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Transient Response Modelling of a Vehicle

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Abstract

This assignment focused on the transient response modelling of a vehicle using advanced motorsport dynamics. A 1DOF sprung mass damper (SMD) system was built and validated from first principles, followed by the development of a 4DOF SMD model in Simulink and further expansion to a 7DOF model. Results were correlated to provided simulator data, and further tests were conducted while changing variables. The study found that while lumped mass models can be useful for analysing vehicle performance and control, they are not detailed enough to accurately depict the nonlinear behaviour of the suspension system or the intricacies of the vehicle's structure. The Simulink model can be used as a starting point for vehicle setup with the ability to look at the effects of different ride frequencies and damping coefficients. However, further validation and a more detailed model are recommended for fine-tuning.

1. Introduction

Vehicle dynamics studies the characteristic behaviour of a vehicle in response to a change in its external environment. Progress in vehicle dynamics plays a fundamental role in the development of the automotive industry. Since the early 1990's there has been exhaustive research regarding the handling stability and ride comfort of vehicles (Yang, Li, & Lu, 2013). The vehicle development process has evolved due to the advancement of dynamics simulation capabilities. Often at the expense of accuracy, theoretical analysis methods were utilised to provide fast and cost-effective alternatives to physical testing. Due to the advancement of computational capabilities and computer simulation technologies, automotive testing in a virtual simulation environment has become a common practice (Na, Jang, Yoo, & Kim, 2015).

1.1. Aim

To build a 7DOF Spring-Mass-Damper model of the Formula Ford Race Car to characterise its transient response and behaviour.

1.2. Objective

The objective of this assignment is for the student to demonstrate their knowledge and show a critical understanding of the processes governing the dynamic behaviour of a vehicle, its suspension, tyres, and driver interaction while in motion. Furthermore, to display cognitive skills to use advanced data acquisition interpretation techniques to acquire dynamic data to validate complex vehicle models.

1.3. Methodology

The methodology for this assignment is as follows:

- Conduct research into vehicle dynamics and modelling processes therein.
- From first principles, build and validate a 1DOF SMD system.
- Build and validate a 4DOF SMD model in Simulink.
- Develop the 4DOF model into a 7DOF model.
- Correlate the model's response to provided simulator data by comparing results.
- Perform further tests on the model while changing variables.
- Discuss results and limitations. Conclude.

2. Theory

Design phase simulation allows engineers to adapt and react to problems and issues much earlier, giving the advantage of reduced costs and development times, more reliable validation, and better-performing vehicles.

2.1. Simulation Methods

Broad simulation methods are static and quasi-static, transient, and HIL. Static and quasi-static simulation refers to time-independent models. A transient analysis is an analysis of time response and requires the material to have a density to calculate the acceleration loads on the vehicle mass. A hardware-in-the-loop (HIL) system consists of a computer simulation of the vehicle and its surroundings with an active steering system with a real-time controller (Skill-Lync, 2020).

2.2. ADAMS and Lumped Mass Models

Automated Dynamic Analysis of Mechanical Systems (ADAMS) is a standard simulation tool for automotive applications. However, obtaining all the data necessary to define and execute the computational model is often unfeasible. In such cases, simplification is necessary. A simple modelling method for vehicle analysis is the lumped mass vehicle model (Na, Jang, Yoo, & Kim, 2015).

Despite simplifying the actual vehicle architecture, analysing vehicle vibration characteristics and the effects of structural parameters on vehicle performance is possible. Mass, spring, and damper elements comprise the lumped parameter modelling of a vehicle with their possible movement defining the model's degrees of freedom. To analyse the vehicle's vertical, pitch and roll vibrations, the lumped mass vehicle model requires a minimum of 7 DOF and is usually three-dimensional (Kim & Ro, 2002). An advantage of the Lumped Mass models is the simplicity and ability to obtain analytic descriptions and design active or semi-active controls (Yang, Li, & Lu, 2013). This model is widely used to study vehicle handling dynamics undergoing emergency collision avoidance, with MathWorks providing a reference application per the ISO 3888-2 standard (MathWorks, 2022), (ISO, 2011).

2.3. Base Performance Simulator

Driver-in-the-loop simulators comprise complex physics, mechanics, and hardware to mimic real-life driving scenarios (Sasena & Abhinav, 2018). Based on rFactor, BPS car models can be developed and manipulated to mimic any vehicle. Live telemetry is transmitted and recorded, allowing for analysis (BPS, 2022). The rFactor physics engine is a proprietary technology developed by Image Space Incorporated (ISI) and is used as the foundation for their racing simulation games (IMAGE SPACE INCORPORATED, 2019). rFactor's physics engine is considered to be one of the most realistic and accurate in the racing simulation genre. As such, it has been licensed by other companies to use in their racing simulators.

