar (AARB) is the ideal recommendation. The AARB will be installed on the 4S4 in the soft setting to maintain optimal ride comfort (soft springs and low damping). It should be noted that the best option was almost definitely roll control with active suspension, and that this would also be a good alternative (e.g. 4S4 with rapid height adjustment at each strut).

5.1 Conclusion

Drawing from the evidence presented in this chapter, the following conclusions are drawn:

- Based on the hypothesis, it was determined that a vehicle's handling skills might be enhanced by taking one or more of the following actions:
 - Lower the car's CG point.

- Tighten and/or dampen the suspension.
- To improve the tyres' vertical load transfer, install one more system.
- Numerous approaches to solving the issue statement were presented in the literature.
- To find the best solution for the issue statement, all of the solutions that could be located in the literature were weighed against the specified criteria. The most appropriate option was found to be the active anti-roll bar solution, which will be used for the remainder of the study.
- Using the 4S4's soft suspension (soft springs and low damping) to absorb road irregularities and control body roll with an active anti-roll bar was the solution developed when it was determined to concentrate on the problem of increasing handling without losing ride comfort on rough roads (AARB).

6. Passive and Active Anti Roll Bar System

6.1 Passive Anti roll Bar system

The most widely utilised ARBs in automobiles nowadays are passive ones. Usually shaped like a U, a passive ARB is made of a metal bar. ARBs with as few bends as possible are ideal, although the precise shape is frequently complicated for packing reasons (Gohl et al. 2004).

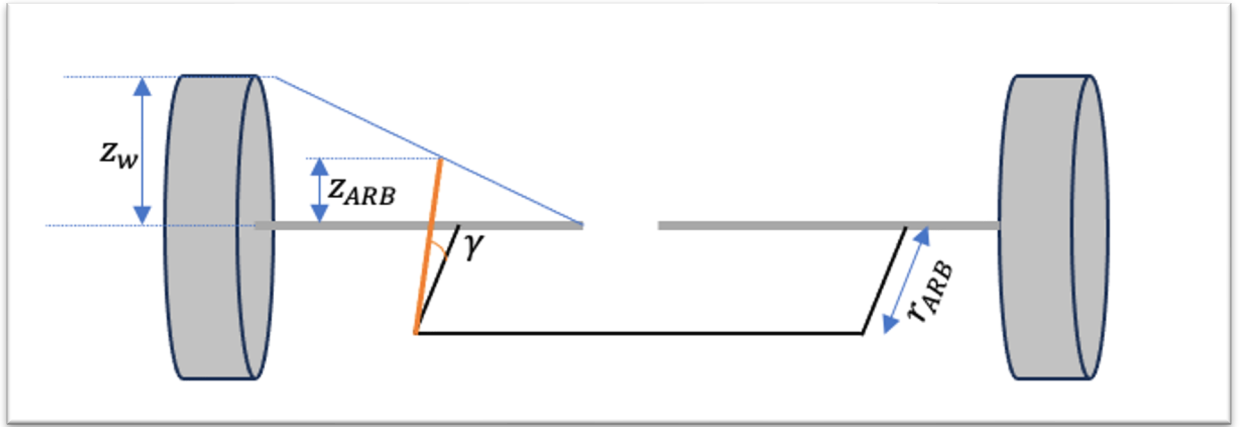


Figure 19 A basic U-shaped ARB.

To counteract roll motion, the ARB's capacity is determined by its torsional stiffness, geometry, and point of connection to the chassis. In this thesis, the ARB is seen as strictly U-shaped for simplicity's sake; refer to Figure 19. Moreover, the difference in vertical motion between the wheel z_w and the component under study—in this case, the ARB z_{ARB} is explained by the idea of motion ratio. The suspension kinematics, the ARB's location, and its orientation are all responsible for this discrepancy (Gohl et al. 2004). Equation 29 states that the motion ratio (i_{ARB}) connects the vertical movements and associated forces.

$$i_{ARB} = \frac{z_{ARB}}{z_w} = \frac{F_{ARB, w}}{F_{ARB}}$$

Equation 29 Motion ratio of wheel travel and anti-roll bar arm travel

The forces generated at each of the two ARB ends are denoted by F_{ARB} in equation 29, and the corresponding forces acting on the wheel are denoted by $F_{ARB, w}$. It is assumed that the ARB has an arm length r_{ARB} and torsional stiffness k_γ . Exciting the left wheel while keeping

the right wheel immobile, as shown in Figure 19, serves as an example of how the ARB works. As a result, the ARB torques, creating forces at the ends that act on the wheels. Equation 30 is used to find the corresponding force acting on each wheel $F_{ARB, w}$. Note that the same equation can be used with τ_{ARB} by replacing the torque generated by the ARB with the product of the torsional stiffness k_γ and the torsion angle γ .

$$F_{ARB, w} = \frac{k_\gamma \gamma}{2r_{ARB}} i_{ARB} = \frac{\tau_{ARB}}{2r_{ARB}} i_{ARB}$$

Equation 30 Torque equation of the anti-roll bar

6.2 Active Anti-roll bar system

Anti-roll bars regulate the vehicle's roll stiffness separately from its vertical (riding) stiffness. This makes it possible for the anti-roll bar to regulate both the vehicle's body roll angle and the vertical load transfer between the left and right wheels. To enhance the effectiveness of a passive anti-roll bar, active anti-roll bars are composed of an actuator and an anti-roll bar. This enables the anti-roll bar to engage with the car only when necessary.

The application of an active anti-roll bar for SUVs was studied by (Everett et al. 2000). To achieve this, a hydraulic cylinder that was directly proportionate to the vehicle's lateral acceleration was used to actuate an anti-roll bar. The control system for the cylinder was open loop. This study involved only simulations. Three assessments were carried out: the mild ramp input test, the severe ramp input test, and the steady state cornering test. Both a fully loaded vehicle and the driver alone were used for all of the tests. During the tests, the body roll angle of the vehicle was measured in order to quantify its handling. 24% improvement for the loaded vehicle and a fifty-five percent improvement for the vehicle with the driver alone was seen in the steady state cornering test results. In the case of the driver-only car, the moderate ramp input test demonstrated a 70% improvement and the

loaded vehicle, 33% improvement; in the case of the two scenarios, the severe ramp input test similarly demonstrated a 70% and 31% improvement. By placing the accelerometer, which measures the lateral acceleration, further in front of the CG point, different findings were obtained. 0.75 metres in front of the CG point was the ideal distance. The accelerometer was placed in the ideal position for each test. Up to a 0.4 g lateral acceleration, the system was engineered to give the vehicle a nearly 0° roll angle. In the event of further lateral acceleration, the roll angle experiences a rise in tandem with the passive system. This action served as a warning to the driver that the vehicle's friction limit is being approached by the lateral acceleration.

A low-cost active anti-roll suspension for passenger cars was developed by (DARLING and HICKSON 1998). Two hydraulic cylinders were used to operate the two anti-roll bars (one at the front and one at the back) that made up this suspension system. At the vehicle's centre of gravity, the lateral acceleration is monitored and used to regulate the hydraulic cylinders in a direct proportion. This study only used simulations. A step steer test was conducted at a speed of 22.2 m/s (80 km/h) with a steering wheel input of 30 degrees. The vehicle's performance throughout the test was evaluated by measuring the body roll angle. The results of this test indicated an 82% improvement between the active and passive systems. Additionally, it was demonstrated that this technology increases the tyre camber by 34% throughout the test, improving the tyre's cornering capabilities.

Two hydraulic actuators were used to actuate the two anti-roll bars in (Darling et al. 1992) active anti-roll bar system. To forecast the system's reaction, simulations were run. In order to validate the simulations, the device was ultimately installed in a Ford Fiesta Mark II. Two experiments were carried out. In the first test, which was a steady state handling test, the vehicle's speed was increased while the steering angle remained unchanged. The vehicle's

lateral acceleration during this test varied between 0.5 and 8 m/s^2 . A dynamic handling test was the second test. A step input of magnitudes 90° , 180° , 270° , and 360° was applied to the steering wheel while the car was being driven at a steady speed during the test. By calculating the vehicle's body roll angle during test manoeuvres, the handling performance of the car was graded. The body roll during the steady state test improved by more than 80%, while the peak body roll angle during the dynamic handling test improved by over 80%, according to the data.

An anti-roll bar driven by a hydraulic cylinder makes up the active torsion bar system created by (Cimba et al. 2006). The hydraulic cylinder is released when roll control is not required, preventing the torsion bar from amplifying abnormalities in a single tyre. The technology was meant to reduce body roll up to a lateral acceleration of 0.5g. After doing simulations, the system was constructed to confirm the results of the simulations. Three tests were carried out to ascertain the system's response and to evaluate the hydraulic system's maximum hose length before it significantly affects the system. These included slalom, fishhook, and step steer tests. The body roll angle of the car was measured during the testing in order to quantify the vehicle's performance. The system demonstrated a 73 percent overall improvement in body roll angle, and it was found that a hose length of up to 24 metres could be employed before affecting the system's responsiveness.

In order to improve handling and comfort, (Danesin et al. 2003) created an active roll control system. Two hydraulic actuators operated each of the two anti-roll bars that made up this system. An Alfa Romeo 166–3.0 V6 was used to test the technology and conduct simulations. Two experiments were carried out to evaluate the system's response. The first test involved maintaining a constant speed for the car while providing the steering wheel with a sine input. The second test was a step steer test, in which the steering wheel is given

a step input while maintaining a constant speed. The vehicle's yaw rate and side slip angle were used to measure how well it performed during the testing. During the step steer test, the system showed a 20% reduction in the yaw rate and a 45% peak improvement in the side slip angle at 100 km/h with a step steering input of 100° over the baseline vehicle.

