

Calculus

Calculus

Differentiation

A straight line has a constant gradient. This means that the rate of change of y with respect to x is a constant.

Calculus

Differentiation

Consider $y = 3x + 1$

If $x=1$ then $y=4$

If $x=2$ then $y=7$

If $x=3$ then $y=10$

If $x=4$ then $y=?$

Calculus

Differentiation

This can also be drawn as a graph, which will look like this:

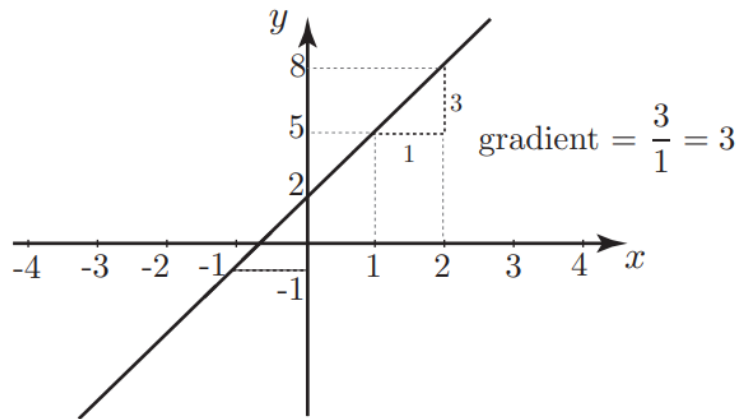


Figure 1. A graph of the straight line $y = 3x + 2$.

Calculus

Differentiation

This can also be drawn as a graph, which will look like this:

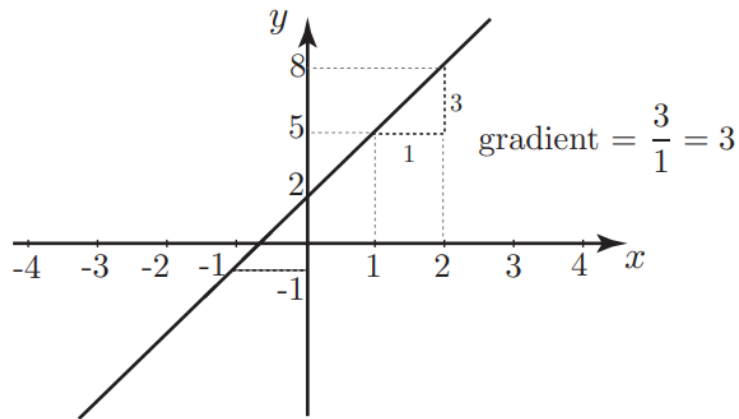


Figure 1. A graph of the straight line $y = 3x + 2$.

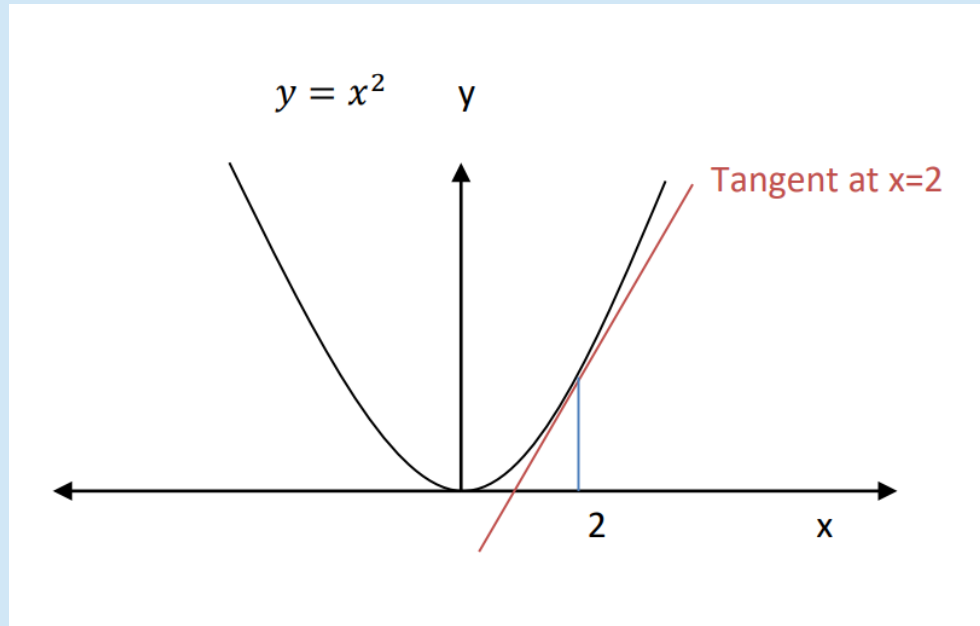
We can calculate the gradient of this line as follows. We take two points and calculate the change in y divided by the change in x .