3. Simulink Models

The following section shows the development and validation of a 1,4, and 7 DOF model, with the 7DOF model used for further analysis. A 1DOF SMD model is limited to capturing vertical motion. In a quarter car model or 2DOF SMD model, the sprung mass m_s represents 25% of the vehicle body, and the unsprung mass m_u , a wheel. The 2DOF model allows for the study of vertical vibrations (Jazae, 2014). A half-car model allows for studying a vehicle's roll or pitch by modelling half the sprung mass and two unsprung wheels. The roll, pitch, and vertical movement of the sprung mass and four unsprung wheels are modelled in a 7DOF model.

3.1. 1DOF - The mass-spring-damper model

3.1.1. Equations and Model

The mass-spring-damper model is a simple idealisation applied to various vibrating physical systems (Zbarsky, 2023). Applying Newton's 2nd Law of Motion produces Equation 3-1, and Figure 3-1 shows the system.

$$F = m\ddot{x} + c\dot{x} + kx \quad (3-1)$$

k refers to spring stiffness in N/m, c is the damping coefficient in N·s/m, m is the object mass measured in kg, F is the externally applied force in N, and the time-dependent expression is $F(t)$. The time-dependent behaviour of the system is solved using the following variables: t is the time since the simulation started in seconds; x and $x(t)$ are displacements and the time-dependent displacement of the mass from the reference equilibrium position in meters; $\dot{x}$ is the mass velocity in m/s; $\ddot{x}$ is mass acceleration in m/s².

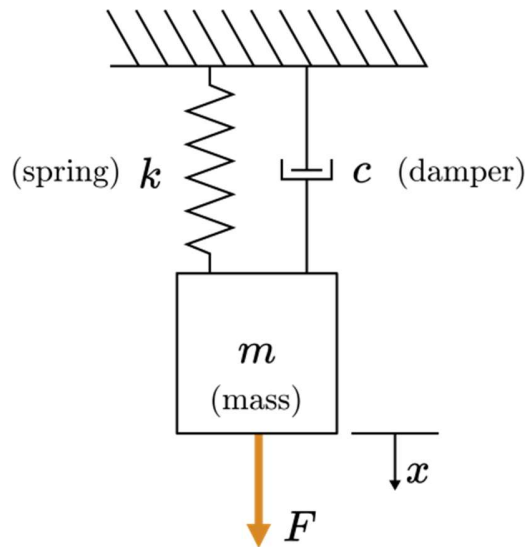


Figure 3-1: Schematic of the mass-spring-damper system (Zbarsky, 2023)

The mass will begin to vibrate when a force is applied to the system. Over time, the vibration diminishes as mechanical energy is converted to heat by the damper. Figure 3-2 shows the MathWorks model used to validate the student model in Figure 3-3. The model made by the student contains spring and damper force gain blocks that evaluate the mass's position and velocity for an applied signal (displacement) to determine the necessary force to apply in real time.

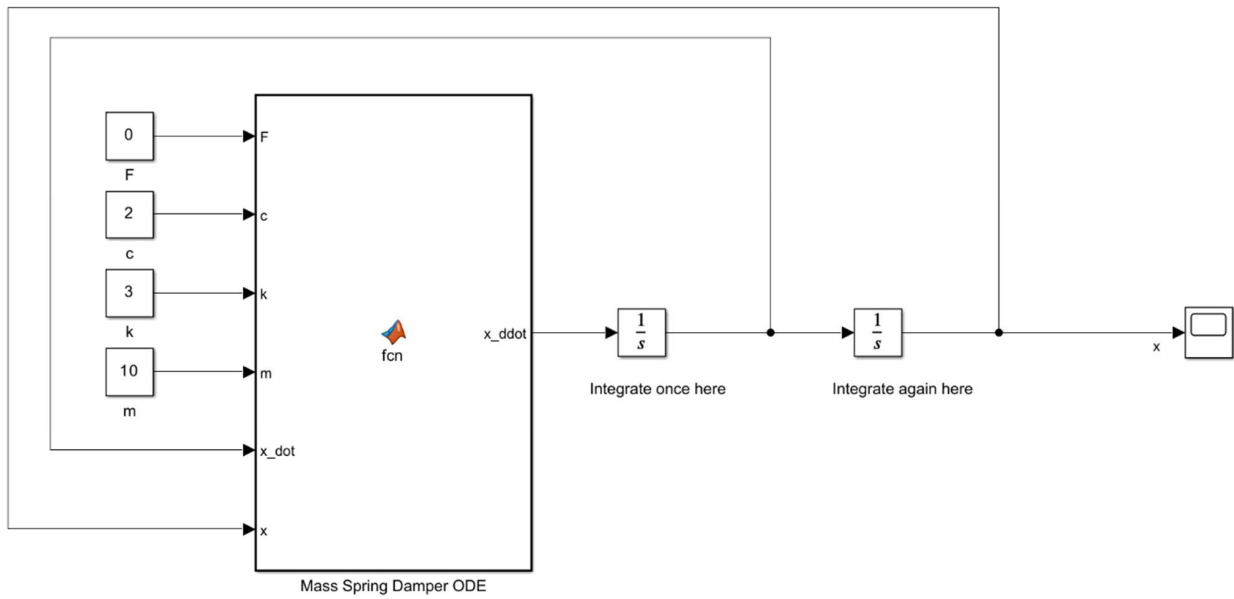


Figure 3-2: MathWorks MSD (Zbarsky, 2023)