Two electro-mechanical torque actuators drive two anti-roll bars in (Sorniotti 2006) electro-mechanical active roll control system. Electronic stability control (ESC) was also used in conjunction with the active roll control system to evaluate how the vehicle responded to it. Simulators and experimental experiments were conducted. An severe step steer manoeuvre was used to test the system, and the body roll angle and yaw rate were used to evaluate the system's performance. In this test, using the active roll control system lowered the vehicle's body roll angle by 33%, and using the active roll control system with ESC reduced it by 42%.

A reliable roll motion control system was created by (Kim and Park 2004). Two linear electro-mechanic actuators that drive two anti-roll bars comprise this system. The system was controlled using a closed loop control system that uses the vehicle's lateral acceleration as an input. The hardware in the loop method was used to simulate the system. A test bench was built and included into the simulation in order to evaluate and ascertain the characteristics of the anti-roll bar system. The test manoeuvre that is used is step steer. The body roll angle was used to gauge how well the method worked. This technology showed a 73 percent reduction in the body roll angle at 100 km/h and an 85 percent improvement at 50 km/h in the step steer manoeuvre.

7. Control theory

There are various approaches to managing the active ARB's behaviour. The theory underlying the various controllers used in this thesis, as well as the active ARB plant model that the controllers are supposed to regulate, are explained in this part.

7.1 Active anti-roll bar plant model

A simulation-based s-function based on Simulink is utilised to replicate the behaviour of the physical electromechanical active ARB under study.

It is known what the plant model's inputs and outputs are, though. Wheel motion-induced disturbances and the intended torsion angle serve as the input. Delivered actuator torque, which is correlated with wheel forces based on equation 30, is the plant model's output.

Actual anti-roll bar torsion angle is the second output. As the input can be viewed as the intended torque divided by the stiffness of the system, the inputted torsion angle is, in an ideal scenario, proportionate to the torque output with the stiffness of the system.

For the purpose of comprehending the potential effects on the overall operation of the system, one must be aware of the fundamental workings of the ARB plant model. Limiting the output torque is the first and most significant non-linearity in the plant model. A specific maximum torque output can only be produced by the actuator. In addition, a frequency sweep reveals that this maximum torque output decreases with increasing frequency, indicating that the controller's bandwidth is frequency dependent. And last, hysteresis effects may arise from the actuator's friction, which is likewise modelled, adding to the plant model's nonlinearity.

7.2 PID controller

In industrial applications, PID controllers are frequently utilised. Its name derives from the way it determines the control signal by using a feedback signal and calculating the error between the feedback and the setpoint. Specifically, it makes use of the integral of the error, the derivative of the error, and a proportionate part. Figure 2.8 displays a general control method for an arbitrary plant that is PID-controlled.

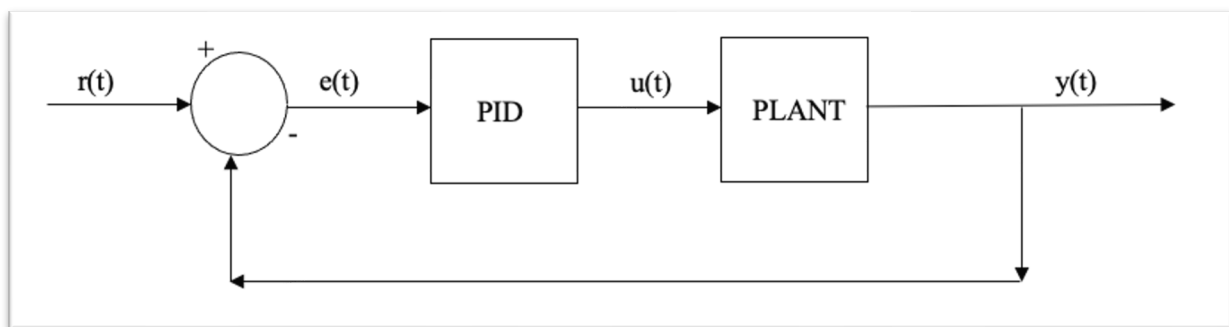


Figure 20 An example of a PID controller.

The error $e(t)$, or the difference between the reference $r(t)$ and the output y , is used to determine which control signal, $u(t)$, to send to the plant in order to regulate the system (t) . Selecting the appropriate parameter for control requires an understanding of how the PID controller influences the plant. Applying a PID controller to the system given in equation 6, where the plant model is regarded as the vehicle in this case, will demonstrate this.

Equation 31 is found by converting this equation into the Laplace domain.

$$\begin{aligned}
 I_{xx}s^2\phi(s) + 0.5w^2(K_{sf} + K_{sr})\phi(s) + 0.5w^2(C_{sf} + C_{sr})s\phi(s) \\
 = -\{0.5wK_{sf}(Z_{ufl} - Z_{ufr}) + 0.5wC_{sf}(\dot{Z}_{ufl} - \dot{Z}_{ufr}) \\
 + 0.5wK_{sr}(Z_{url} - Z_{urr}) + 0.5wC_{sr}(\dot{Z}_{url} - \dot{Z}_{urr}) + m_s h_{cgM} a_y\}
 \end{aligned}$$

Equation 31 Laplas domain equation of roll angle

When a PID controller with the gains K_P , K_I , and K_D is introduced, equation 32 represents the corresponding control signal that the controller produces in the Laplace domain.

$$u(s) = \left(K_P + K_I \frac{1}{s} + K_D s \right) e(s)$$

Equation 32 PID equation

A PID controller is used in equation 33 to minimise the roll rate $\dot{\phi}$, or to set the setpoint to zero. In this instance, the idealised active ARB plant is simplified to a simple stiffness $k_{\phi, AARB}$ and disregards the integrated controller.

$$\begin{aligned} I_{xx} s^2 \phi(s) + 0.5w^2 (K_{sf} + K_{sr}) \phi(s) + 0.5w^2 (C_{sf} + C_{sr}) s \phi(s) \\ + k_{\phi, AARB} \left(K_P + K_I \frac{1}{s} + K_D s \right) s e(s) \\ = -\{0.5wK_{sf}(Z_{ufl} - Z_{ufr}) + 0.5wC_{sf}(\dot{Z}_{ufl} - \dot{Z}_{ufr}) \\ + 0.5wK_{sr}(Z_{url} - Z_{urr}) + 0.5wC_{sr}(\dot{Z}_{url} - \dot{Z}_{urr}) + m_s h_{cgM} a_y\} \end{aligned}$$

Equation 33 Laplas transformation Roll angle equation with active anti roll bar system.

Equation 34 is obtained by rewriting equation 33. This indicates that increasing the roll rate using a PID controller improves the system's damping as well as its inertia through the derivative component and stiffness through the integral part.

$$\begin{aligned} (I_{xx} + K_D k_{\phi, AARB}) s^2 \phi(s) + (0.5w^2 (K_{sf} + K_{sr}) + K_I k_{\phi, AARB}) \phi(s) \\ + (0.5w^2 (C_{sf} + C_{sr}) + K_P k_{\phi, AARB}) s \phi(s) \\ = -\{0.5wK_{sf}(Z_{ufl} - Z_{ufr}) + 0.5wC_{sf}(\dot{Z}_{ufl} - \dot{Z}_{ufr}) \\ + 0.5wK_{sr}(Z_{url} - Z_{urr}) + 0.5wC_{sr}(\dot{Z}_{url} - \dot{Z}_{urr}) + m_s h_{cgM} a_y\} \end{aligned}$$

8. Simulation

Simulations were utilised to determine several important design parameters, forecast the system's performance, and assess the viability of the suggested solution. The route type and manoeuvres executed will be covered in this chapter, after which the simulation model will be examined. The model's improved and verified parameters will be discussed, and at the end, the simulations that will be run to evaluate the suggested fix will be showcased.

8.1 Simulink vehicle 7 DOF model

A model of the Morgan Plus Four model two-seater sports car was constructed and simulated using the MATLAB/Simulink R2023b software suite as shown in the figure 20. This model has 7 degrees of freedom. To do this, predefined variables were added to Simulink. In MATLAB and Simulink, these variables are utilised to do calculations. MATLAB and Simulink were used to calculate and solve the spring characteristics, damper characteristics, AARB characteristics, and control of the vehicle dynamics.

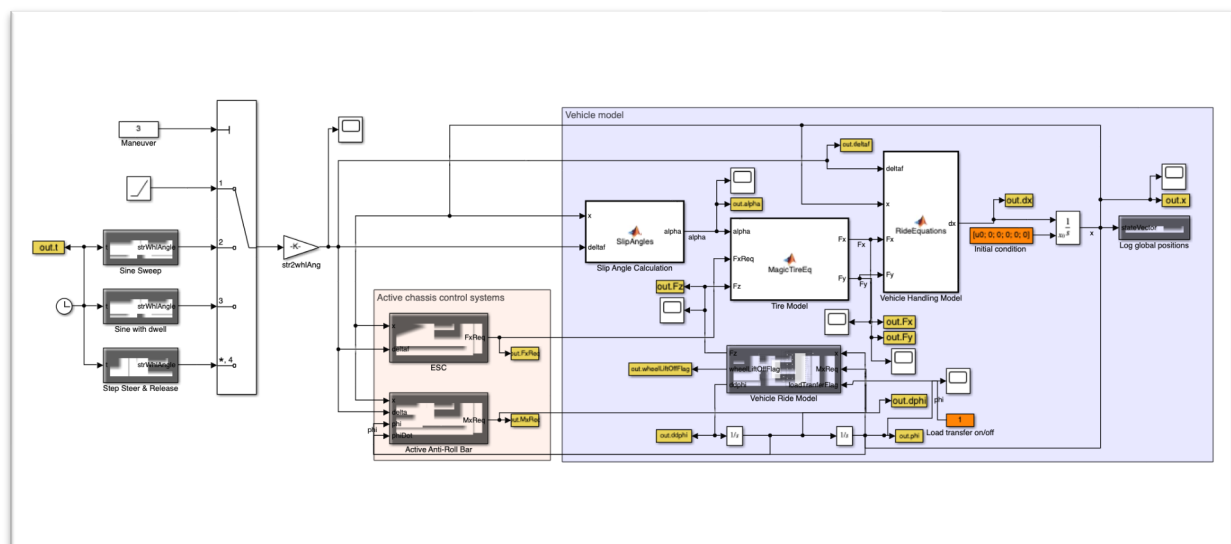


Figure 21 Morgan Plus Four Full Car Simulink Model

The Morgan Plus Four car's parameters are shown in table 5.