$$\text{Gradient} = \frac{8 - 5}{2 - 1} = \frac{3}{1} = 3$$

Calculus

Differentiation

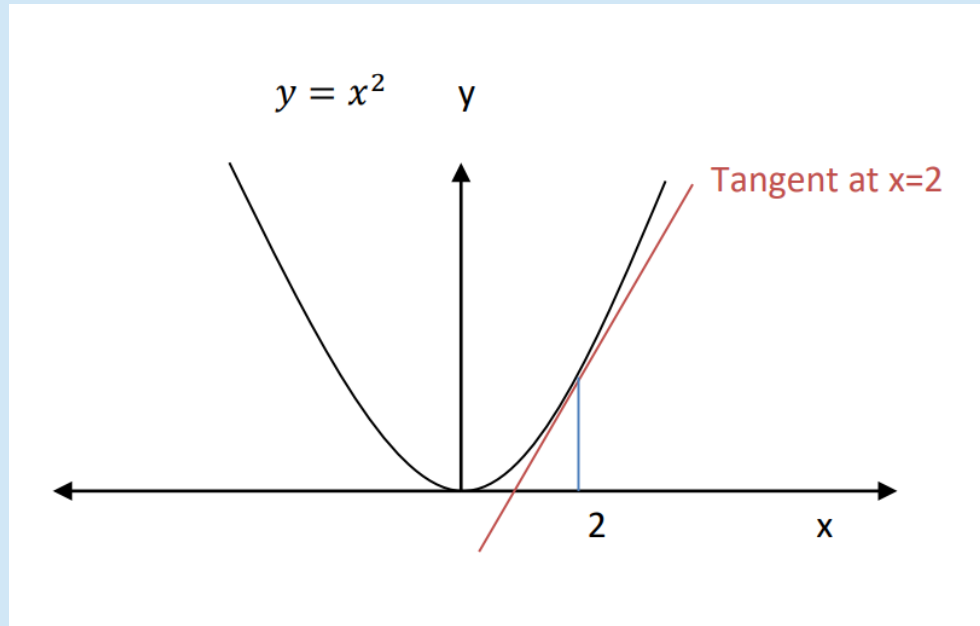
If instead of a straight line we had a curve, for example $y = x^2$.



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Differentiation

If instead of a straight line we had a curve, for example $y = x^2$.