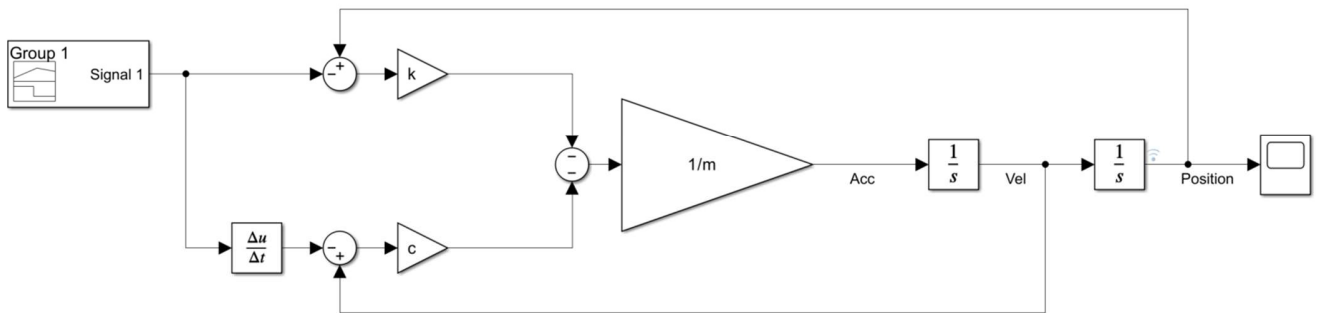


Figure 3-3: Student 1DOF MSD Simulink Model

3.1.2. Validation

Validation of the student model is done by using the MathWorks model and setting the following initial conditions:

- $m = 10 \text{ kg}$
- $F = 0 \text{ N}$
- $V_0 = -3 \text{ m/s}$
- $X_0 = 2 \text{ m}$

Figures 3-4 and 3-5 show the results of mass position vs time for the respective models, validating the student model.

