

$$\begin{aligned}
 \text{b) } f(x) &= \frac{4}{x(1+4\ln x)^2} & u &= 1+4\ln x \\
 & & \frac{du}{dx} &= \frac{4}{x} & dx &= \frac{du \cdot x}{4} \\
 & & & & & \\
 & \int \frac{4}{x u^2} \cdot \frac{du \cdot x}{4} \\
 & = \int \frac{1}{u^2} du = -\frac{1}{u} \\
 & = -\frac{1}{1+4\ln x}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) (i) } \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x-1}} & \quad u = 3e^{2x} \quad \frac{du}{dx} = 6e^{2x} \quad v = \frac{2(e^x-1)^{1/2}}{3} \\
 & \quad \frac{du}{dx} = 6e^{2x} \\
 \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x-1}} &= 3e^{2x} \left(\frac{2(e^x-1)}{3} \right) - \int 6e^{2x} \left(\frac{2(e^x-1)}{3} \right) \\
 &= 2e^{2x}(e^x-1) - \int 4e^{3x} - 4e^{2x} \quad u = 2e^x \\
 & \quad \frac{du}{dx} = 2e^x \quad dx = \frac{du}{2e^x} \\
 &= e^{3x} - e^{2x} - \frac{4}{3}e^{3x} + 2e^{2x} \\
 &= -\frac{1}{3}e^{3x} + e^{2x} \\
 &= \left[e^{2x} - \frac{1}{3}e^{3x} \right]_{\ln 2}^{\ln 5} = \left(\cancel{12} - \frac{50}{3} - \frac{4}{3} \right) \\
 &= -\frac{54}{3} = -18
 \end{aligned}$$