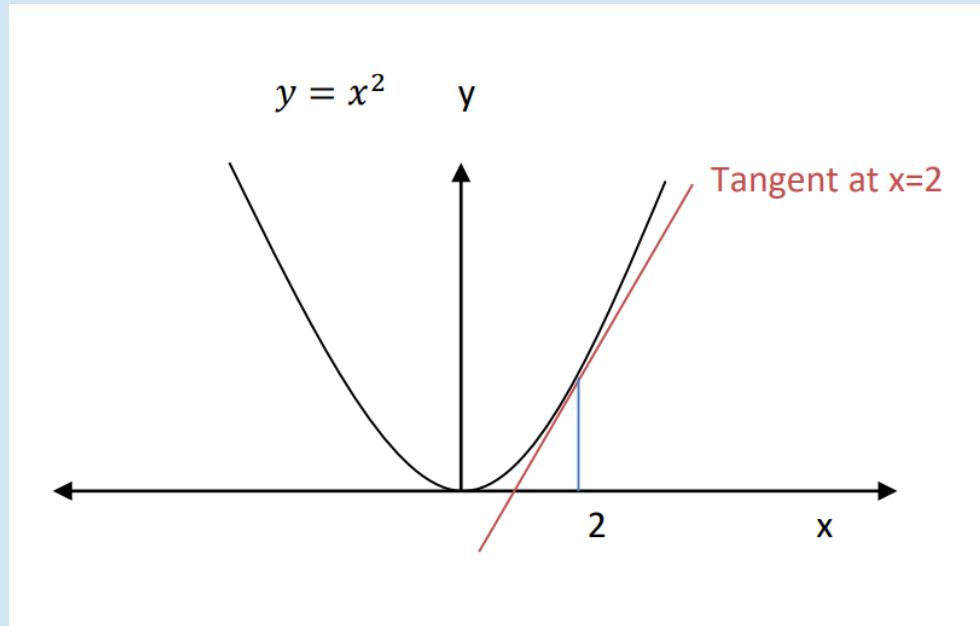
Once again the slope of the curve at $y = x^2$ at the point $x = 2$ is given by:

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

Calculus

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If instead of a straight line we had a curve, for example $y = x^2$.



In calculus, change in y can be written as dy and change in x as dx so we can say that:

$$\text{Gradient} = \frac{dy}{dx}$$

Calculus

Differentiation

Therefore, the meaning of $\frac{dy}{dx}$ is that it represents the slope of a curve at any point on the curve.

To determine $\frac{dy}{dx}$ we use a list of standard derivatives. Starting with:

$$\text{If } y = ax^n \text{ then } \frac{dy}{dx} = (a \times n)x^{(n-1)}$$

Calculus

Differentiation

There are other useful standard derivatives:

$$y = \sin(ax) \text{ then } \frac{dy}{dx} = a \times \cos(ax)$$

Calculus

Differentiation

There are other useful standard derivatives:

$$y = \cos(ax) \text{ then } \frac{dy}{dx} = -a \times \sin(ax)$$

Calculus

Application of Differentiation

Velocity and Acceleration:

If a body moves a distance, x metres in a time of t seconds then:

Distance $x = f(t)$

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Application of Differentiation

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Calculus

Application of Differentiation

Velocity and Acceleration:

If a body moves a distance, x metres in a time of t seconds then:

Distance $x = f(t)$

Velocity $v = f'(t)$ or $\frac{dx}{dt}$

Acceleration $a = \frac{dv}{dt} = f''(t) = \frac{d^2x}{dt^2}$

Calculus

Application of Differentiation

The distance, x metres moved by a car in a time, t seconds is given by:

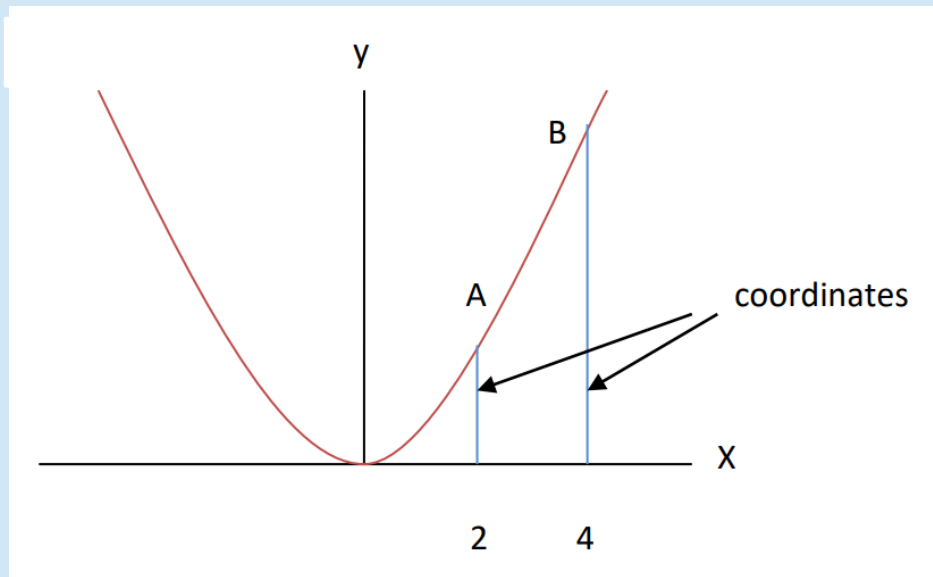
$$x = 3t^3 - 2t^2 + 4t - 1$$

Determine the distance, velocity and acceleration when $t=1.5$ and $t=2.5$ s

Calculus

Integration

Integration is basically the opposite of differentiation.



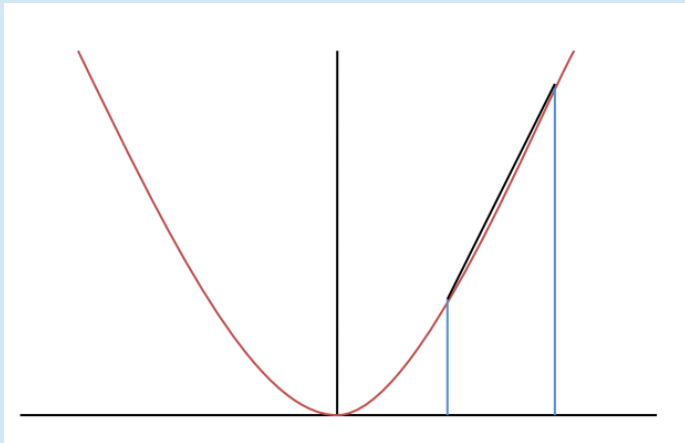
One of its main uses is to find the area under a curve.

Consider the curve $y = x^2$. We want to find the area underneath the curve between $x = 2$ and $x = 4$.

Calculus

Integration

We could turn the area into a trapezium, but this will overestimate the area.



To improve the accuracy, we could split the area up into multiple smaller trapeziums. The greater the number of trapeziums, the more accurate the result will become.

Calculus

Integration

Integration is essentially the same concept, except we're dividing the area into an infinite number strips.

Its width along the x axis will be very small and is called dx , the height along the y axis will be called y .

We can think of the strip as being virtually rectangular in shape so the area is $y \cdot dx$.

Calculus

Integration

The total area will be the sum of all these small rectangles.

$$Area = \sum_2^4 y \cdot dx$$

When this theory was developed the symbol for sum of was:

$$\int_2^4 y \cdot dx$$

Calculus

Integration

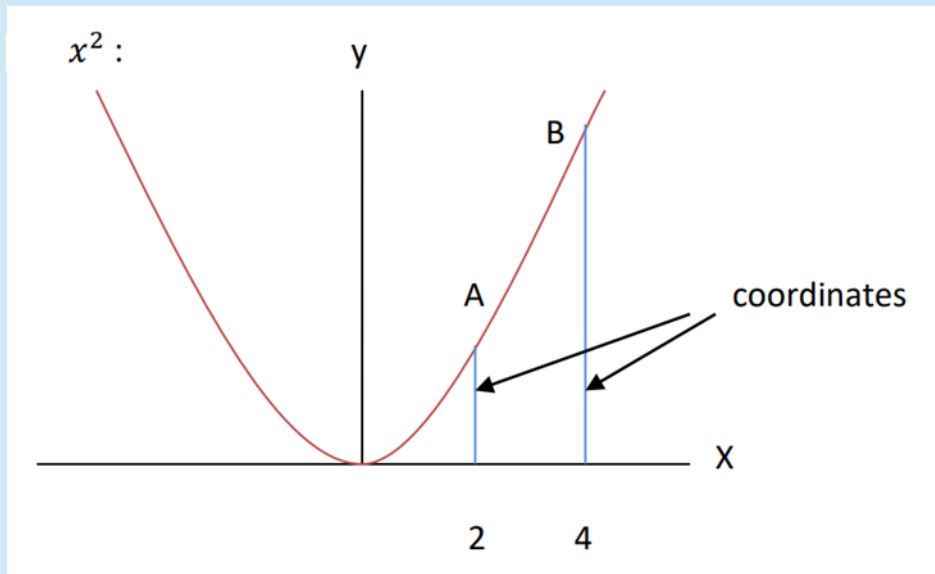
Integrals are worked out using standard integrals, the most used one shows us how to integrate a power of x.