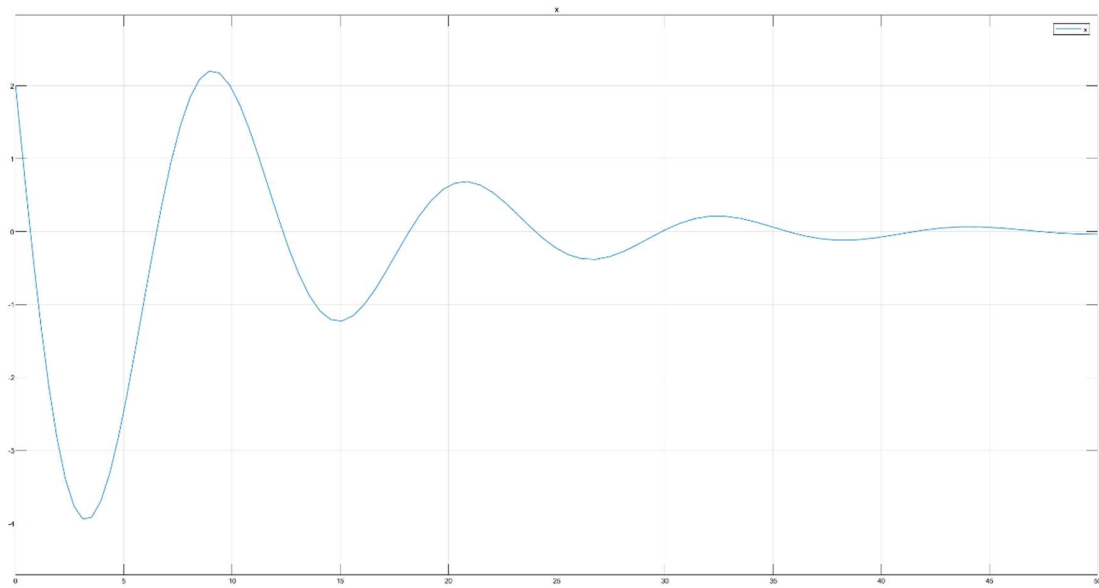


Figure 3-4: Position vs Time graph of MathWorks model

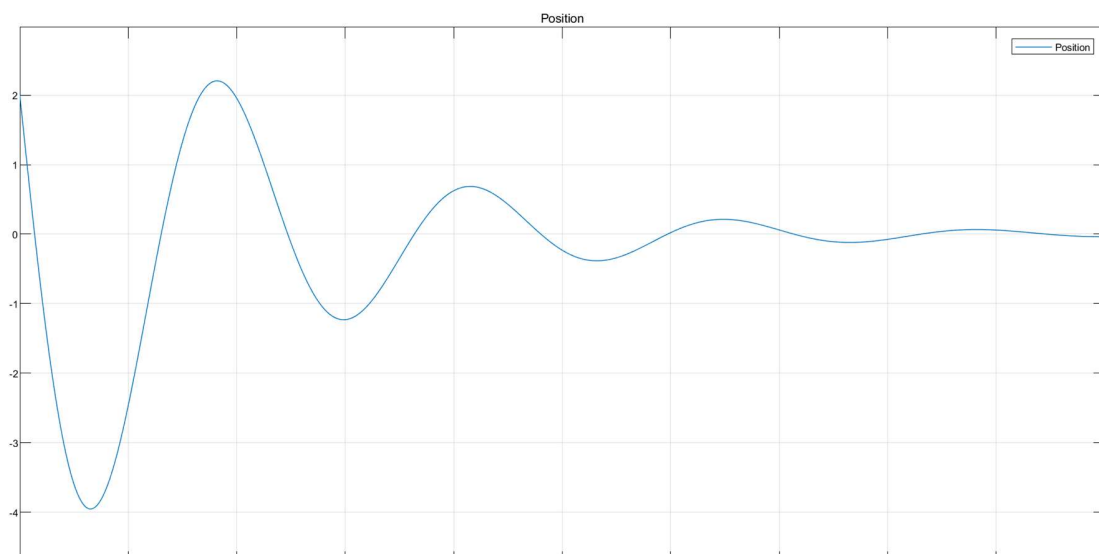


Figure 3-5: Position vs Time graph of Student Model

3.2. 4DOF - Half Car Model

3.2.1. Equations and Model

A half-car model is a simplified mathematical representation of a vehicle's dynamic behaviour used to analyse and predict a car's suspension and handling characteristics. It consists of a single-axle model, representing one-half of the vehicle, and is used to simulate the vehicle's behaviour in response to road inputs. The half-car model typically includes mathematical representations of the vehicle's suspension system, tires, and road surface. Engineers use this model to analyse the vehicle's dynamic response to different road inputs and to test and optimise the vehicle's suspension system. The half-car model is also used to develop semi-active and active suspension systems where the suspension is adjusted automatically in real-time to optimise the vehicle's handling and ride comfort. KL University conducted similar research (Sastry et al, 2017). Their passive 4DOF model was used for validation against the student-built Simulink model. Figure 3-6 shows a diagram of the KL University half-car-passive suspension model, and Figure 3-7 shows the student's model.

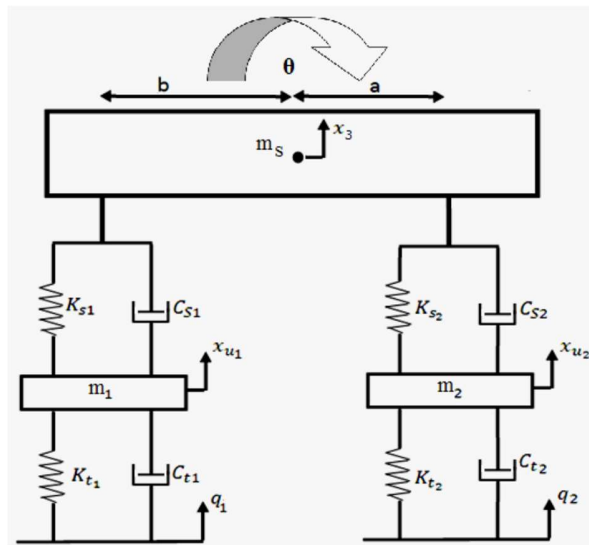


Figure 3-6: diagram the KL University half-car-passive suspension model (Sastry et al, 2017)

