



12 Pages

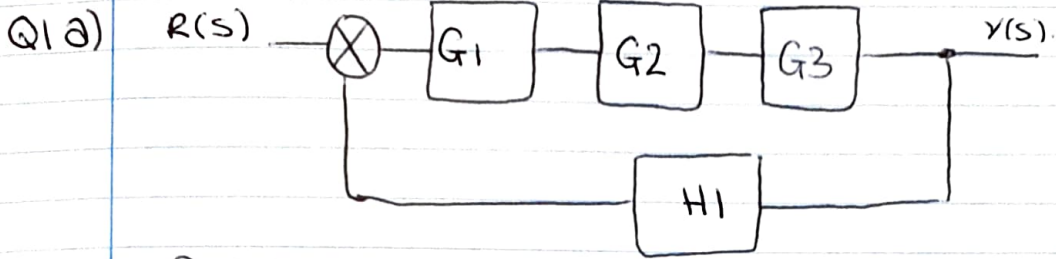
1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
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8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

SURNAME: PERRY

FORENAME(S): MATTHEW

1	Student Number	2024421
2	Title of Examination Paper	Control Engineering
3	Module Code	SGEES009
4	Date and Time of Examination	18/5/23 09:30 - 12:00
5	Examination Venue	CSG
6	Seat Number	33
7	Number of Answer Books handed in	

[illegible]



$$G_1(s) = K_P$$

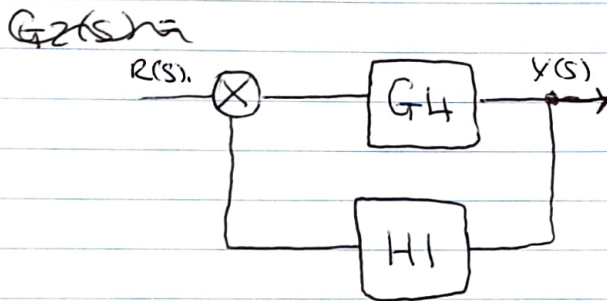
$$G_2(s) = \frac{K_1 \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$

X

$$G_3(s) = \frac{K_2}{\tau_1 s + 1} = 1$$

$$H_1(s) = \frac{1}{\tau_2 s + 1}$$

b



$$G_4 = G_1 G_2 G_3$$

$$G_4 = K_P \times \frac{K_1 \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \times \frac{K_2}{\tau_1 s + 1}$$

Q2 a)
$$T(s) = \frac{G(s)}{1 + H(s)G(s)}$$

where $G(s) = \frac{25}{s(s+1)}$

$H(s) = 1$

$$T(s) = \frac{25/s(s+1)}{1 + 1(25/s(s+1))}$$

$$T(s) = \frac{25}{s(s+1) + 25} = \frac{25}{s^2 + s + 25}$$

b)
$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n^2 = 25$

$\omega_n = \sqrt{25} = 5 \text{ rad/s}$

$2\zeta\omega_n = 1$

$\zeta = \frac{1}{2\omega_n} = \frac{1}{2 \times 5} = 0.1$

$$\omega = \omega_n \sqrt{1 - \zeta^2} = 5 \sqrt{1 - 0.1^2} = 4.975 \text{ rad/s}$$

$$tr = \frac{\pi - \tan^{-1} \left(\frac{\sqrt{1 - \zeta^2}}{\zeta} \right)}{\omega_n \sqrt{1 - \zeta^2}}$$

$$= \frac{\pi - \tan^{-1} \left(\frac{\sqrt{1 - 0.1^2}}{0.1} \right)}{4.975}$$

$$= \frac{\pi - 1.471}{4.975} = 0.33 \text{ seconds}$$

(4)

(1)

(1)

(2)

(2)

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{5 \sqrt{1-0.12}}$$

$$= \frac{\pi}{4.975} = 0.631 \text{ seconds.}$$

$$\%OS = 100 \times e^{\frac{-3\pi}{\sqrt{1-\zeta^2}}}$$

$$= 100 \times e^{\frac{-0.1\pi}{\sqrt{1-0.12}}} = 100 \times e^{-0.3157}$$

$$= 72.92\%$$

$$2\% \quad t_s = \frac{4}{3\omega_n} = \frac{4}{0.1 \times 5} = 8 \text{ seconds.}$$

number of oscillations

$$\frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} = \frac{2}{\pi} \sqrt{\frac{1}{0.1^2} - 1} = 6.33$$

C poles

$$s^2 + s + 25 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2(a)}$$

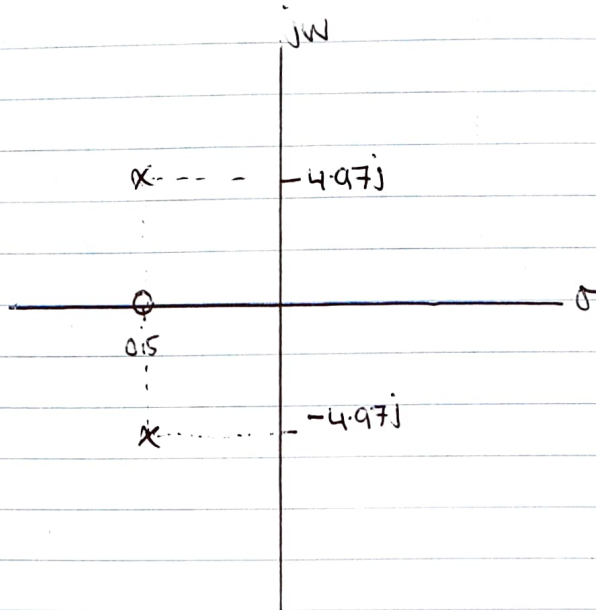
$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(25)}}{2(1)} = \frac{-1 \pm \sqrt{1 - 100}}{2}$$

$$= \frac{-1 \pm \sqrt{-99}}{2} = \frac{-1 \pm 9.94j}{2}$$

$$-0.5 + 4.97j$$

$$-0.5 - 4.97j$$

c)



poles are in left-hand plane
= stable.