Vehicle mass	m	kg	1115
Sprung mass	ms	kg	987

Front unsprung mass	musf	kg	33
Rear unsprung mass	musr	kg	31
Pitch moment of inertia	lyy	kgm ²	1328
Roll moment of inertia	lxx	kgm ²	284
Yaw moment of inertia	lzz	kgm ²	1500.7
front spring stiffness	Kf	N/m	35000
Rear spring stiffness	Kr	N/m	28000
Front damping coefficient	Cf	N/m/s	Variable
Rear damping coefficient	Cr	N/m/s	Variable
Front roll stiffness of anti-roll bar	Karbf	Nm/rad	700
Rear roll stiffness of anti-roll bar	Karbr	Nm/rad	400
Vertical stiffness of front tire	Ktf	N/m	135000
Vertical stiffness of rear tire	Ktr	N/m	133000
Distance between CG to front axle	a	m	1.134
Distance between CG to rear axle	b	m	1.386
Front track width	tf	m	1.492
Rear track width	tr	m	1.492
Wheelbase	w	m	2.52
Anti-roll bar length	larb	m	1.2
Anti-roll bar arm length	larb_arm	m	0.27
Anti-roll bar diameter	darb	m	0.03
Weight distribution Ratio	FR:RR	%	55:45
Steering ratio			18
Height of CG	Hcg	m	0.3943

Table 5 Morgan Plus Four parameters

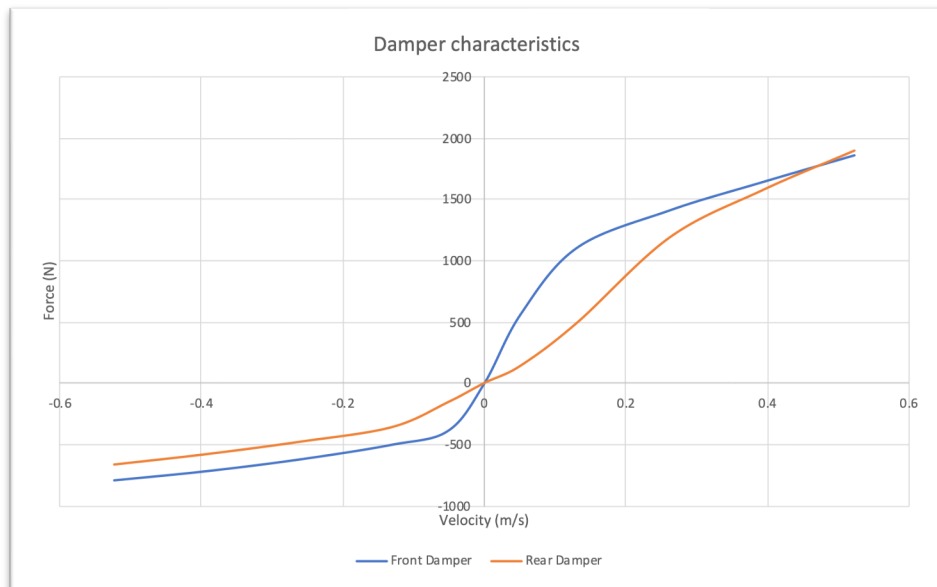


Figure 22 Damper characteristics

8.2 Test Manoeuvres condition for handling

Evaluating handling was done in both the linear and non-linear ranges, under both transient and steady state conditions.

Four manoeuvres were used to assess the handling capability of the active ARBs. The selection of manoeuvres encompassed all aspects of vehicle handling, including both transient and steady state manoeuvres in both linear and non-linear ranges. Stability systems like traction control, ABS, ESC, and so forth were deactivated during these manoeuvres because the evaluation of active ARBs must be conducted independently of other active systems. The selected manoeuvres are visualised in respect to the two dimensions in Table 6.

	Steady state	Transient
Linear	Constant radius	Frequency responded
Non-linear	Constant radius	Sine with dwell

Table 6 Handling performance evaluation and the chosen manoeuvre

8.2.1 Steady state cornering

According to the constant radius manoeuvre, a steady state cornering test was carried out. There are two variations of this test: one that uses many discrete speed steps and another that gradually increases the vehicle's speed until it reaches its limit. The latter option was selected in this case. The guidelines in the ISO standard were followed in selecting the parameters that define the manoeuvre. Table 7 lists all of the specific parameters that were used.

Parameter	Value
Constant radius	100 m
Max allowed deviation from path	± 0.5 m
Max allowed increase lateral acceleration	$0.2 \text{ m/s}^2/\text{s}$

Table 7 ISO 4138:2012 parameters for the constant radius manoeuvre(British Standards Document 2012)

A comparison of several metrics was used to analyse the vehicle's performance in the constant radius manoeuvre. A number of metrics are recommended in ISO 4138:2012, although only a subset of them were thought to be required to assess the performance of the active ARBs. Table 8 lists the metrics that were chosen, along with their meanings and units.

Metric	Definition	Unit
Roll angle gradient	$\dot{\varphi}/a_y$	deg/m/s ²
Understeer gradient	K_{us}	Rad/m/s ²

Table 8 metrics taken from ISO 4138:2012 that were applied to the constant radius cornering manoeuvre performance evaluation.

8.2.2 Frequency response

A frequency response test was conducted in compliance with ISO 7401:2011 (ISO 2011) continuous sine test specifications. The process involved driving the car at a fixed speed in a straight line before applying a continuous sinusoidal steering wheel angle with a rising frequency and maintaining a constant throttle input. Table 9 lists the parameters that define the manoeuvre. Performance evaluation was carried out using one more metric that was deemed significant in addition to the one provided in ISO 7401:2011 (ISO 2011). In table 10., the selected metrics are specified.

Parameter	Value
Speed	100 km/h
Frequency range	0.1 – 10 Hz
Lateral acceleration	4 m/s ²

Table 9 The continuous sinusoidal manoeuvre parameters are defined by ISO 7401:2011(ISO 2011)

Metric	Definition	Unit	Comment
Yaw rate gain	$\dot{\psi}/\delta_{SWA}$	deg/s/deg	In frequency domain
Roll angle gain	$\dot{\varphi}/a_y$	deg/m/s ²	In frequency domain

Table 10 measures chosen from ISO 7401:2011 that are applied to performance assessment.

8.2.3 Sine with dwell

Numerous manoeuvres, including sine steer, steadily increasing steer, constant radius circle, sine with dwell, etc., were assessed. It was discovered that sine with dwell may produce enough yaw and lateral displacement at comparatively small steering degrees. It is therefore possible to conclude that the test best captures the impact of the stiffness distribution between the front and rear axles and is optimal for assessing the vehicle's lateral stability. The method of steadily increasing steer test and recording the steering wheel input at which 0.3g of lateral acceleration is achieved yields an estimate of the nominal steering wheel angle for the move. Table 11 lists the sine with dwell manoeuvre parameters.

Parameter	Value
Frequency	0.7 Hz
Entrance speed	50 mph
Dwell time	0.5 sec
Nominal steer wheel angle	1.5 SWA at 0.3g
Steer wheel angle increment	0.5 SWA at 0.3g
Maximum Steer wheel angle	6.5 SWA at 0.3g or 270 deg

Table 11 Sine with dwell manoeuvre parameters

9. Simulation with Passive anti roll bar

To determine how the passive ARBs affect vehicle handling and to confirm the theoretical results, a series of simulations are run. Moreover, these simulations set the standard for assessing the active ARBs' performance.

9.1 Constant radius cornering

The roll behaviour of a vehicle with and without ARBs is depicted in Figure 23. It is evident that the passive vehicle's roll gradient is constant across the range of lateral accelerations under study, and that the roll gradient increases when the ARBs are removed. This is because the roll stiffness of the vehicle is lowered and is solely attributed to the actions of the dampers and springs.