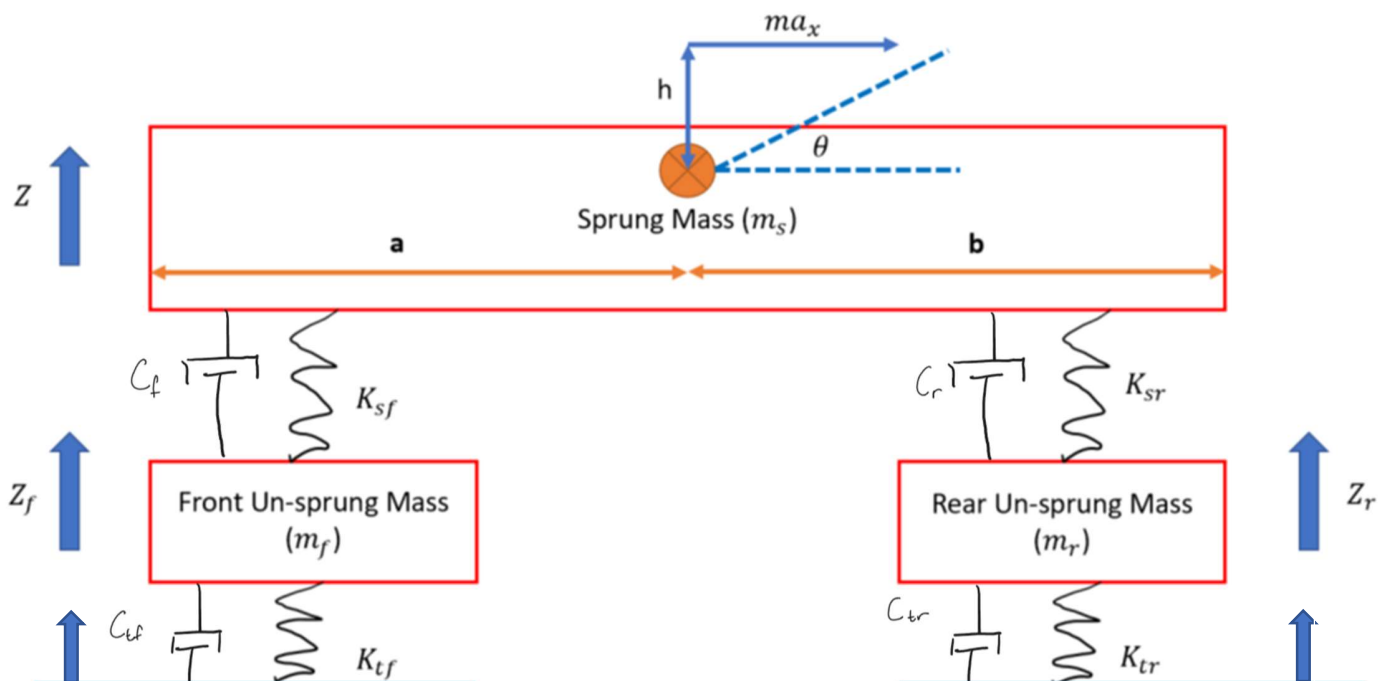


Figure 3-7: Diagram of Student's Half-Car Model

Equations 3-2 and 3-3 describe the vertical motion of the front and rear unsprung masses, respectively. Equation 3-2 represents the acceleration of the front unsprung mass $\ddot{Z}_f$ and Equation 3-3 is the same as equation 3-2 but for the rear unsprung mass $\ddot{Z}_r$. Equation 3-5 represents the acceleration of the pitch $\ddot{Q}$ and is a function of the pitch moment of inertia I_p .

$$\ddot{Z}_f = \frac{-1}{m_f} [K_{tf}(Z_f - Y_f) - K_{sf}(Z - Z_f - a\theta) + C_{tf}(\dot{Z}_f - \dot{Y}_f) - C_f(\dot{Z} - \dot{Z}_f - a\dot{\theta})] \quad (3-2)$$

$$\ddot{Z}_r = \frac{-1}{m_r} [K_{tr}(Z_r - Y_r) - K_{sr}(Z - Z_r + b\theta) + C_{tr}(\dot{Z}_r - \dot{Y}_r) - C_r(\dot{Z} - \dot{Z}_r + b\dot{\theta})] \quad (3-3)$$

$$\ddot{Z} = \frac{-1}{m_s} [K_{sf}(Z - Z_f - a\theta) + C_f(\dot{Z} - \dot{Z}_f - a\dot{\theta}) + K_{sr}(Z - Z_r + b\theta) + C_r(\dot{Z} - \dot{Z}_r + b\dot{\theta})] \quad (3-4)$$

$$\ddot{Q} = \frac{-1}{I_p} [bK_{sr}(Z - Z_r + b\theta) + bC_r(\dot{Z} - \dot{Z}_r + b\dot{\theta}) - aK_{sf}(Z - Z_f - a\theta) - aC_f(\dot{Z} - \dot{Z}_f - a\dot{\theta}) + ma_x h] \quad (3-5)$$

K_{tf} and K_{tr} refer to the front and rear tyre spring coefficients in N/m. C_{tf} and C_{tr} represent the front and rear tyre damping coefficients in N·s/m. K_{sf} and K_{sr} refer to the front and rear spring coefficients [N/m] while C_f and C_r are the front and rear damping coefficients [N·s/m]. "a" and "b" are distances from the centre of mass to the front and rear, respectively [m]. a_x is the longitudinal acceleration that affects the pitch of the vehicle. It is converted to a force by multiplying it by mass m_s and to a torque by further multiplying the product by the height "h". θ is the sprung mass pitch. Y_f and Y_r refer to the ground position at the front and rear of the vehicle. Figure 3-8 shows the Simulink model describing equations 3-2 to 3-5.

