



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2401039
2	Title of Examination Paper	Mathematics
3	Module Code	ACDB4003
4	Date and Time of Examination	12/05/2026 09:30
5	Examination Venue	10310
6	Seat Number	11
7	Number of Answer Books handed in	

Complete, fold and seal
(see note 2 below)

SURNAME: Cramwell

FORENAME(S): Simon

Jac

List the numbers of the questions you have attempted	Office Use
1	9
2	10
3	14
4	14
5	14
6	8
7	10
8	6
Total	78

78%
good

1.

$$\text{i) } 3x - 2x + x = 2x$$

$$4y - 3y = y$$

$$-2 + 22 + 42 = 52$$

$$\text{Ans} = 2x + y + 52$$

$$\text{ii) } x(x+4) = x^2 + 4x$$

$$x^2 + 4x - x + 3$$

$$\text{Ans} = x^2 + 3x + 3$$

Careless here!

$$\text{iii) } 8p^4 \quad 2 \times 4 = 8$$

$$p^3 p = p^4$$

$$q \cdot q^4 \cdot q = q^6$$

$$r \cdot r^2 = r^3$$

$$\text{Ans} = 8p^4 q^6 r^3$$

$$\text{iv) } \frac{48}{12} = 4$$

$$\frac{p^2}{p} = p$$

$$\frac{q^3}{q} = q^2$$

$$\frac{r}{r-3} = r^4$$

$$\text{Ans} = 4pq^2r^4$$

$$\text{v) } F \cdot F = F^2$$

$$g \cdot g = g^2$$

$$h^3 \cdot h^5 = h^8$$

1.

$$v) 32F^2g^2h^8$$

$$16 \times 4 = 64$$

$$h \cdot h = h^2$$

$$F \cdot F^4 = F^5$$

$$\frac{32F^2g^2h^8}{64h^2F^5}$$

$$= \frac{1}{2} F^{-3} g^2 h^6$$

②

2.

$$i) x^2 + 7x + 12 = 0$$

$$(x+4)(x+3) = 0$$

$$x_1 = -4 \quad x_2 = -3$$

⑤

$$ii) LCM = 6$$

$$6 \left(\frac{x-3}{2} \right) = 6 \left(\frac{2x+1}{3} \right)$$

$$3x - 9 = 4x + 2$$

$$x = -11$$

⑤

3. $5^{x+1} = 6$

i)

$$\log 5^{x+1} = \log 6$$

$$(x+1)(\log 5) = \log 6$$

$$x \log 5 + \log 5 = \log 6$$

$$x = \frac{\log 6 - \log 5}{\log 5}$$

$$x = 0.11 \text{ (2dp)}$$

6

ii) $2 \log x = \log(5x-6)$

$$\log x^2 = \log(5x-6)$$

Can't Cancel logs!!

$$x^2 = 5x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x_1 = 2, x_2 = 3$$

3

b)

i) $\log(abc) = \log a + \log b + \log c \therefore$

$$\log(x^2 y z) = \log x^2 + \log y + \log z$$

$$\text{Simplify to } 2 \log x + \log y + \log z$$

3

ii) $\log \frac{a}{b} = \log a - \log b$

3. b)

∴

11

$$\log y^2 + \log x - \log z$$

$$2 \log y + \log x - \log z$$

(2)

4.

i)

$$\text{dot} = a \cdot b$$

$$(1 \times 1) + (2 \times 1) + (3 \times 1)$$

$$= 1 + 2 + 3$$

$$= 6$$

(2)

ii)

$$\text{Angle between} = \frac{\text{dot}}{|a| |b|} = \cos \theta$$

$$|a| = \sqrt{1^2 + 2^2 + 3^2}$$

$$= \sqrt{14}$$

$$|b| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\text{dot} = \frac{6}{\sqrt{14} \sqrt{3}} \cos \theta$$

$$\frac{6}{\sqrt{42}} \cos \theta$$

$$\theta = \cos^{-1} \left(\frac{6}{\sqrt{42}} \right)$$

$$\theta \approx 22.208^\circ$$

$$\theta = 22.21^\circ (2 \text{ dp})$$

good

(2)

4.

iii) IF perpendicular, dot = 0

$$\text{dot } a \cdot c = (1 \times 2) + (2 \times -1) + (3 \times 0)$$

$$\text{dot} = 2 - 2$$

dot $a \cdot c = 0 \therefore$ perpendicular

iv)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 9\mathbf{i}$$

$$\mathbf{i} = +(2 \times 1) - (1 \times 3)$$

$$\mathbf{i} = -1$$

$$\mathbf{j} = (1 \times 1) - (3 \times 1)$$

$$\mathbf{j} = -2$$

$$\mathbf{k} = (1 \times 1) - (2 \times 1)$$

$$\mathbf{k} = -1$$

$$= -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

$$\text{Unit norm} = \frac{\text{cross}}{|\mathbf{d}|}$$

$$|\mathbf{d}| = \sqrt{1^2 + 2^2 + 1^2}$$

$$|\mathbf{d}| = \sqrt{6}$$

$$\text{unit norm} = \frac{-1 - 2 - 1}{\sqrt{6}}$$

Vector not a scalar

5. $\det(A) =$

$$+3 \begin{vmatrix} -1 & 2 \\ -3 & -4 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix}$$