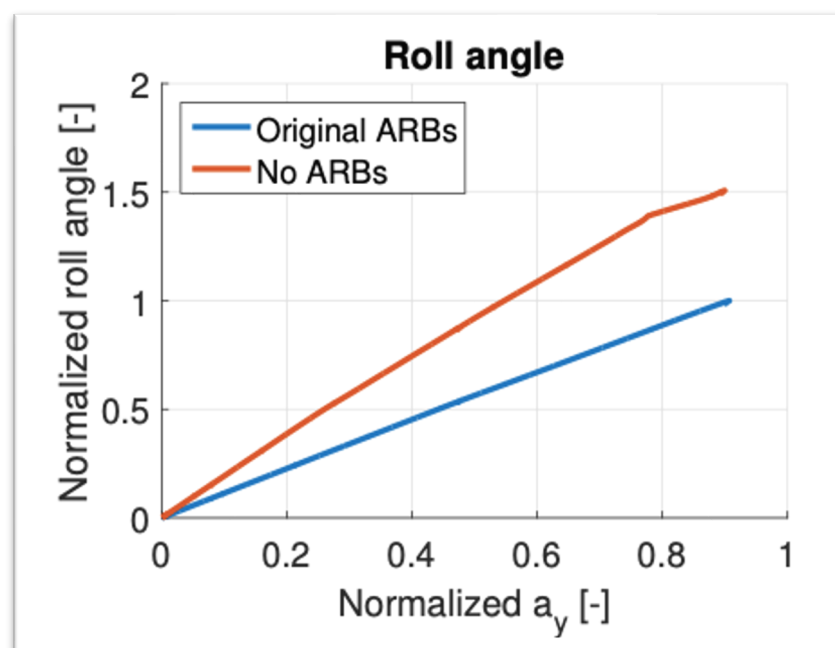


Figure 23 Roll angle vs lateral acceleration

The stiffness distribution between the front and rear axles determines how ARBs affect the vehicle's handling characteristics, as covered in section 4.3, Handling Theory. The understeer coefficient, or K_{us} values, are used to quantify the vehicle's handling characteristics. It is necessary to replicate extreme circumstances with only one ARB at a

time in order to determine the maximum impact that ARBs can have on handling. The handling characteristics for these scenarios are contrasted with the vehicle's default distribution in Figure 24.

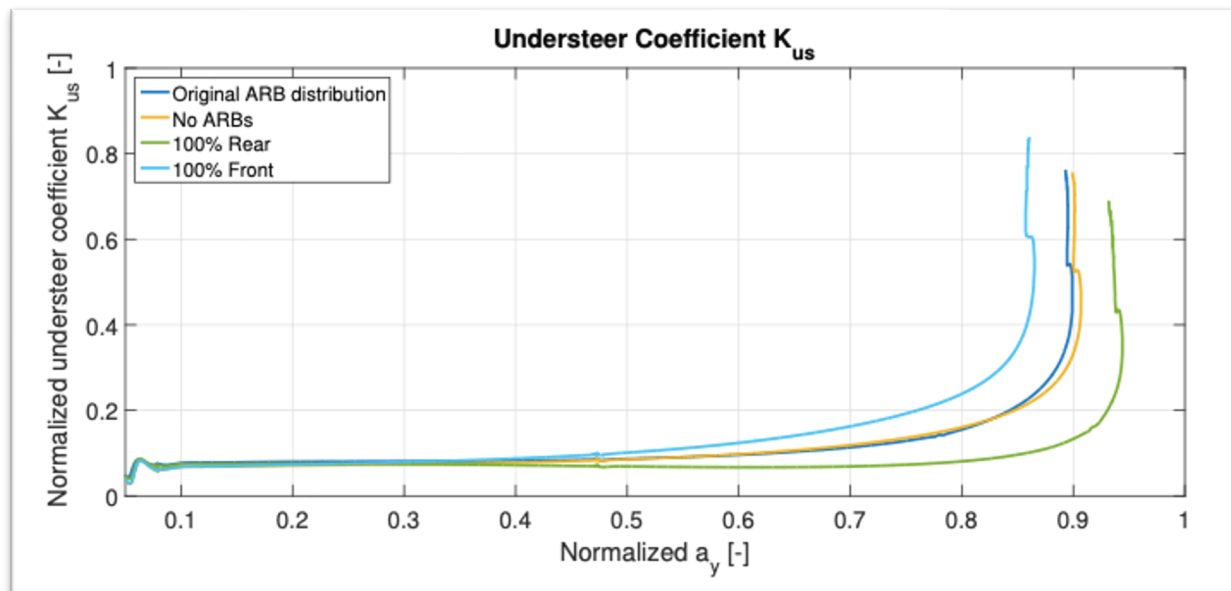


Figure 24 Handling characteristics for different stiffness distribution

Figure 24 shows that the vehicle's understeering behaviour is decreased when the rear axle stiffness is increased. It may turn more quickly and with less steering effort because of the increased weight transfer and consequent reduction of cornering stiffness on the rear axle. Additionally, this modification enables the car to accelerate laterally at higher values. When the front axle becomes more rigid, the front loses cornering stiffness more quickly and requires more steering input to perform the same manoeuvre. The opposite occurs. It's also important to notice that the vehicle's handling characteristics with passive ARB are very comparable to those without ARB. This suggests that the stiffness distribution of the suspension configuration and the stiffness distribution offered by the ARB on the front and rear axles are not all that dissimilar.

9.2 Frequency response

The results of frequency response simulations for a vehicle with ARBs, without ARBs, without front ARBs, and without rear ARBs are displayed in Figure 25 and Figure 26 in terms of normalised roll angle gain, or phase angle over frequency. This is to investigate how the vehicle roll behaviour is affected by the stiffness of the ARB. The test that is outlined in section 8.2.2 is followed while conducting the simulations. All outcomes are adjusted using the steady state roll angle gain of the original vehicle.

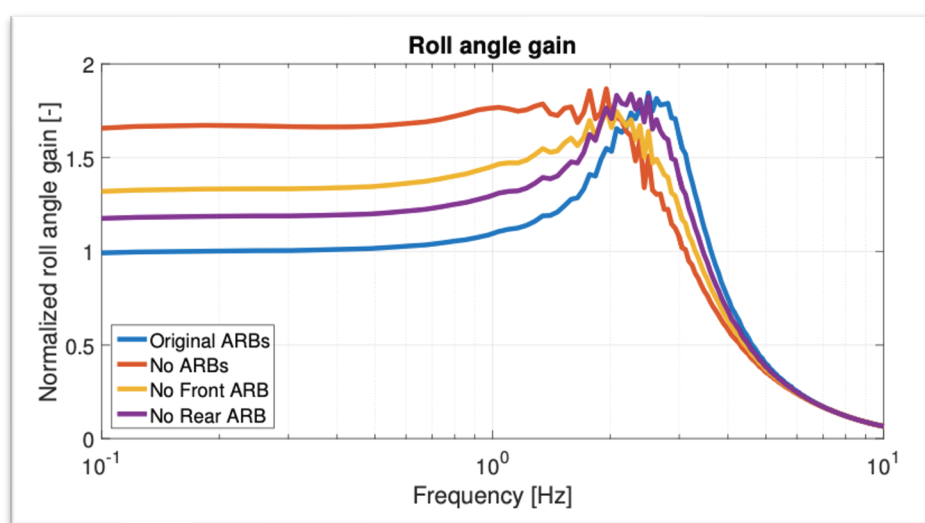


Figure 25 Roll angle vs frequency

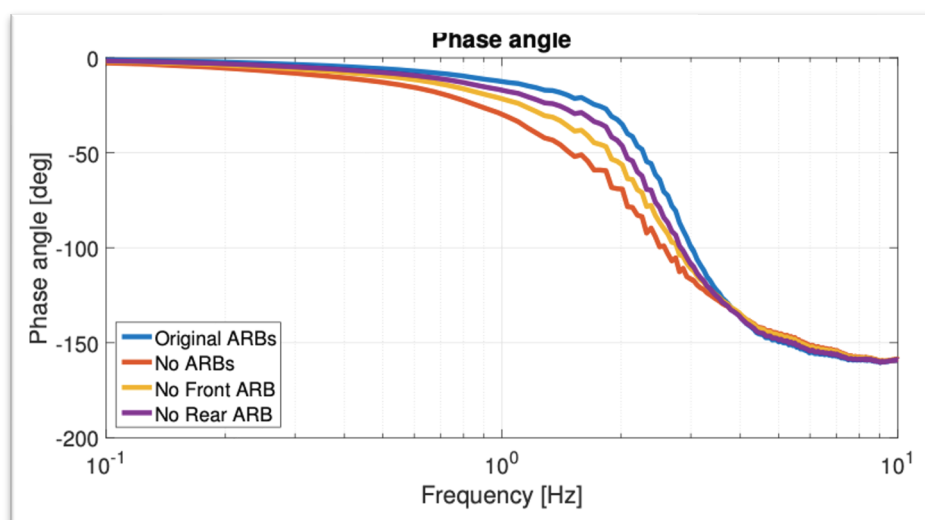


Figure 26 Phase angle vs frequency

Reduced roll stiffness is found to improve roll angle gain, shift the eigenfrequency to a lower frequency, and increase damping ratio; these conclusions are corroborated by the

simulation results. As predicted, the original vehicle has the maximum roll stiffness among the four simulations, followed by the vehicle without a rear ARB. ARB-free vehicles also have the lowest rigidity, with front-wheel drive vehicles coming in second. Roll damping follows the same hierarchy: vehicles without ARBs are almost completely damped, meaning that there is very little roll overshoot relative to the steady state value. It's also important to note that, as would be predicted, the higher roll damping causes the phase angle in Figure 26 to increase, which lengthens the interval between the peak of the lateral acceleration and the peak of the roll angle.

Finally, a coherence analysis of the data reveals that all simulations in the investigated frequency range had coherence levels above 98 percent. As a result, there is a high level of validity for the results.

Figure 27 and Figure 28 illustrates how the ARBs affect the yaw behaviour during a frequency response manoeuvre for the same vehicle settings. The findings demonstrate that, for all ARB combinations, in a stable condition, the yaw rate gain—see Figure 27—is lower than in the original vehicle. This demonstrates that the yaw rate gain is generally decreased when the roll stiffness is decreased. A possible explanation for this could be the understeer behaviour of the vehicle, which occurs when the suspension geometry of the vehicle rolls and causes kinematic toe angle changes. This understeers the vehicle more and reduces the yaw rate gain. To determine that this is the true cause, though, more research is necessary. Moreover, in the steady state, there is no discernible impact of the roll stiffness distribution on the yaw rate gain. The modest lateral accelerations are assumed to be the cause of this, meaning that the tyres operate within the linear range of the cornering stiffness. The reverse behaviour is observed around the eigenfrequency of the yaw rate, i.e., a higher yaw rate gain is achieved with lower roll stiffness and higher roll damping.

