

Q1) (a)

$$(i) y = \frac{x^2}{2x+1}$$

$$\frac{d}{dx} = \left(\frac{x^2}{2x+1} \right)$$

$$\text{Apply quotient rule: } \left(\frac{f}{g} \right)' = \frac{f' \times g - g' \times f}{g^2}$$

$$= \frac{\frac{d}{dx} (x^2) (2x+1) - \frac{d}{dx} (2x+1) x^2}{(2x+1)^2}$$

$$\frac{d}{dx} (x^2) = 2x$$

$$\frac{d}{dx} (2x+1) = 2$$

$$= \frac{2x(2x+1) - 2x^2}{(2x+1)^2}$$

$$\text{Expand brackets: } = 4x^2 + 2x$$

$$4x^2 + 2x - 2x^2 = 2x^2 + 2x$$

$$= \frac{2x^2 + 2x}{(2x+1)^2}$$

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Q1) (a)

(iii) $y = 3x^{3/2} + \frac{3}{x^2} - 6x$

$$\frac{d}{dx} \left(3x^{3/2} + \frac{3}{x^2} - 6x \right)$$

Apply: $(f \pm g)' = f' \pm g'$ $\frac{3}{x^2}$

$$= \frac{d}{dx} (3x^{3/2}) + \frac{d}{dx} (x^{-2}) - \frac{d}{dx} (6x)$$

$$\frac{d}{dx} (3x^{3/2}) = \frac{9x^{1/2}}{2}$$

$$\frac{d}{dx} \left(\frac{3}{x^2} \right) = \frac{6}{x^3}$$

$$\frac{d}{dx} (6x) = 6$$

$$= \frac{9x^{1/2}}{2} - \frac{6}{x^3} - 6$$

Combine the two fractions: $\frac{9x^{1/2}}{2} - \frac{6}{x^3} = \frac{9x^{1/2} - 12}{2x^3}$

$$= \underline{\underline{\frac{9x^{1/2} - 12}{2x^3} - 6}}$$

Q1) (a)

(iv) $y = \sec^3 4x$

$$\frac{d}{dx} (\sec^3(4x))$$

Apply Chain rule :

$$= 3(\sec(4x))^2 \frac{d}{dx} (\sec(4x))$$

$$\frac{d}{dx} (\sec(4x)) = \sec(4x) \tan(4x) \times 4$$

$$= 3(\sec(4x))^2 \sec(4x) \tan(4x) \times 4$$

$$\text{Simplify: } 3 \sec^2(4x) \sec(4x) \tan(4x) \times 4 = 12 \sec^3(4x) \tan(4x)$$

$$= \underline{\underline{12 \sec^3(4x) \tan(4x)}}$$

Q1) (a)

(v) $y = (x + \sin 2x)$

$$\frac{d}{dx} (x + \sin 2x)$$

Apply : $(f \pm g)' = f' \pm g'$ rule

$$= \frac{d}{dx} (x) + \frac{d}{dx} (\sin(2x))$$

$$\frac{d}{dx} (x) = 1$$

$$\frac{d}{dx} (\sin(2x)) = \cos(2x) \times 2$$

$$= \underline{\underline{1 + \cos(2x) \times 2}}$$

Q1) (a)

$$(vi) \quad y = \frac{1-10x}{(2x-1)^2}$$

$$\frac{d}{dx} \left(\frac{1-10x}{(2x-1)^2} \right)$$

Apply the quotient rule: $\left(\frac{f}{g} \right)' = \frac{f' \times g - g' \times f}{g^2}$

$$= \frac{\frac{d}{dx} (1-10x) (2x-1)^2 - \frac{d}{dx} ((2x-1)^2) (1-10x)}{((2x-1)^2)^2}$$

$$\frac{d}{dx} (1-10x) = -10$$

$$\frac{d}{dx} ((2x-1)^2) = 4(2x-1)$$

$$= \frac{(-10)(2x-1)^2 - 4(2x-1)(1-10x)}{((2x-1)^2)^2}$$

$$\text{Simplify: } \frac{(-10)(2x-1)^2 - 4(2x-1)(1-10x)}{((2x-1)^2)^2} = - \frac{2(-10x-3)}{(2x-1)^3}$$

$$= - \frac{2x(-10x-3)}{(2x-1)^3}$$

Q2) (a)