$$2\zeta\omega_n = 2 \times 0.1 \times 5 = 1$$

$$\omega_n^2 = 25 \quad \omega_n = \sqrt{25} = 5.$$

2

- Q3a) i) All co-efficients are present and none are negative = STABLE ✓
 ii) All co-efficients are present but one is negative = UNSTABLE ✓
 iii) None are negative but one coefficient is missing so at best = CRITICALLY STABLE ✓

b).
$$G_0(s) = \frac{3}{(s+2)} \times \frac{1}{s(s+4)}$$

$$= \frac{3}{s(s+2)(s+4)} = \frac{3}{(s^2 + 2s)(s+4)}$$

$$= \frac{3}{(s^2 + 2s)s(s^2 + 2s)4} = \frac{3}{s^3 + 2s^2 + s^24 + 8s}$$

$$G_0(s) = \frac{3}{s^3 + 6s^2 + 8s} \quad \checkmark$$

$$H(s) = K.$$

$$G(s) = \frac{G_0(s)}{1 + H(s)G_0(s)}.$$

$$= \frac{3/s^3 + 6s^2 + 8s}{1 + K(3/s^3 + 6s^2 + 8s)}$$

$$= \frac{3}{s^3 + 6s^2 + 8s + 3K} \quad \checkmark$$

where $a_n = 1$
 $a_{n-1} = 6$
 $a_{n-2} = 8$
 $a_{n-3} = 3K$
 $a_{n-4} = 0$
 $a_{n-5} = 0.$

a_n	a_{n-2}	a_{n-4}
a_{n-1}	a_{n-3}	a_{n-5}
b_1	b_2	b_3
c_1	c_2	c_3

$$b_1 = an^{-2} - \left(\frac{an}{an^{-1}} \right) \times an^{-3}$$

$$= 8 - \left(\frac{1}{6} \right) \times 3k$$

$$= 7.5k.$$

$$8 - \frac{k}{2}$$

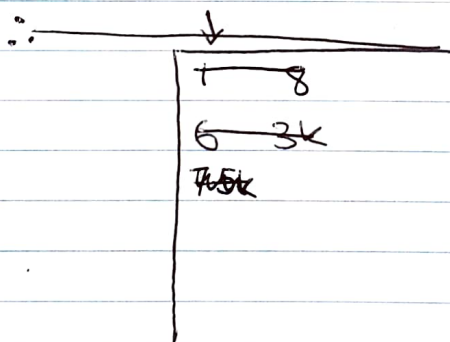
~~an~~	~~an⁻²~~	~~an⁻⁴~~
~~an⁻¹~~	~~an⁻³~~	~~an⁻⁵~~
b1	b2	b3
c1	c2	c3.

$$b_2 = an^{-4} - \left(\frac{an}{an^{-1}} \right) \times an^{-5}$$

$$= 0 - \left(\frac{1}{6} \right) \times 0 = 0$$

$$c_1 = an^{-3} - \left(\frac{an^{-1}}{b_1} \right) \times b_2$$

$$= 3k - \left(\frac{6}{7.5k} \right) \times 0 = 3k. /$$



$$c_2 = an^{-5} - \left(\frac{an^{-1}}{b_1} \right) \times b_3$$

$$c_2 = 0 - \left(\frac{6}{7.5k} \right) \times 0 = 0$$

7

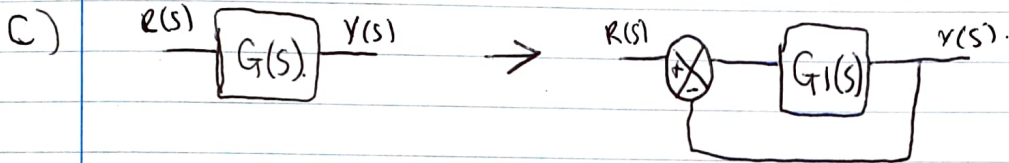
	CI	
	↓	
s^3	1	8
s^2	6	$3k$
s^1	$7.5k$	0
s^0	$3k$	0

error carried forward

no sign change in CI so system is stable.

$$k > 0$$

k must not be a negative value.



$$G(s) = \frac{G_1(s)}{1 + G_1(s)}$$

$$G(s) [1 + G_1(s)] = G_1(s)$$

$$G(s) + G(s)G_1(s) = G_1(s)$$

$$G(s) = G_1 - G(s)G_1(s)$$

$$G(s) = G_1(s) [1 - G(s)]$$

$$G_1(s) = \frac{G(s)}{1 - G(s)}$$

$$G_1(s) = \frac{3/s^3 + 6s^2 + 8s + 3k}{1 - (3/s^3 + 6s^2 + 8s + 3k)}$$

$$= \frac{3}{s^3 + 6s^2 + 8s + 3(k-1)}$$

$$class = 0$$

(1)

(5)

Q4a)

$$\frac{60}{(s+4)} = \frac{z}{z - e^{-2T}}$$

$$= \frac{z}{z - e^{-60T}}$$

(1)