$$y = ax^n \quad \int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

Calculus

Integration

Integrals are worked out using standard integrals, the most used one shows us how to integrate a power of x .

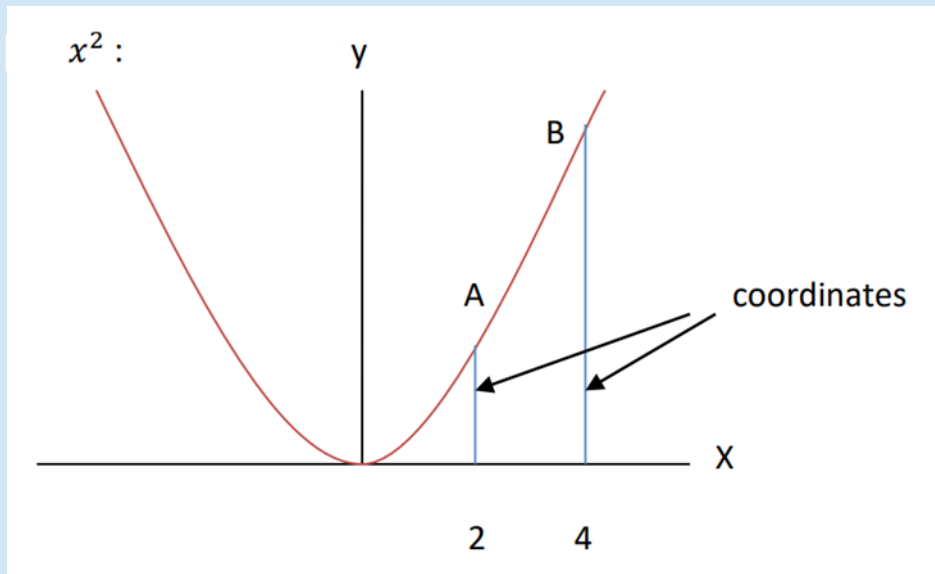


$$\int_2^4 x^2 \cdot dx = \left[\frac{x^{2+1}}{2+1} + c \right]_2^4 = \left[\frac{x^3}{3} + c \right]_2^4$$

Calculus

Integration

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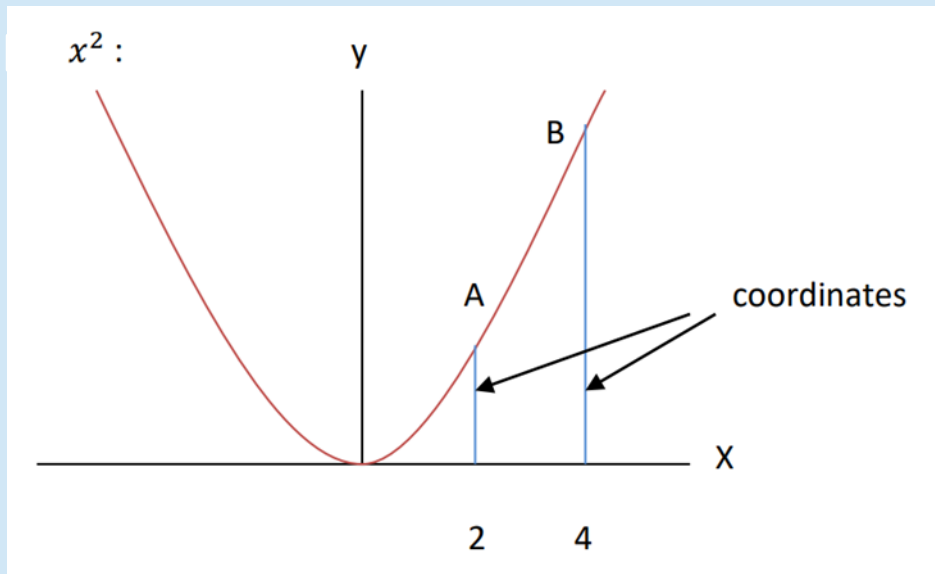