(i) $\int \frac{2x}{(4+3x^2)} dx$

Remove constant: $\int a \times f(x) dx = a \times \int f(x) dx$

$$= 2 \times \int \frac{x}{4+3x^2} dx$$

$$= 2 \times \int \frac{1}{6u} du$$

Remove constant: $\int \frac{1}{u} du = \ln|u|$

$$= 2 \times \frac{1}{6} \times \int \frac{1}{u} du$$

use the common integral: $\int \frac{1}{u} du = \ln|u|$

$$= 2 \times \frac{1}{6} \ln|u|$$

substitute $u = 4 + 3x^2$ back in: $= 2 \times \frac{1}{6} \ln|4 + 3x^2|$

simplify: $2 \times \frac{1}{6} \ln|4 + 3x^2| = \frac{1}{3} \ln|4 + 3x^2|$

$$= \frac{1}{3} \ln|4 + 3x^2|$$

$$= \underline{\underline{\frac{1}{3} \ln|4 + 3x^2| + C}}$$

Q2) (a)

(ii) $\int_0^4 e^{\frac{1}{2}x} dx$

Substitute u : $= \int_0^2 e^u \times 2 du$

Remove constant: $\int a \times f(x) dx = a \times \int f(x) dx$

$= 2 \times \int_0^2 e^u du$

Use the common integral: $\int e^u du = e^u$

$= 2 [e^u]_0^2$

~~ans~~ $= \underline{\underline{2(e^2 - 1) = 12.7781}}$

Q2) (a)

(iii) $\int_0^4 \frac{5x+13}{(2x+1)(x+4)} dx$

Find the partial fraction of : $\frac{5x+13}{(2x+1)(x+4)} = \frac{3}{2x+1} + \frac{1}{x+4}$

$$= \int_0^4 \frac{3}{2x+1} + \frac{1}{x+4} dx$$

Apply the sum rule : $\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$

$$= \int_0^4 \frac{3}{2x+1} dx + \int_0^4 \frac{1}{x+4} dx$$

$$\int_0^4 \frac{3}{2x+1} dx = 3 \ln(3)$$

$$\int_0^4 \frac{1}{x+4} dx = \ln(2)$$

$$= \underline{\underline{3 \ln(3) + \ln(2)}}$$

Q2) (b)

$$f(x) = \frac{4}{x(1+4\ln x)^2} dx$$

$$f(x) : \frac{4 dx}{x + 8x \ln(x) + 16x \ln^2(x)}$$

$$f(-x) : \frac{4 d(-x)}{-x - 8x \ln(-x) - 16x \ln^2(-x)}$$

$$-f(x) = \frac{4 dx}{x + 8x \ln(x) + 16x \ln^2(x)}$$

$$= \underline{\underline{\frac{4 dx}{x + 8x \ln(x) + 16x \ln^2(x)}}}$$

Q3) a)

$$(i) \left(\frac{1+i}{1-i} \right)^3$$

$$= \frac{(1+i)^2}{(1-i)(1+i)}$$

$$= \frac{1^2 + i^2 + 2(i)i}{1^2 - i^2}$$

$$i^2 = -1$$

$$Z = \frac{1-1+2i}{1+1}$$

$$Z = \frac{2i}{2}$$

$$Z = i$$

a + ib form: a = 0 and b = 1

Q3 (b)

$$z = (4 + 7i)(3 - 2i)$$

Apply complex arithmetic rule

$$a=4, b=7, c=3, d=-2$$

$$(4 \times 3 - 7(-2)) + (4(-2) + 7 \times 3)i$$

$$4 \times 3 - 7(-2) = 26$$

$$4(-2) + 7 \times 3 = 13$$

$$= \underline{\underline{26 + 13i}}$$

Q3) (c)

$1+3i$ and $1+i$

$$\frac{1+3i}{1+i}$$

Apply complex arithmetic rule:

$$a=1, b=3, c=1, d=1$$

$$= \frac{(1 \times 1 + 3 \times 1) + (3 \times 1 - 1 \times 1)i}{1^2 + 1^2}$$

$$\text{Refine : } = \frac{4+2i}{2}$$

$$\text{Simplify : } \frac{4+2i}{2} = 2+i$$

$$= \underline{\underline{2+i}}$$