

University of Wales Trinity Saint David

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Analytical Methods Coursework

Analytical Methods (ACHA4000)

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$$\begin{aligned}
 1 \quad & (x+6)^2 + 2x = x^2 - 5 \\
 & (x+6)(x+6) + 2x = x^2 - 5 \\
 & x^2 + 6x + 6x + 36 + 2x = x^2 - 5 \\
 & 6x + 6x + 36 + 2x = -5 \\
 & 14x = -5 - 36 \\
 & 14x = -41 \\
 & x = \frac{-41}{14} = -2.93
 \end{aligned}$$

$$\begin{aligned}
 ii \quad & 2^{x+5} = 6^x \\
 & \log(2^{x+5}) = \log 6^x \\
 & (x+5)\log 2 = x\log 6 \\
 & 0.30x + 1.51 = 0.78x \\
 & 1.51 = 0.78x - 0.30x \\
 & 1.51 = 0.48x \\
 & \frac{1.51}{0.48} = x \quad x = \frac{1.51}{0.48} = 3.15
 \end{aligned}$$

$$\begin{aligned}
 iii \quad & 2\log(x+3) = \log(x+5) + \log(x+8) \\
 & \log(x+3)^2 = \log(x+5)(x+8) \\
 & (x+3)(x+3) = (x+5)(x+8) \\
 & x^2 + 3x + 3x + 9 = x^2 + 8x + 5x + 40 \\
 & 6x + 9 = 13x + 40 \\
 & 9 - 40 = 13x - 6x \\
 & -31 = 7x \\
 & -\frac{31}{7} = x \quad x = -\frac{31}{7} = -4.43
 \end{aligned}$$

$$\begin{aligned}
 iv \quad & x - \frac{1 - \frac{6x}{5}}{3} = \frac{4 - \frac{x}{3}}{12} + \frac{5}{4} \\
 & x - \frac{5 - 6x}{15} = \frac{12 - x}{36} + \frac{5}{4} \quad \times 540 \\
 & 540x - 540 \times \frac{5 - 6x}{15} = 540 \times \frac{12 - x}{36} + 540 \times \frac{5}{4} \\
 & 540x - 36(5 - 6x) = 15(12 - x) + (135 \times 5) \\
 & 540x - 180 + 216x = 180 - 15x + 675 \\
 & 540x + 216x + 15x = 180 + 180 + 675 \\
 & 771x = 1035 \\
 & x = \frac{1035}{771} = 1.34
 \end{aligned}$$

2.

$$2x + 4y - 3z = 120 \quad (1)$$

$$5x + 2y + z = 140 \quad (2)$$

$$x - 3y + 5z = 100 \quad (3)$$

$$x = 3y - 5z + 100$$

Sub x to (1)

$$2(3y - 5z + 100) + 4y - 3z = 120$$

$$6y - 10z + 200 + 4y - 3z = 120$$

$$10y - 13z = -80 \quad (4)$$

Sub x to (2)

$$5(3y - 5z + 100) + 2y + z = 140$$

$$15y - 25z + 500 + 2y + z = 140$$

$$17y - 24z = -360 \quad (5)$$

$$(4) \times 17$$

$$170y - 221z = -1360 \quad (6)$$

$$(5) \times 10$$

$$170y - 240z = -3600 \quad (7)$$

$$(6) - (7) \quad -19z = -2240$$

$$z = \frac{-2240}{-19} = 117.89$$

Sub z to (4)

$$10y - (13 \times 117.89) = -80$$

$$10y - 1532.57 = -80$$

$$10y = 1452.57$$

$$y = 145.26$$

Sub y and z to (3)

$$x - (3 \times 145.26) + (5 \times 117.89) = 100$$

$$x - 435.78 + 589.45 = 100$$

$$x - 435.78 = -489.45$$

$$x = -53.67$$

Check

$$(2 \times -53.67) + (4 \times 145.26) - (3 \times 117.89) = 120.03$$

$$(5 \times -53.67) + (2 \times 145.26) + 117.89 = 140.06$$

$$3 \quad 0.6x^2 + 3.4x = 7.4$$

$$0.6x^2 + 3.4x - 7.4 = 0$$

$$\frac{-3.4 \pm \sqrt{3.4^2 - 4 \times 0.6 \times -7.4}}{2 \times 0.6}$$

$$\frac{-3.4 \pm \sqrt{11.56 + 17.76}}{1.2}$$

$$\frac{-3.4 + \sqrt{29.32}}{1.2}$$

$$x = 1.68$$

$$\frac{-3.4 - \sqrt{29.32}}{1.2}$$

$$x = -7.35$$

$$4. \quad y = 5x^3 - 7x^2 + 3x - 1$$

$$\frac{dy}{dx} = 5(3x^2) - 7(2x) + 3 - 0$$

$$ii. \quad y = -\frac{1}{x^5}$$

$$y = -1x^{-5}$$

$$\frac{dy}{dx} = 5x^{-6}$$

$$iii. \quad y = e^{\frac{x}{2}} + \cos \frac{x}{6} + \sin 2x$$

$$\frac{dy}{dx} = \frac{1}{2}e^{\frac{x}{2}} - \frac{1}{6}\sin \frac{x}{6} + 2\cos 2x$$