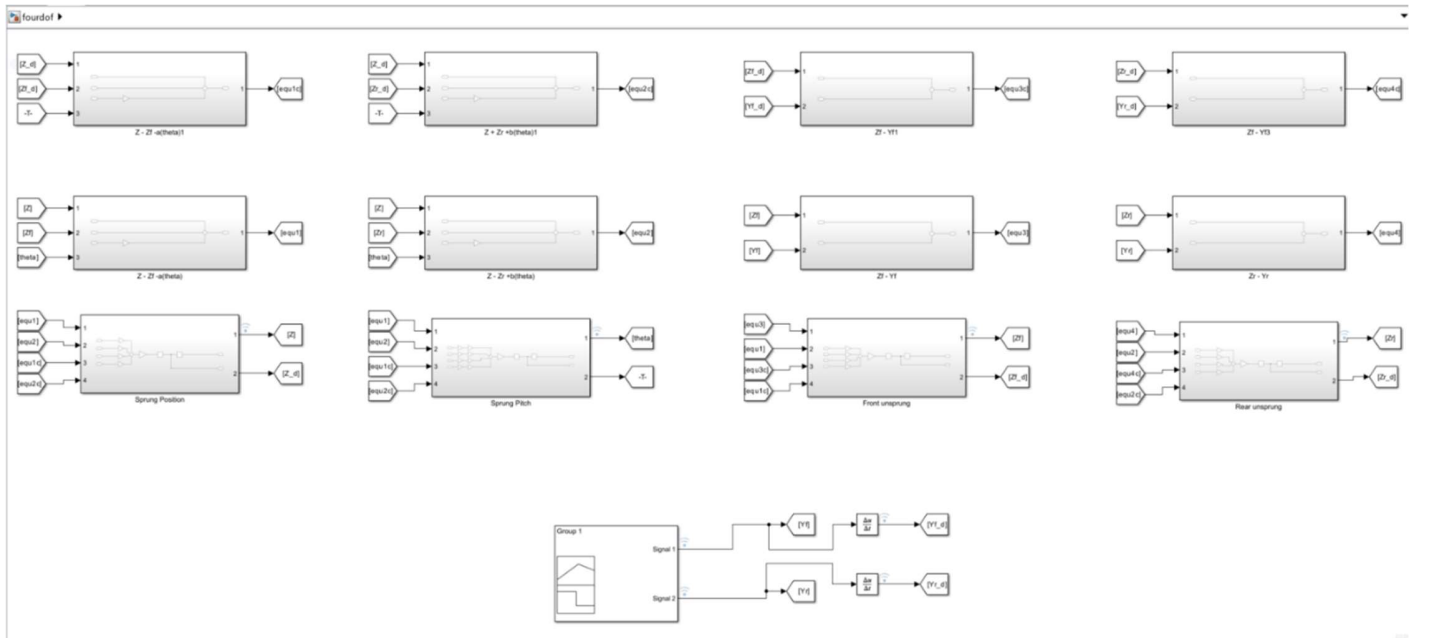


Figure 3-8: Simulink model describing Half-Car Model

3.2.2. Validation

The student has validated their 4 degrees of freedom model against a passive suspension model from KL University in Figures 3-9 to 3-12. The results showed that the displacements of the sprung mass vs time graphs were almost identical, and the pitch vs time showed the same trends but with an opposite sign convention, indicating that the student's model agrees with the model from KL University and can be used in further analysis. The MATLAB code (with vehicle estimations) and the input signal used are provided in Appendix A.

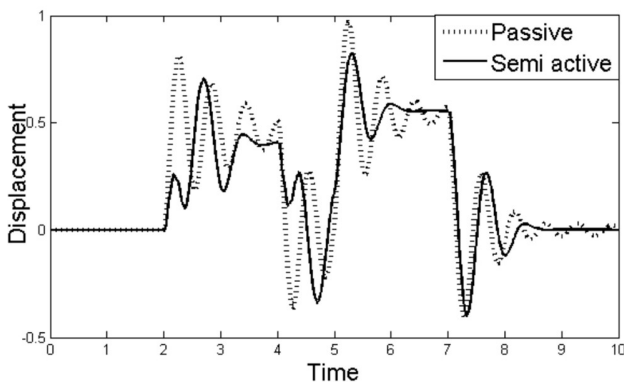


Figure 3-9: Displacement vs Time results (Sastry et al, 2017)

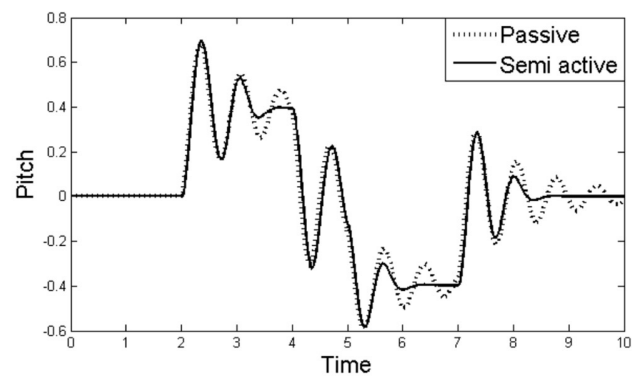


Figure 3-10: Pitch vs Time results (Sastry et al, 2017)