$$3(4 - -6) - 2(-8 - 2) + 1(-6 - -1)$$

$$3(10) - 2(-10) + 1(-5)$$

$$= 30 + 20 - 5$$

$$= 45$$

$$\det A = 45$$

Now

$$\begin{aligned} &+ \begin{vmatrix} -1 & 2 \\ -3 & -4 \end{vmatrix} - \begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} \\ &- \begin{vmatrix} 2 & 1 \\ -3 & -4 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix} - \begin{vmatrix} 3 & 2 \\ 1 & -3 \end{vmatrix} \\ &+ \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} - \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} \end{aligned}$$

$$= \begin{aligned} &(4 - -6) - (-8 - 2) + (-6 - -1) \\ &\quad 10 \quad + 10 \quad - 5 \\ &- (-8 - -3) + (-12 - 1) - (-9 - 2) \\ &\quad + 5 \quad - 13 \quad + 11 \end{aligned}$$

$$\begin{aligned} &(4 - -1) - (6 - 2) + (-3 - 4) \\ &\quad + 5 \quad - 4 \quad - 7 \end{aligned}$$

5.

$$\begin{pmatrix} 10 & 10 & -5 \\ 5 & -13 & 11 \\ 5 & -4 & -7 \end{pmatrix}$$

cofactor = C

$$C^T = \begin{pmatrix} 10 & 5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} C^T$$

$$\text{Ans} = \frac{1}{45} \begin{pmatrix} 10 & 5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{pmatrix}$$

24

6.

$$AX = b$$

$$A^{-1}AX = A^{-1}b$$

$$X = A^{-1}b$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

A X b

After way around here!

we know from 5 cofactor ∴

$$\frac{1}{45} \begin{pmatrix} 10 & 5 & 5 \\ 10 & -13 & -4 \\ -5 & 11 & -7 \end{pmatrix} \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$(60 + 15 - 30) = 45$$

$$(60 - 39 + 24) = 45$$

$$(-30 + 33 + 42) = 45$$

$$6. \frac{1}{45} \begin{pmatrix} 45 \\ 45 \\ 45 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \checkmark$$

$$\therefore x = 1, y = 1, z = 1 \checkmark$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \checkmark$$

8

7.

$$i) y = 4x^{10}$$

$$\frac{dy}{dx} = 4 \checkmark$$

need $\frac{dy}{dx}$ in all these!

1

$$ii) y = \frac{1}{x} + 8x^{\frac{1}{2}}$$

$$y = x^{-1} + 8x^{\frac{1}{2}} \checkmark$$

$$\frac{dy}{dx} = -x^{-2} + 4x^{-\frac{1}{2}} \checkmark$$

1

$$iii) y = \frac{1}{5x^4} + 7x^3$$

$$y = x^{-4} + 7x^3 \checkmark$$

$$\frac{dy}{dx} = -4x^{-5} + 21x^2 \checkmark$$

1

$$iv) y = \sin 2x$$

$$\frac{dy}{dx} = 2 \cos 2x \checkmark$$

1

$$v) y = x^3 \cos(x)$$

$$y = u \cdot v \checkmark$$

$$= \frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx} \checkmark$$

$$u = x^3 \quad v = \cos x$$

$$\frac{dy}{dx} = x^3 (-\sin x) + \cos x (3x^2) \checkmark$$

$$y = -x^3 \sin x + 3x^2 \cos x$$

$$= x^2 (\dots)$$

1

vii) chain rule :-

$u^o = \text{outer}$

$$v = 5x + 3$$

$$y = 10(5x + 3)^9 \cdot 5$$

$$\frac{dy}{dx} = 10(5x + 3)^9 \cdot 5$$

$$\frac{dy}{dx} = 15(5x + 3)^9$$

vii) $y = e^x + \ln x - 3$

$$\frac{dy}{dx} = e^x + \frac{1}{x}$$

b) $u = x - 3$
 $v = 1 + x^2$

$$y = \frac{(1 + x^2)(1) - (x - 3)(2x)}{(1 + x^2)^2}$$

$$\frac{dy}{dx} = \frac{1 + x^2 - 2x^2 + 6x}{(1 + x^2)^2}$$

$$y = \frac{-x^2 + 6x + 1}{(1 + x^2)^2}$$

If $x = 0$

$$y = \frac{1 + 0^2 - 2(0)^2 + 6(0)}{(1 + 0^2)^2}$$

$$\frac{dy}{dx} = \frac{1}{1}, \frac{dy}{dx} = 1$$

8.

$$i) \int 2x + 5x^4 dx$$

$$\frac{2x^2}{2} + \frac{5x^5}{5} + C$$

$$= x^2 + x^5 + C$$

①

$$ii) \frac{3x^3}{3} + 5e^x + C$$

$$= x^3 + 5e^x + C$$

②

$$iii) x^{-1} - 4$$

$$\frac{x^0}{0} - 4x + C$$

$$\frac{1}{0} - 4x + C$$

$$= -4x + C$$

$$= \ln x - 4x + C$$

①

$$iv) \int 3x^3 + 4x^2 dx$$

$$\frac{3x^4}{4} + \frac{4x^3}{3} + C$$

$$= \frac{3}{4}x^4 + \frac{4}{3}x^3 + C$$

①

$$v) \int_0^1 \left(\frac{4}{3}x^4 + \frac{2}{3}x^2 + C \right) dx$$

no c's in definite integral

$$(1^4 + 1^2) - (0^4 + 0^2)$$

$$= 2$$

①

This is a good effort, well Done!