$$iv. \quad y = (x^7 + 4)^5 (3x^2 - 1)^4$$

$$\frac{dy}{dx} = 5(x^7 + 4)^4 (7x^6) 4(3x^2 - 1)^3 (6x)$$

$$v. \quad y = \frac{(5x^2 + 3x + 1)^5}{e^x}$$

$$\frac{dy}{dx} = \frac{(e^x)5(5x^2 + 3x + 1)^4(10x + 3) - (5x^2 + 3x + 1)^5(e^x)}{(e^x)^2}$$

$$vi. \quad y = \sqrt{\frac{5x-1}{8x+5}} = \left(\frac{5x-1}{8x+5}\right)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{5x-1}{8x+5}\right)^{-\frac{1}{2}} = \frac{1}{2} \left(\frac{(8x+5)(5) - (5x-1)(8)}{(8x+5)^2}\right)^{-\frac{1}{2}}$$

$$vii. \quad y = (10x + \sqrt[3]{x^5 + 1})^5$$

$$\frac{dy}{dx} = 5(10x + \sqrt[3]{x^5 + 1})^4 (10 + \sqrt[3]{5x^4})$$

$$x = \frac{dx}{dt} = \frac{d^2x}{dt^2} = \frac{d^3x}{dt^3} = \frac{d^4x}{dt^4} = \frac{d^5x}{dt^5} = \frac{d^6x}{dt^6} \quad S$$

Velocity Acceleration Jerk Snap Crackle Pop

6

i: $9 \sin x = 2$

$$\sin x = \frac{2}{9}$$

$$x = \sin^{-1}\left(\frac{2}{9}\right)$$

$$x = 12.84^\circ$$

$$x = 180 - 12.84 = 167.16^\circ$$

ii: $5 \sin^2 x - 0.3 \cos x = 2$

$$\sin^2 x + \cos^2 x = 1$$

$$5 \sin^2 x - 0.3 \cos x - 2 = 0$$

$$\sin^2 x = 1 - \cos^2 x$$

$$5(1 - \cos^2 x) - 0.3 \cos x - 2 = 0$$

$$5 - 5 \cos^2 x - 0.3 \cos x - 2 = 0$$

$$-5 \cos^2 x - 0.3 \cos x + 3 = 0$$

$$5 \cos^2 x + 0.3 \cos x - 3 = 0$$

$$x = 1$$

$$\cos x = \frac{-0.3 \pm \sqrt{0.3^2 - 4 \times 5 \times -3}}{2 \times 5}$$

$$= \frac{-0.3 \pm 7.75}{10}$$

$$\cos x = 0.745$$

$$x = \cos^{-1}(0.745)$$

$$x = 41.84^\circ$$

$$360 - 41.84 = 318.16$$

$$\cos x = -0.805$$

$$\cos x = 0.805$$

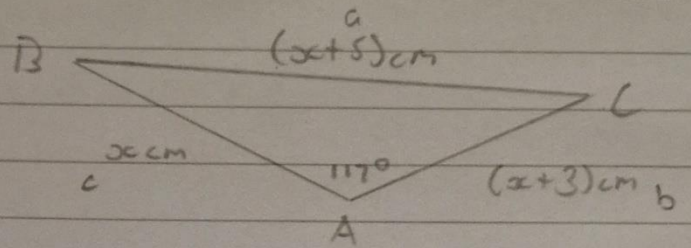
$$x = \cos^{-1}(0.805)$$

$$x = 36.39^\circ$$

$$180 - 36.39 = 143.61^\circ$$

$$180 + 36.39 = 216.39^\circ$$

7.



$$(x+5)^2 = (x+3)^2 + (x)^2 - 2 \times (x+3)(x) \cos 117$$

$$(x+5)(x+5) = (x+3)(x+3) + x^2 - 2(x+3)(x) \cos 117$$

$$x^2 + 5x + 5x + 25 = x^2 + 3x + 3x + 9 + x^2 - 2x^2 + 3x \cos 117$$

$$x^2 + 10x + 25 = 2x^2 + 9x + 9 - 2x^2 \cos 117$$

$$x^2 + 10x + 25 = 9x + 9 \cos 117$$

$$x^2 + 10x = 9x - 16 \cos 117 \quad ; \quad c = 1.91 \text{ cm}$$

$$x^2 + x = -16 \cos 117$$

$$b = (1.91 + 3) = 4.91 \text{ cm}$$

$$x + x = \sqrt{-16 \cos 117}$$

$$a = (1.91 + 5) = 6.91 \text{ cm}$$

$$2x = \sqrt{-16 \cos 117}$$

$$x = \frac{\sqrt{-16 \cos 117}}{2}$$

$$x = 1.91 \text{ cm}$$

$$ii. \quad \frac{1}{2} \times 1.91 \times 4.91 \sin(117) = 4.18 \text{ cm}^2$$