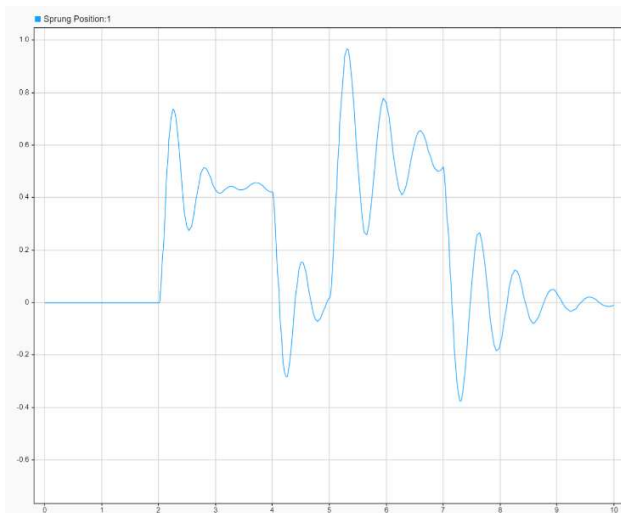


Figure 3-11: Student model Sprung Mass Position vs Time

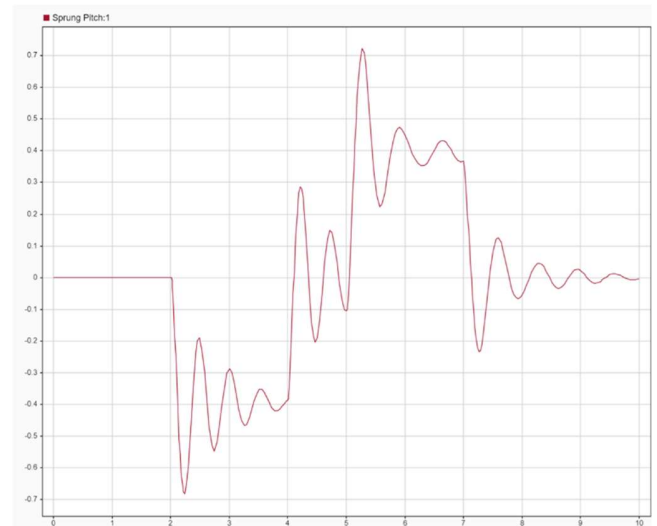


Figure 3-12: Student model Sprung Mass Pitch vs Time

3.3. 7DOF model – Full Car Model

3.3.1. Equations and Model

Equations 3-6 to 3-12 describe the 7DOF SMD model in Figure 3-13. The naming convention from the 4DOF model is carried over with a subscript 1 denoting the right and a two denoting the left side of the vehicle. "c" and "d" refer to the left and right perpendicular distances to the centre of mass from each unsprung mass. Subscripts "r" and "f" denote front and rear. KAR_f and KAR_r are anti-roll springs that rely on roll angle, ϕ , and are only applied to the unsprung masses and the roll equation. Following the equations is an in-depth analysis of the 7DOF Simulink model. The MATLAB code is in Appendix C.