Additionally, it is seen that the yaw rate eigenfrequency remains unchanged in response to variations in roll stiffness.

Finally, for all four simulations, the coherence between the steering wheel angle signal and the yaw rate signal was good, above 99 percent over the whole frequency range under study.

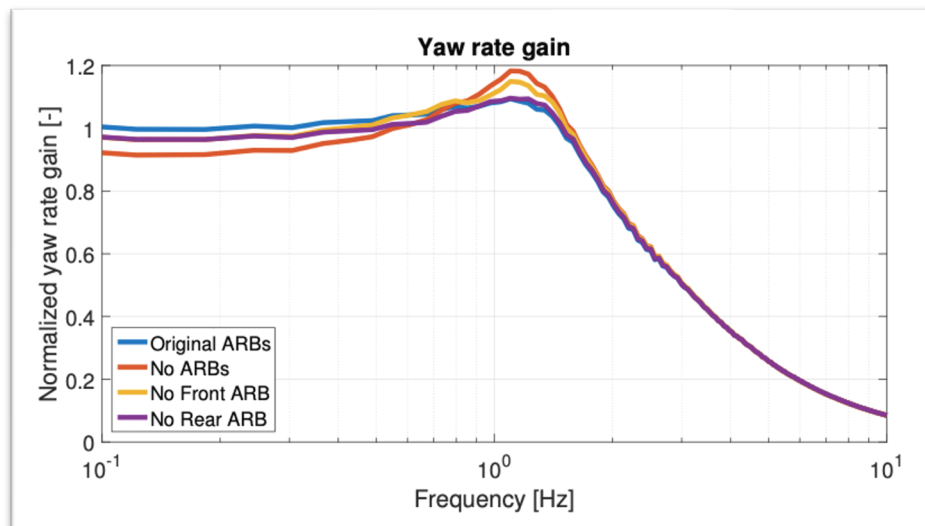


Figure 27 Yaw rate gain

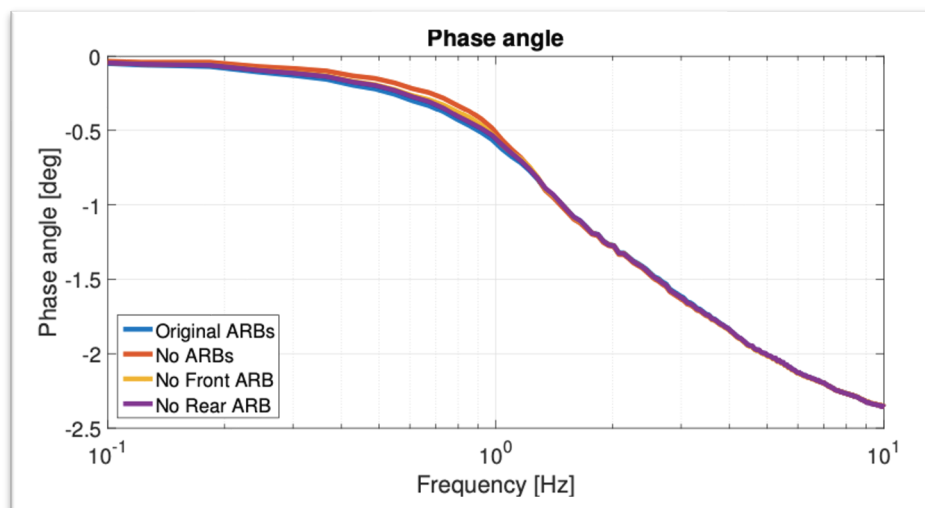


Figure 28 Phase angle

9.3 Sine with Dwell

Sine with dwell tests are conducted with and without ARB in order to investigate the impact of ARB on handling in non-linear transient settings. The greatest steering angle for each

measure that is within the bounds of the standard is found by gradually increasing the SWA.

The impact of ARB on a vehicle's roll and yaw behaviour is depicted in Figures 29 and 30, respectively.

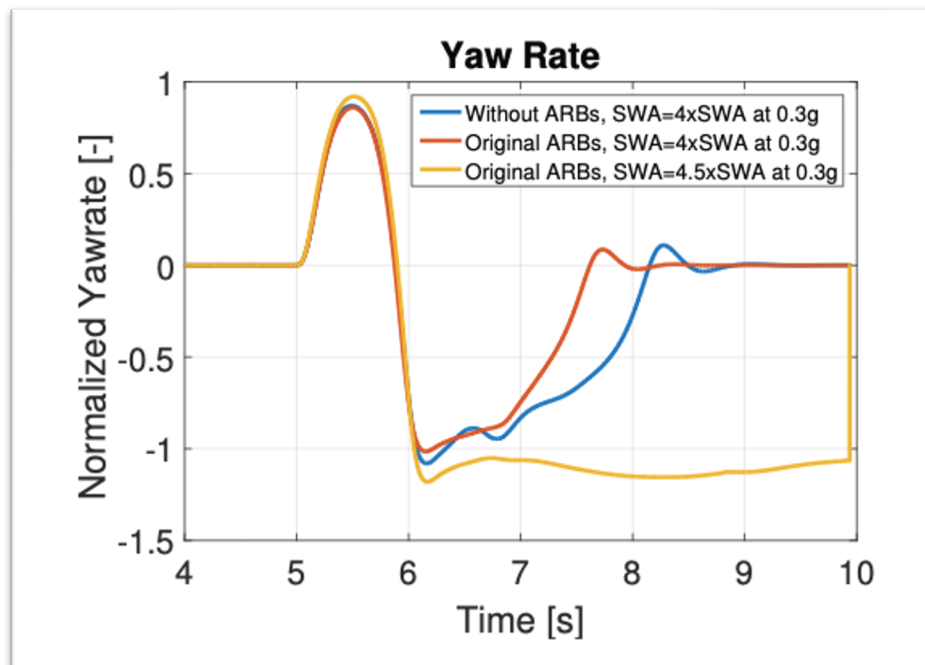


Figure 29 Yaw rate behaviour

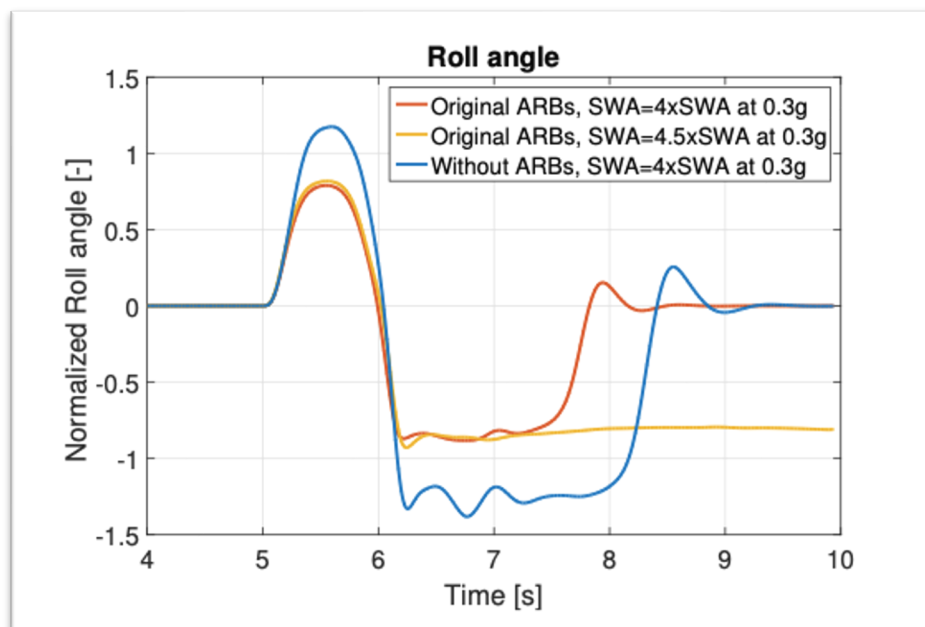


Figure 30 Roll angle behaviour

Figure 29 shows that the introduction of ARBs significantly improves yaw damping at a steering input of four times SWA at 0.3g. In the subsequent test, the steering angle is

increased to 4.5 times SWA at 0.3g. In this test, the vehicle experiences excessive yaw motion because passive ARBs are unable to offer enough lateral stability. Comparable findings are reached regarding the vehicle's roll performance, as seen in Figure 30.

It is evident from the findings shown in Figure 30 that the vehicle equipped with ARBs rolls more than the vehicle without ARBs. Additionally, a direct dependence of roll angle behaviour over yaw rate behaviour is noted when employing ARBs.

10. Simulation with Active Anti-Roll bar

The latest findings from applying the designed control method to the current vehicle model are shown in this chapter. Presentation of simulation data is made from both the handling and ride comfort models. The purpose of this is to demonstrate the capabilities of active anti-roll bars (ARBs) with the selected control technique and the degree to which the vehicle's suspension geometry influences it. Also illustrated and demonstrated are the shortcomings of active ARBs as well as the present control approach.

10.1 Ride comfort

A single-sided cosine wave is used to simulate ride comfort scenarios. To comprehend the constraints of the active ARBs and the control approach, the controller performance in these movements is assessed and examined.

10.1.1 Single side cosine wave

To assess the overall controller's effectiveness and demonstrate the differences in the vehicle's response to the active ARBs and the original ARBs, a series of single-sided cosine wave road disturbance tests are simulated. Additionally, the impact of test speed on the controller and active ARB combined performance is examined.

