

1.

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Module Code: ACH1A4000 / ACMA4006

Analytic Mathematics

Question 1

a)

b)

$$(i) \frac{3x-3}{5} = 4\left(\frac{-5x+6}{3}\right)$$

$$-9x+9 = -25x+30$$

$$-9x+9-9 = -25x+30-9$$

$$-9x = -25x+21$$

$$-9x+25x = -25x+21+25x$$

$$16x = 21$$

$$\frac{16x}{16} = \frac{21}{16}$$

$$x = \frac{21}{16}$$

$$(ii) \frac{(4x-1)}{3} - \frac{x}{4} = \frac{(4x+3)}{2}$$

$$4(4x-1) - 3x = 6(4x+3)$$

$$16x-4-3x = 24x+18$$

$$13x-4 = 24x+18$$

$$-4 = 24x+18-13x$$

$$-4-18 = 11x$$

$$-22 = 11x$$

$$\frac{-22}{11} = x$$

$$-2 = x$$

$$x = -2$$

$$(iii) 2\log(5x+3) = \log(x+1) + \log(x+2)$$

$$2\log(5x+3) + (x+1) = \log(x+2)$$

$$x^2 + 8x + 15 = x^2 + 2x$$

$$0 = 9x^2 + 8x + 9x^2 - 13x$$

using quadratic formula.

$$0 = 8x + 9x^2 - 13$$

$$8x^2 + 9x^2 - 13 = 0$$

$$x = \frac{-9x^2 \pm \sqrt{9x^2 - 4 \times 8 \times -13}}{2 \times 8}$$

$$x = \frac{-9x \pm \sqrt{497}}{16}$$

$$x = -3.43$$

$$x = +14.57$$

$$(iv) 7^x \times 3^{x+2} = 5^{x-1}$$

$$7^x \times 3^x \times 3^2 = 5^x \times 5^{-1}$$

$$7^x \times 3^x \times 9 = 5^x \times 1/5$$

$$(7 \times 3)^x \times 9 = 1/5 \times 5^x$$

$$9 \times 21^x = 1/5 \times 5^x$$

$$9 \times (21/5)^x = 1/5$$

$$9 \times 21^x = 1/5 \times 5^x$$

$$9 \times \left(\frac{21}{5}\right)^x = 1/5$$

$$\left(\frac{21}{5}\right)^x = 1/45$$

$$x = -2.65$$

g

Question 1

c)

(i) $7R^2 + 24R + 5$

$$7R^2 + 15R + 9R + 5$$

$$3R \times (9R + 5) + 9R + 5$$

$$(9R + 5) \times (3R + 1)$$

3.

$$\begin{aligned}
 & \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 & \alpha = \frac{-24 \pm \sqrt{24^2 - 4 \times 21 \times 5}}{2 \times 21} \\
 & \alpha = \frac{-24 \pm \sqrt{504 - 420}}{42} \\
 & \alpha = \frac{-24 \pm \sqrt{84}}{42} \\
 & \alpha = \frac{-24 \pm 2\sqrt{21}}{42} \\
 & \alpha = \frac{-12 \pm \sqrt{21}}{21}
 \end{aligned}$$

(ii) CF=9

$$9 \left(\frac{81Q^2}{9} + \frac{-144U^2}{9} \right)$$

$$9(9Q^2 - 16U^2)$$

$$9((3Q)^2 - (4U)^2)$$

$$9(3Q+4U)(3Q-4U)$$

$$(iii) \quad fr - fy + ur + uy \\ + (r-y) + u(r-y)$$

$$(r-y)x(f+u)$$

d)

$$(i) x^2 + 9x + 8 = 0$$

$$(x+1)(x+8) = 0$$

$$x_1 = -8$$

$$x_2 = -1$$

4.