$$= \left[\frac{x^3}{3} + c \right] = \left[\frac{4^3}{3} + c \right] = \left[\frac{64}{3} + c \right]$$

Calculus

Integration

Integrals are worked out using standard integrals, the most used one shows us how to integrate a power of x .



$$= \left[\frac{x^3}{3} + c \right] = \left[\frac{2^3}{3} + c \right] = \left[\frac{8}{3} + c \right]$$

Calculus

Integration

$$= \left[\frac{64}{3} + c \right] - \left[\frac{8}{3} + c \right]$$

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Integration

$$= \left[\frac{64}{3} + c \right] - \left[\frac{8}{3} + c \right]$$

$$= \left[\frac{64}{3} \right] - \left[\frac{8}{3} \right] = \frac{56}{3}$$

Calculus

Integration

There are two types of integral: Definite (with limits)

e.g. $\int_{-1}^2 (x^2 + 4)dx$ - Definite Numerical Answer

Calculus

Integration

There are two types of integral: Definite (with limits) and Indefinite (without limits)

e.g. $\int (x^2 + 4) dx$ – Answer must contain '+c' at the end.

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There are other useful standard integrals:

$$y = \sin(ax) \text{ then } \int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

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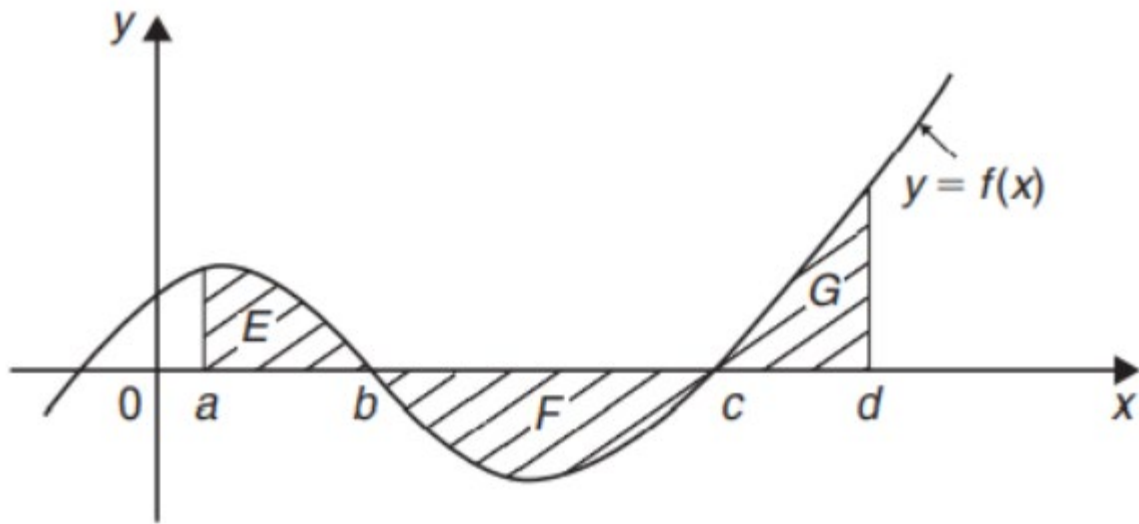
$$y = \cos(ax) \text{ then } \int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

Calculus

Integration:

There are many problems in engineering where the area under a curve needs to be determined.