10.1.1.1 Controller performance

The evaluation of the single-sided cosine wave test performance involves simulating at a test speed of 60 km/h while activating the entire stiffness controller. This indicates that the roll angle, roll damping, and feed forward controllers are all employed with a tuning that is effective for all test cases considered in this thesis, including handling and ride comfort test cases. Additionally, the load transfer distribution used in these simulations is fixed and matches the vehicle's load transfer distribution with the original ARBs.

The roll angle for a single-sided cosine wave test at a test speed of 60 km/h is displayed as a function of time in Figure 31. The roll angle is shown to be lowered to almost -1 degree as opposed to -3.7 degrees for the car equipped with passive ARBs.

Additionally, since the roll overshoot observed for the passive vehicle is eliminated entirely, the roll damping is also increased.

Figure 32 shows the roll rate for the same simulation as a function of time. The findings demonstrate that the controller causes slight fluctuations in the roll rate signal and lowers peak roll speeds.

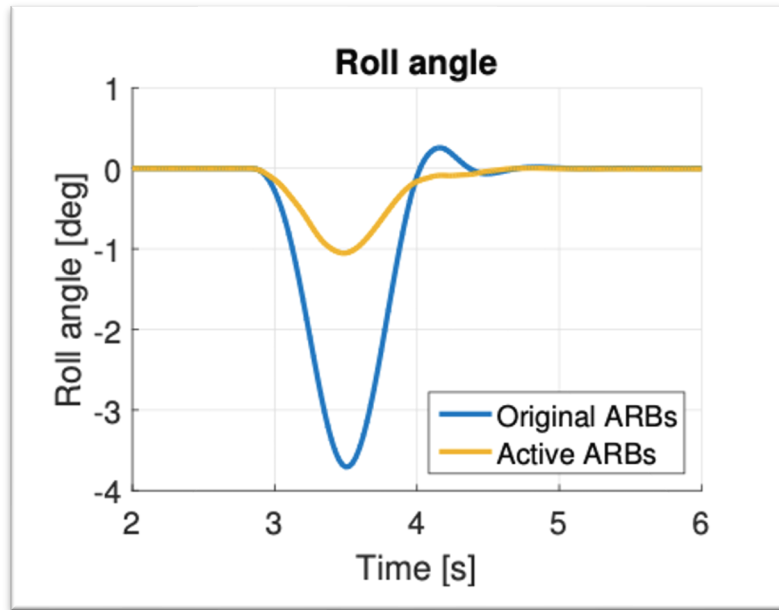


Figure 31 Roll angle speed at 60km/h

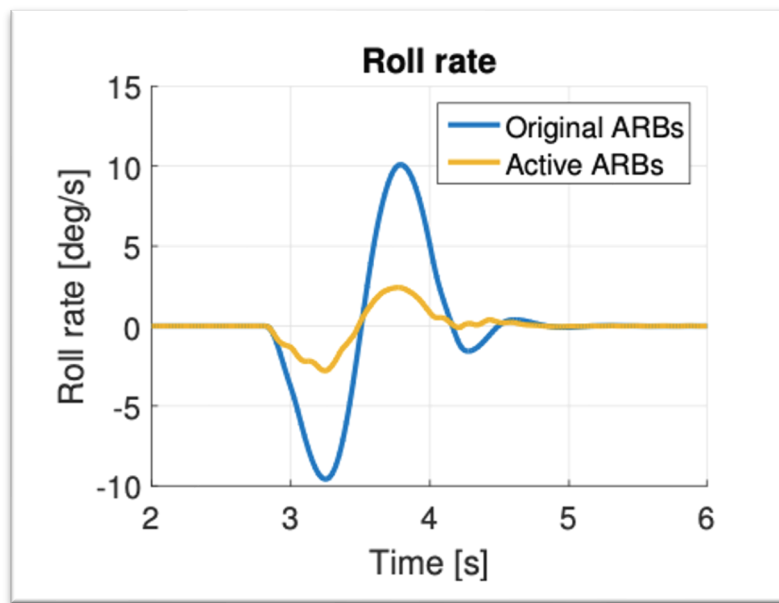


Figure 32 Roll speed at 60km/h.

Figure 33 displays the roll acceleration. In comparison to the vehicle equipped with the original ARBs, the frequency composition is pushed towards higher frequencies, but generally the controller reduces the roll acceleration's RMS values, as can be seen from the oscillations that are also present in this instance. Moreover, Figure 34 vehicle body's peak vertical acceleration is marginally lessened by the active ARBs.

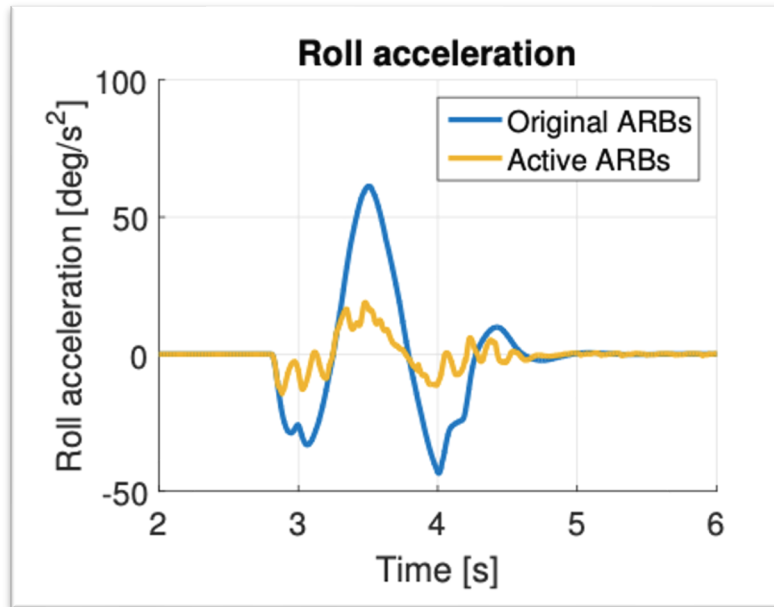


Figure 33 Roll acceleration at 60km/h.

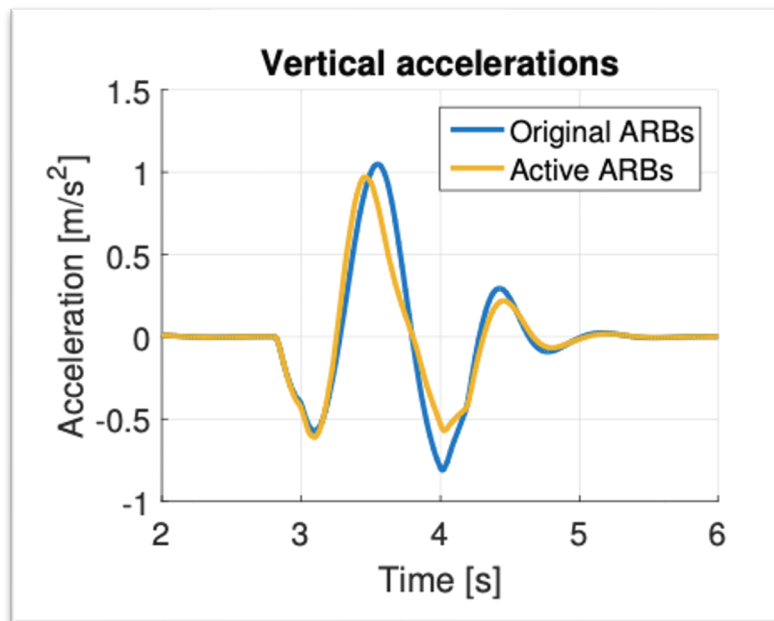


Figure 34 Vertical acceleration at 60km/h.

When compared to passive ARBs, Figure 35 illustrates how the active ARBs are reducing the vehicle's roll rate and roll angle in terms of wheel forces. It is evident that, at first, the front active ARB functions in the opposite manner from the passive ARB. While the passive ARB causes greater roll by acting in the opposite way, the active ARB lessens vehicle roll by forcing the left front wheel into the ground and raising the right front wheel. Initially, the

vehicle's roll motion is being reduced by the active and passive ARBs on the rear axle.

Because of the vehicle's roll angle, the passive ARBs reverse the direction of the forces given to the wheels after the first phase. On the other hand, the active ARB counteracts the roll overshoot by keeping the forces in the same direction for a longer duration and changing them toward the conclusion of the dip.