$$(ii) 12x^2 - 5x - 3 = 0$$

$$12x^2 + 4x - 9x - 3 = 0$$

$$4x(3x+1) - 3(3x+1) = 0$$

$$(3x+1)(4x-3) = 0$$

$$x_1 = \underline{\underline{-\frac{1}{3}}}$$

$$x_2 = \underline{\underline{\frac{3}{4}}}$$

$$3x+1=0$$

$$x = -\frac{1}{3}$$

$$4x-3=0$$

$$x = \frac{3}{4}$$

$$(iii) 2x^2 + 17x + 30 = 0$$

$$2x^2 + 12x + 5x + 30 = 0$$

$$2x^2 + 12x + 5x + 30 = 0$$

$$2x(x+6) + 5(x+6) = 0$$

$$(x+6)(2x+5) = 0$$

$$x = -6$$

$$x = \underline{\underline{-\frac{5}{2}}}$$

$$(iv) x^2 - 256 = 0$$

$$x^2 = 256$$

$$x = \pm 16$$

$$\begin{pmatrix} x=16 \\ x=-16 \end{pmatrix}$$

$$c) (x+5)^2 - x^2 - 32 = (x+1)(x+2) - x^2$$

$$(x+5)^2 - 32 = (x+1)(x+2)$$

$$x^2 + 10x + 25 - 32 = x^2 + 2x + x + 2$$

$$x^2 + 10x - 7 = x^2 + 3x + 2$$

$$x^2 + 10x - 7 = x^2 + 3x + 2$$

$$10x = 3x + 2 + 7$$

$$10x - 3x = 9$$

$$7x = 9$$

$$x = \frac{9}{7}$$

$$= \frac{9}{7}$$

Question Two

a)

~~$$y = 8x^3 - 7x^2 + 10x + 10$$~~

~~$$y = 8x^3 - 7x^2 + 0x + 10$$~~

~~$$y = 10$$~~

~~$$(ii) y = (x+3)(x-4)$$~~

~~$$0 = (x+3)(x-4)$$~~

~~$$(x+3)(x-4) = 0$$~~

~~$$x+3=0$$~~

~~$$x-4=0$$~~

$$x = \frac{3}{4}$$

$$x = \frac{1}{2}$$

(iii)

~~$$y = 8x^3 + 10x$$~~

~~$$y = 8x^3 + 10x$$~~

No solution for $x=0$
so there is no y-intercept

NO

Q6 Question Two

$$(i) y = 8x^3 - 2x^2 + x + 10$$

$$y = \frac{d}{dx} (8x^3 - 2x^2 + x + 10)$$

$$y = \frac{d}{dx} (8x^3) + \frac{d}{dx} (-2x^2) + \frac{d}{dx} (x) + \frac{d}{dx} (10)$$

$$y = 8 \times 3x^2 - 2 \times 2x + 1 + 0$$

$$y = 24x^2 - 4x + 1$$

$$(ii) y = (5x - 3) \times (6x - 1)$$

$$y = \frac{d}{dx} ((5x - 3) \times (6x - 1))$$

$$y = \frac{d}{dx} (30x^2 - 5x - 18x + 3)$$

$$y = \frac{d}{dx} (30x^2 - 23x + 3)$$

$$y = \frac{d}{dx} (30x^2) + \frac{d}{dx} (-23x) + \frac{d}{dx} (3)$$

$$y = 30 \times 2x - 23 + 0$$

$$y = 60x - 23$$

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$$(iii) y = \frac{3}{\sqrt{x}} + \ln(x)$$

$$y = \frac{d}{dx} \left(\frac{3}{\sqrt{x}} + \ln(x) \right)$$

$$y = \frac{d}{dx} \left(\frac{3}{\sqrt{x}} \right) + \frac{d}{dx} (\ln(x))$$

$$y = -3x^{-\frac{1}{2}} + \frac{1}{x}$$

$$y = \frac{-3 + 2\sqrt{x}}{2x\sqrt{x}}$$

$$(iv) y = 5\sin\left(\frac{x}{2}\right) - 6\cos(3x) - \tan\left(\frac{x}{3}\right)$$

$$y = \frac{d}{dx} \left(5\sin\left(\frac{x}{2}\right) - 6\cos(3x) - \tan\left(\frac{x}{3}\right) \right)$$

$$y = \frac{d}{dx} \left(5\sin\left(\frac{x}{2}\right) - 6\cos(3x) + \tan\left(\frac{x}{3}\right) \right)$$

$$y = \frac{d}{dx} \left(5\sin\left(\frac{x}{2}\right) \right) + \frac{d}{dx} (-6\cos(3x)) + \frac{d}{dx} \left(\tan\left(\frac{x}{3}\right) \right)$$

$$y = 5\cos\left(\frac{x}{2}\right) \times \frac{1}{2} - 6 \times (-\sin(3x) \times 3) + \sec^2\left(\frac{x}{3}\right) \times \frac{1}{3}$$

$$y = \frac{5\cos\left(\frac{x}{2}\right)}{2} + 18\sin(3x) + \frac{\sec^2\left(\frac{x}{3}\right)}{3}$$