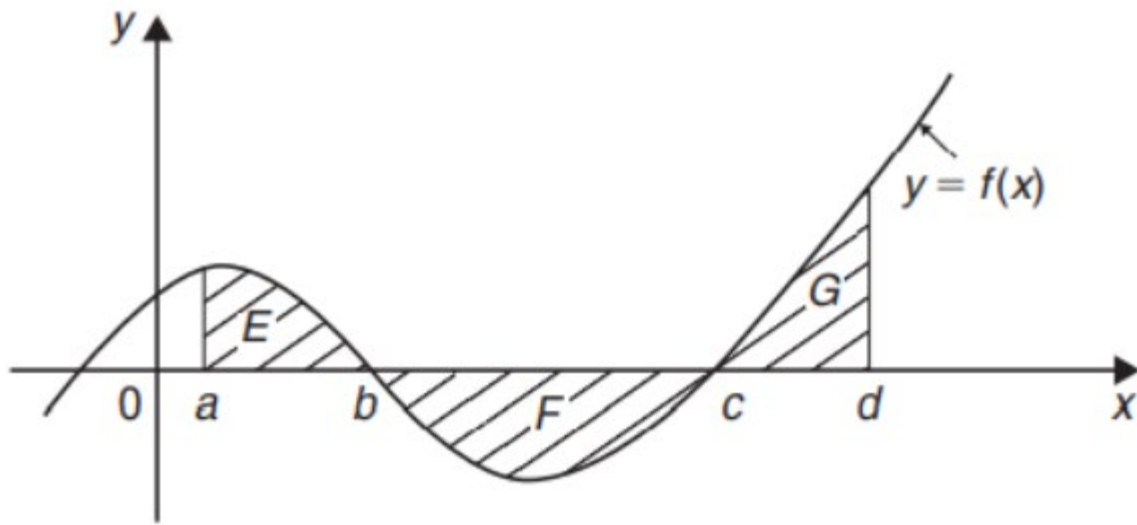
For the curve shown, the total shaded area is given by (area E + area F + area G).



Calculus

Integration:

There are many problems in engineering where the area under a curve needs to be determined.



For the curve shown, the total shaded area is given by (area E+ area F +area G).

$$= \int_a^b f(x)dx - \int_b^c f(x)dx + \int_c^d f(x)dx$$

Note: this is NOT the same as

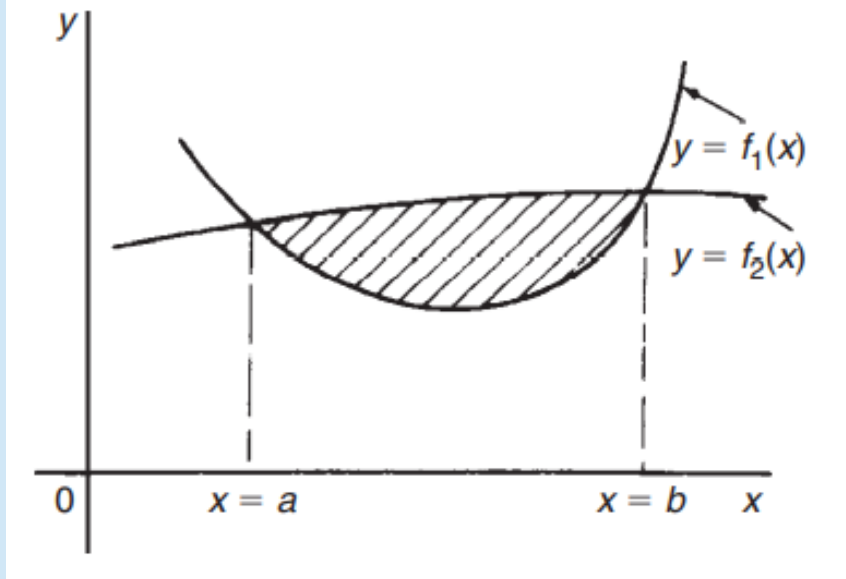
$$\int_a^d f(x)dx$$

Calculus

Integration:

There are many problems in engineering where the area under a curve needs to be determined.

The area between curves may be determined by using integration.



$$= \int_a^b f_2(x) dx - \int_b^c f_1(x) dx$$