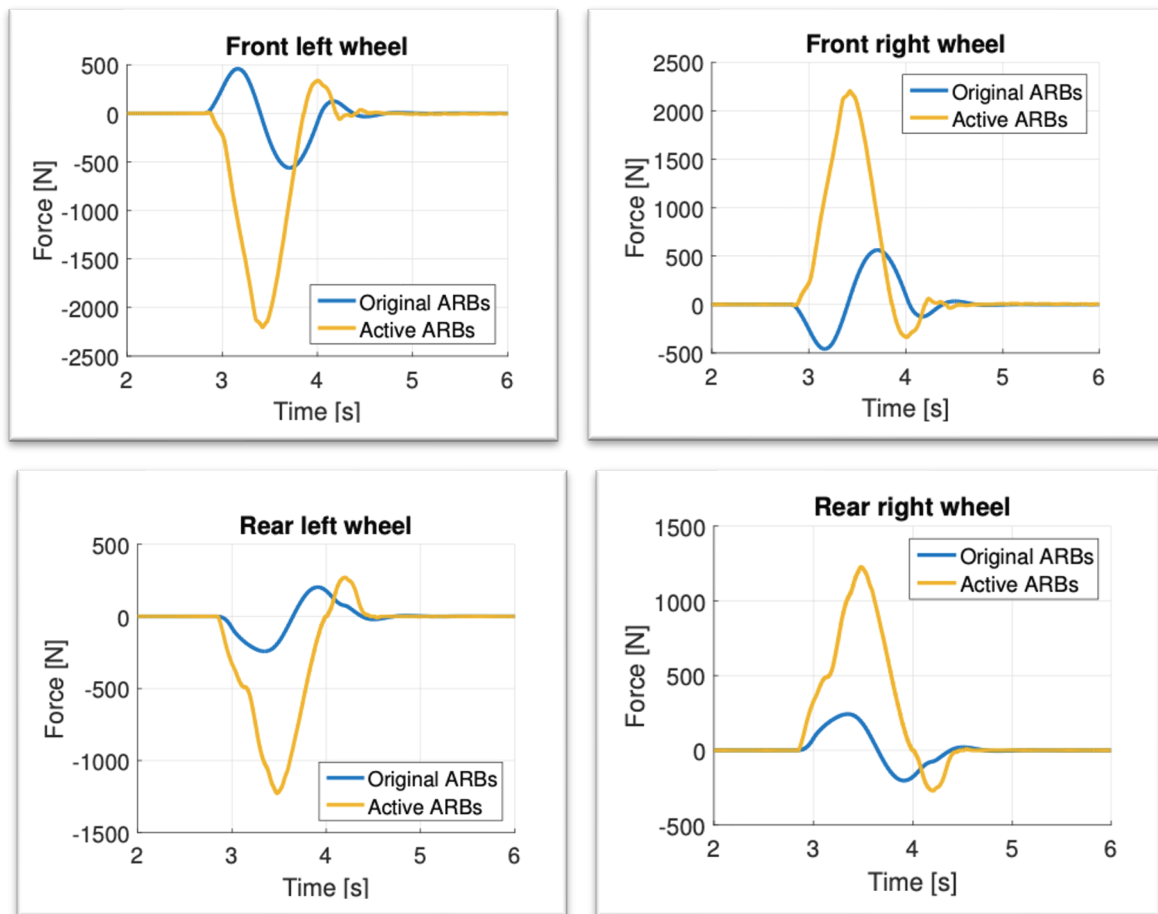


Figure 35 Wheel forces at 60km/h.

10.1.1.2 Dependency on speed

The performance of the active ARBs is observed to be influenced by test speed in the single-sided cosine wave test. The roll angle for a test speed of 30 km/h is displayed as a function of time in Figure 36(a). Here, the tiny roll overshoot is eliminated, and the roll angle is decreased. The findings at a test speed of 90 km/h are displayed in Figure 36(b). Here, it is evident that while the active ARBs lessen the roll angle, the roll overshoot is not eliminated.

This is caused by a combination of the active ARB limited torque production, lengthy response times, and absence of roll damping. This suggests that the roll damping controller should be tuned based on speed for optimal results.

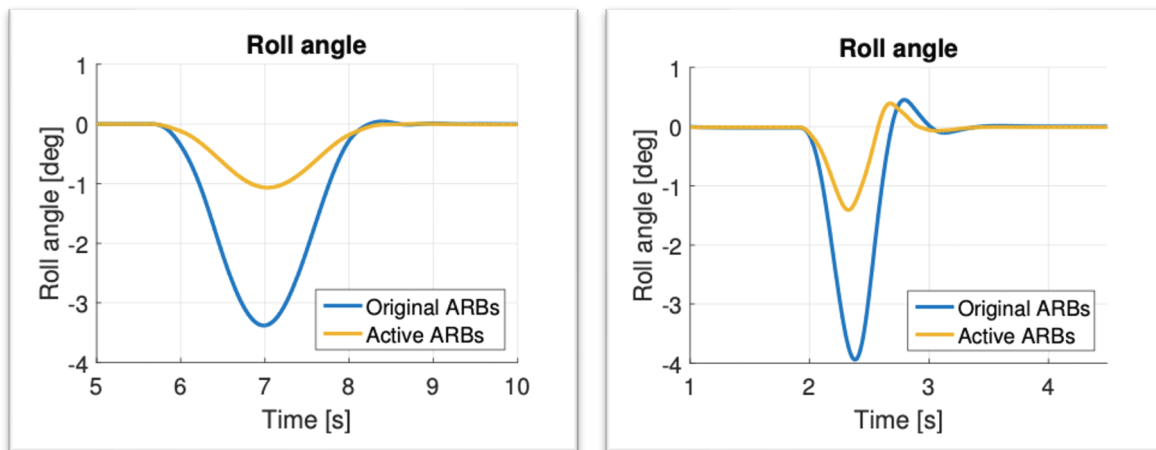


Figure 36 Roll angle at 30 mph (left hand side figure) and 90 mph (right hand side figure)

10.2 Handling performance

- **Yaw performance**

Handling simulation test conducting Sine with Dwell manoeuvre. The results are compared to passive ARB as presented in Figures 37 and 38.

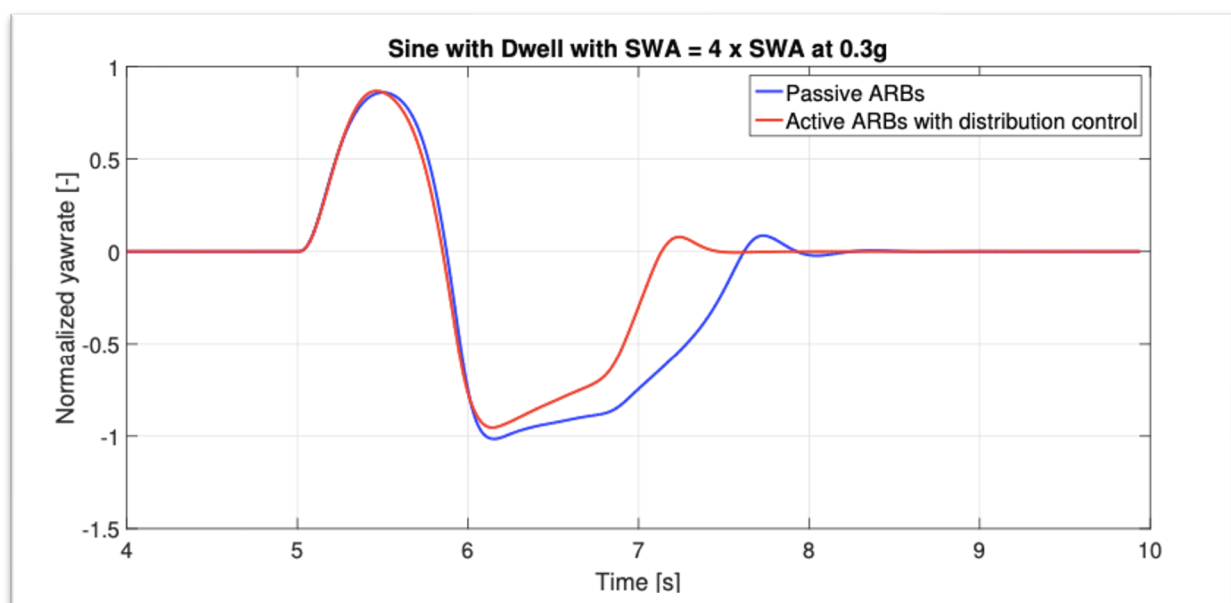


Figure 37 Normalized yaw rate for a $SWA=4 \times SWA$ at 0.3g

Figure 37 shows that peak yaw rate during dwell phase is decreased and yaw damping is also noticeably better for the specified understeer reference. The measurements also show this. The same manoeuvre is repeated with a larger steering angle in light of the improved stability and yaw performance (see Figure 38).

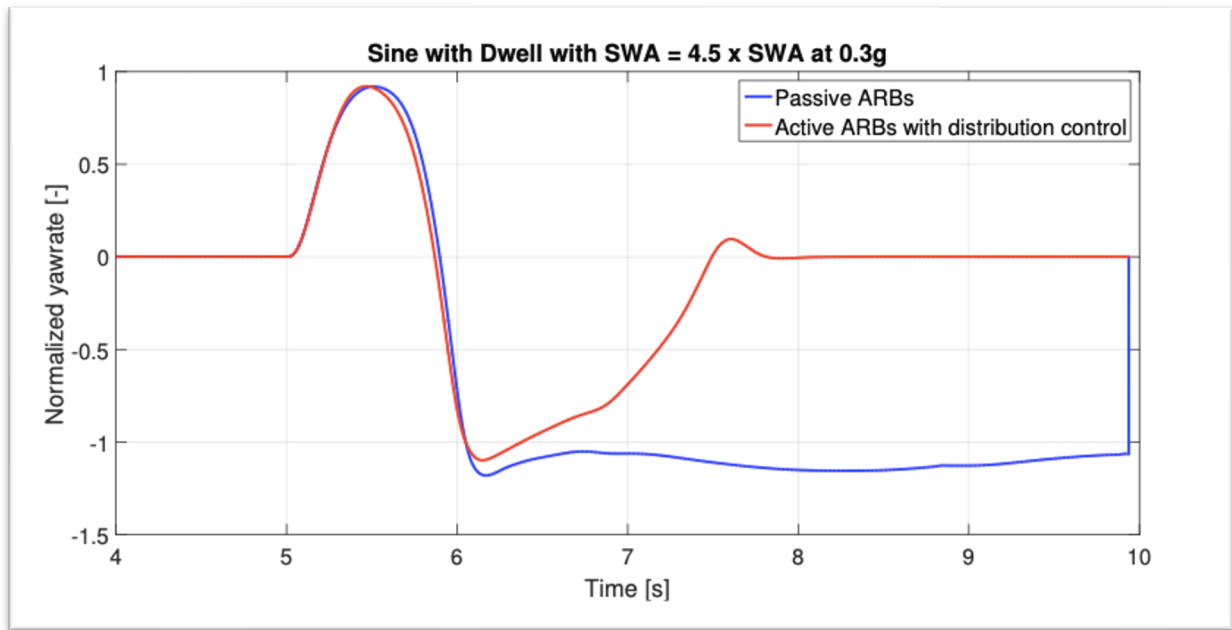


Figure 38 Normalized yaw rate for a $SWA=4.5 \times SWA$ at 0.3g

Figure 38 shows that the car equipped with passive ARBs is unable to execute the manoeuvre with a greater steering amplitude. This is a result of ARBs' incapacity to offer adequate lateral stability. Conversely, distribution control and active ARBs enable the vehicle to effectively execute the manoeuvre within the specified metric limitations. It is an obvious illustration of the car's enhanced handling qualities.