9.4

(iv)

$$y = 5 \sin \frac{x}{2} + 6 \cos 3x - \tan \frac{x}{3}$$

$$y = 5 \sin \left(\frac{0}{2} \right) + 6 \cos \left(3 \cdot 0 \right) - \tan \left(\frac{0}{3} \right)$$

$$y = 5 \sin(0) + 6 \cos(0) - \tan(0)$$

$$y = 5 \times 0 + 6 \times 1 - \tan(0)$$

$$y = 6 \times 1 - 0$$

$$y = 6 - 0$$

$$y = 6$$

INCORRECT

(v)

$$y = e^{7x} \tan 3x$$

~~$$y = e^{7x} \tan(3x)$$~~

$$y = \frac{d}{dx} (e^{7x} \times \tan(3x))$$

$$y = \frac{d}{dx} (e^{7x}) \times \tan(3x) + e^{7x} \times \frac{d}{dx} (\tan(3x))$$

$$y = e^{7x} \times 7 \tan(3x) + e^{7x} \times \sec(3x)^2 \times 3$$

$$y = 7e^{7x} \times \frac{\sin(3x)}{2\cos(3x)^2} + 3e^{7x}$$

$$y = \frac{7e^{7x} \times \sin(3x) + 6e^{7x}}{2\cos(3x)^2}$$

(vi)

$$y = \frac{\sin^2 x}{e^{3x}}$$

$$y = \frac{d}{dx} \left(\frac{\sin(2x)}{e^{3x}} \right)$$

$$y = \frac{d}{dx} (\sin(2x)) \times \frac{1}{(e^{3x})^2} - \sin(2x) \times \frac{d}{dx} (e^{3x})$$

$$y = \frac{2\cos(2x) - 3\sin(2x)}{e^{3x}}$$

$$(vii) y = \sqrt[3]{4x^3 - 2x + 1}$$

$$y = \frac{d}{dx} (4x^3 - 2x + 1)$$

$$y = \frac{d}{dx} (x^{\frac{1}{3}}) \times \frac{d}{dx} (4x^3 - 2x + 1)$$

$$y = \frac{1}{3} \times (4x^3 - 2x + 1)^{-\frac{2}{3}} \times (4 \times 3x^2 - 2)$$

$$y = \frac{12x^2 - 2}{3\sqrt[3]{(4x^3 - 2x + 1)^2}}$$

$$b) 4^{th} \text{ derivative of } y = \sin 2x$$

$$y = \frac{d}{dx} (\sin(2x))$$

$$y = \frac{d}{dx} (\sin) \times \frac{d}{dx} (2x)$$

$$y = \cos(x) \times \frac{d}{dx} (2x)$$

$$y = \cos(x) \times 2$$

$$x = 2x$$

$$y = \cos(2x) \times 2$$

$$y = 2\cos(2x)$$

$$\underline{y = 2\cos(2x)}$$

11.

c)

$$d) \begin{aligned} y &= 3x^2 + 16x + 30 \\ y &= 2x^2 + 4x + 3 \end{aligned}$$

$$y = 3x^2 + 16x + 30 = 2x^2 + 4x + 3$$

$$x = -9$$

$$x = -3$$

$$y = 2x(-9)^2 + 4x(-9) + 3$$

$$x = -3$$

$$y = 2x(-9)^2 + 4x(-9) + 3$$

$$y = 2x(-3)^2 + 4x(-3) + 3$$

$$y = 129$$

$$y = 2x(-3)^2 + 4x(-3) + 3$$

$$y = 129$$

$$y = 9$$

$$(x_1, y_1) = (-9, 129)$$

$$(x_2, y_2) = (-3, 9)$$