- **Roll performance**

Roll angles are compared with those obtained with passive ARBs in order to examine the roll reduction capabilities of active ARBs during a sine with dwell manoeuvre, as shown in Figure 39. In terms of peak roll angle reduction, Figure 39 shows that the best results are obtained when active ARBs are used without distribution control. When compared to passive ARBs,

the roll damping is also noticeably superior at the end of the manoeuvre. Distribution control reduces the roll reduction capabilities of active anti-roll bars (ARBs) but improves roll damping because the vehicle returns to its initial state more quickly and smoothly. The test result with the increased input steering angle, or 4.5 times the steering angle at 0.3g, is shown in Figure 40.

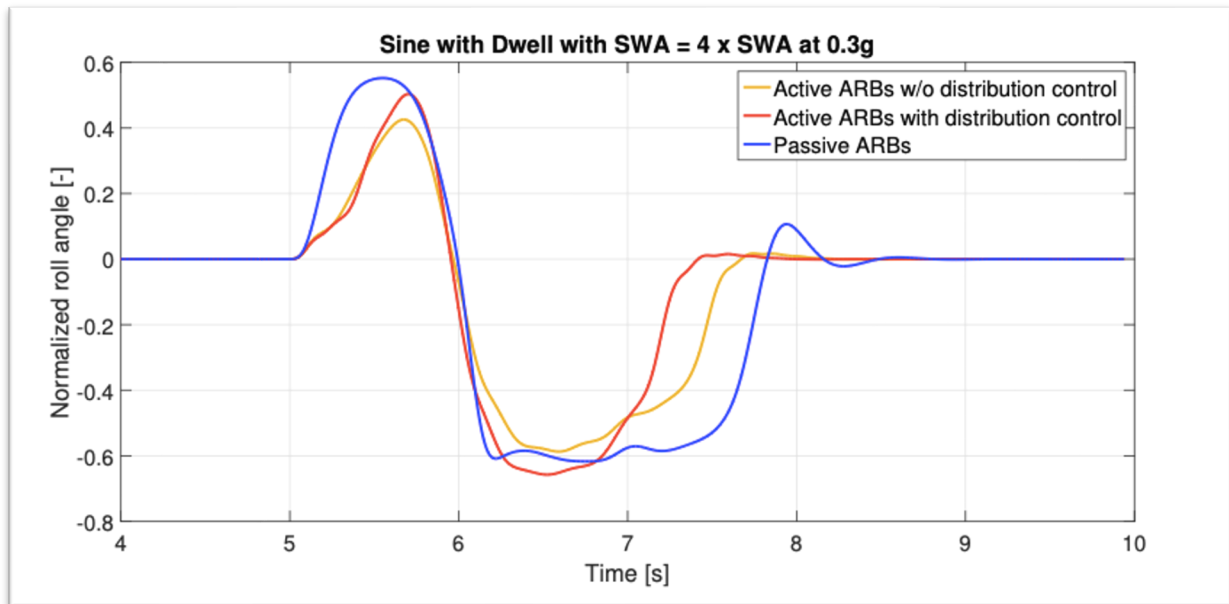


Figure 39 Roll angle for a $SWA=4 \times SWA$ at 0.3g

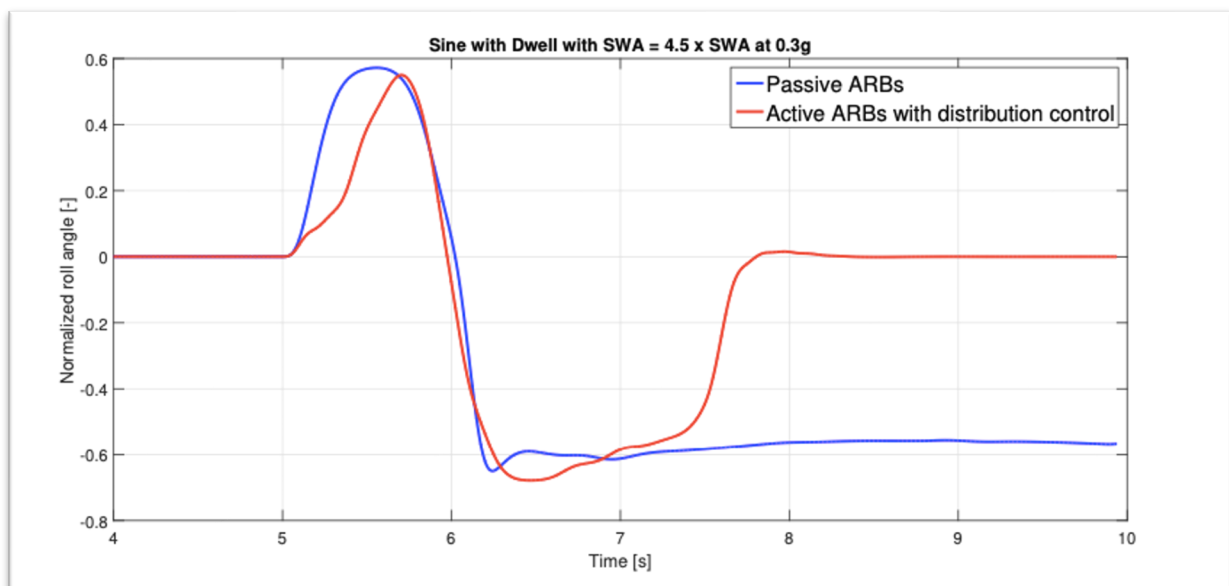


Figure 40 Roll angle for a $SWA=4.5 \times SWA$ at 0.3g

Figure 40 shows that the vehicle can now successfully complete the manoeuvre with higher steering inputs and no discernible loss in roll reduction capacity when employing active ARBs with distribution control as opposed to passive ARBs.

11. Discussion

Active ARBs are thought to have good potential for enhancing ride comfort in low frequency road excitations like the single-sided cosine wave. For all test speeds, there was potential to decrease the peak roll angle caused by the road excitations, and for 30 and 60 km/h, there was opportunity to increase roll damping. Therefore, in these circumstances, the roll damping control approach now in use is deemed adequate. Both the controller's tuning and the active ARB maximum torque and response time appear to have an impact on the roll damping. It can be inferred that a speed-dependent roll damping controller tuning is likely required to achieve optimal roll damping performance, as the roll overshoot was not mitigated at high speeds by the current controller tuning in conjunction with the active ARB plant model.

Active ARBs have demonstrated significant promise in affecting a vehicle's handling characteristics. Passive ARBs were used to evaluate handling performance in steady-state settings, and the results demonstrated a good theoretical working range. The handling characteristics were significantly altered by fully adjusting the rigidity of the front and rear axles, particularly at extreme lateral accelerations. Although the bicycle model made it easier to simplify the creation of the distribution controller for active ARBs, the SMC controller handles the majority of the non-linearities, which can lead to some inefficiency in the control strategy. To make the control strategy function, several ad hoc measures must be put in place, such as utilising "unit delay" blocks to identify zero crossover and avoiding

the use of sign functions within the SMC. Afterwards, distribution is made directly proportional to the controller's output rather than using tyre stiffness curves, which would have yielded more precise information on the link between lateral forces and load transfer, particularly in the non-linear range.

A simulation involving dwell manoeuvres was used to assess performance in non-linear ranges. A reference understeer curve was established, based on the active ARB's capacity under steady-state conditions, however it cannot be fully validated. With better yaw and roll damping, lower yaw rates were possible for a given understeer reference, since SUVs are nominally required to produce understeer at large lateral accelerations. Limitations with the front axle suspension provide a trade-off between the improvement in roll and lateral stability, as also shown during steady-state testing.

The trade-off necessitates the implementation of a clever method that gives the requirements priority. For instance, how much of a sacrifice on roll angle may be made in order to get the intended distribution. In real-world scenarios, the existing strategy—which prevents the distribution ratio from falling below a predetermined lateral acceleration value—is probably not going to hold up well.

12. Conclusion

The goal of creating a control algorithm and assessing its handling and ride comfort performance was achieved. This allowed for the investigation of how active ARBs can be employed to increase customer value. It was possible to reflect the potential that active ARBs can provide in enhancing handling and ride behaviours in the strategies designed for roll stiffness and distribution control. The successful integration of these controllers was essential to the functioning of the active ARB. Roll reduction and controlling understeer behaviour at high lateral accelerations are two areas where active ARBs show the most promise. Active ARBs are able to insulate the car from low frequency road excitations while also preventing roll overshoot in ride comfort ratings. More research is still required to determine how well high frequency road excitations operate. Active ARBs demonstrate several options when managing assessments. On the other hand, active ARBs have no impact on high frequency yaw behaviour at low lateral accelerations (up to 4 m/s²). When active ARBs are evaluated for non-linear transient conditions, the vehicle's on-limit handling stability is significantly improved, and there is a noticeably smaller trade-off between handling and ride quality.

The thesis concludes that the suspension design has a significant impact on how well active ARBs operate. Because active ARBs can produce a wide range of vehicle behaviours, it is possible to distinguish between various driving modes in a vehicle and customise the driving dynamics of the car with them.

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