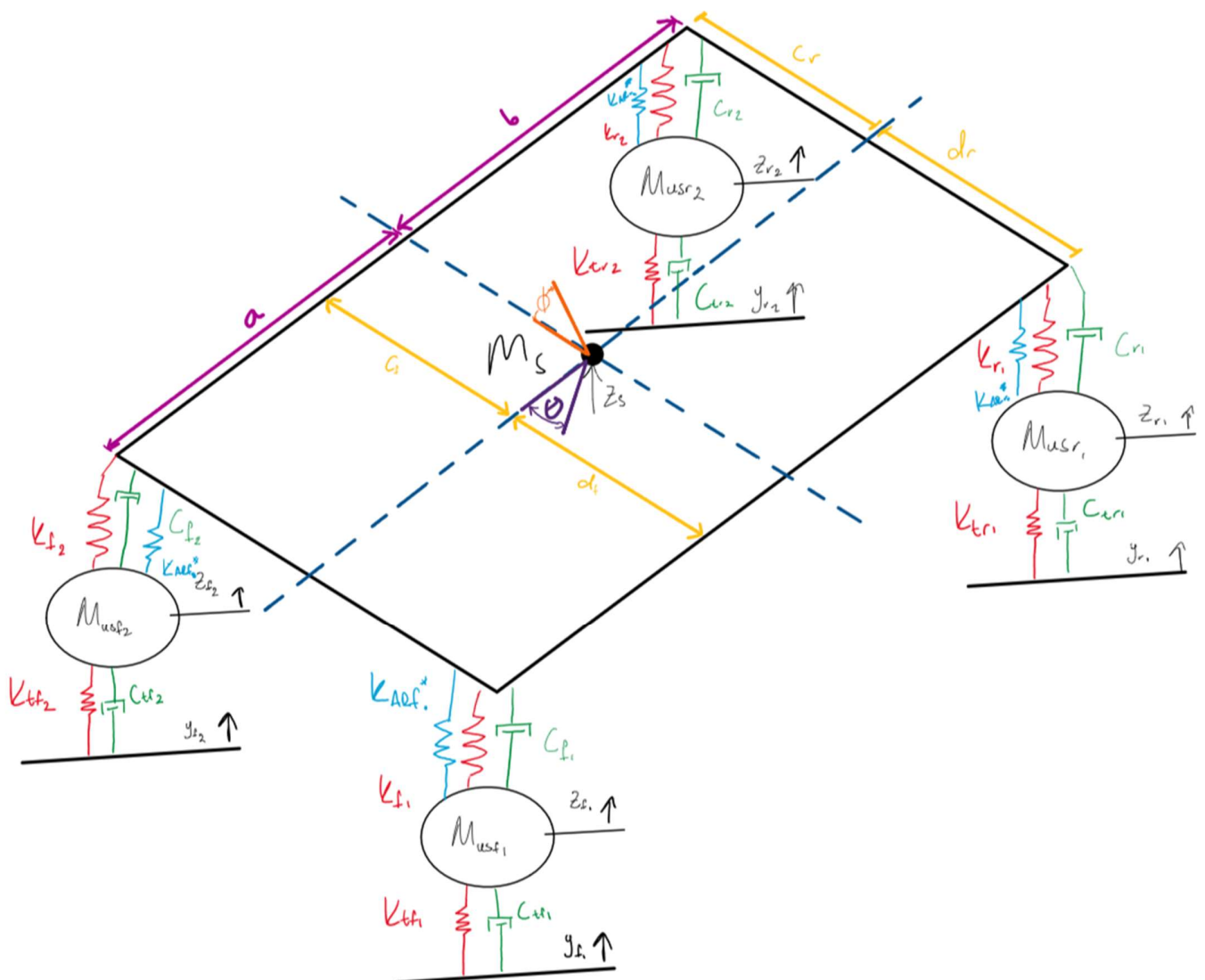


Figure 3-13: Diagram of 7DOF Model

$$\ddot{Z}_{f1} = \frac{-1}{m_{f1}} [K_{tf1}(Z_{f1} - Y_{f1}) - K_{sf}(Z_s - Z_{f1} - a\theta - d_f\dot{\varphi}) + C_{tf1}(\dot{Z}_{f1} - \dot{Y}_{f1}) - C_{f1}(\dot{Z}_s - \dot{Z}_{f1} - a\dot{\theta} - d_f\dot{\varphi}) + K_{ARf}(-d_f\dot{\varphi})] \quad (3-6)$$

$$\ddot{Z}_{f2} = \frac{-1}{m_{f2}} [K_{tf2}(Z_{f2} - Y_{f2}) - K_{sf}(Z_s - Z_{f2} - a\theta + c_f\dot{\varphi}) + C_{tf2}(\dot{Z}_{f2} - \dot{Y}_{f2}) - C_{f2}(\dot{Z}_s - \dot{Z}_{f2} - a\dot{\theta} + c_f\dot{\varphi}) + K_{ARf}(c_f\dot{\varphi})] \quad (3-7)$$

$$\ddot{Z}_{r1} = \frac{-1}{m_{r1}} [K_{tr1}(Z_{r1} - Y_{r1}) - K_{sr}(Z_s - Z_{r1} + b\theta - d_r\dot{\varphi}) + C_{tr1}(\dot{Z}_{r1} - \dot{Y}_{r1}) - C_{r1}(\dot{Z}_s - \dot{Z}_{r1} - a\dot{\theta} - d_r\dot{\varphi}) + K_{ARr}(-d_r\dot{\varphi})] \quad (3-8)$$

$$\ddot{Z}_{r2} = \frac{-1}{m_{r2}} [K_{tr2}(Z_{r2} - Y_{r2}) - K_{rf}(Z_s - Z_{r2} + b\theta + c_r\dot{\varphi}) + C_{tr2}(\dot{Z}_{r2} - \dot{Y}_{r2}) - C_{r2}(\dot{Z}_s - \dot{Z}_{r2} + b\dot{\theta} + c_r\dot{\varphi}) + K_{ARr}(c_r\dot{\varphi})] \quad (3-9)$$

$$\begin{aligned} \ddot{Z}_s = \frac{-1}{m_s} [& K_{sf}(Z_s - Z_{f1} - a\theta - d_f\dot{\varphi}) + C_{f1}(\dot{Z}_s - \dot{Z}_{f1} - a\dot{\theta} - d_f\dot{\varphi}) \\ & + K_{sf}(Z_s - Z_{f2} - a\theta + c_f\dot{\varphi}) + C_{f2}(\dot{Z}_s - \dot{Z}_{f2} - a\dot{\theta} + c_f\dot{\varphi}) \\ & + K_{sr}(Z_s - Z_{r1} + b\theta - d_r\dot{\varphi}) + C_{r1}(\dot{Z}_s - \dot{Z}_{r1} - a\dot{\theta} - d_r\dot{\varphi}) \\ & + K_{rf}(Z_s - Z_{r2} + b\theta + c_r\dot{\varphi}) + C_{r2}(\dot{Z}_s - \dot{Z}_{r2} + b\dot{\theta} + c_r\dot{\varphi})] \end{aligned} \quad (3-10)$$

$$\begin{aligned} \ddot{\theta} = \frac{-1}{I_p} [& bK_{sr}(Z_s - Z_{r1} + b\theta - d_r\dot{\varphi}) + bC_{r1}(\dot{Z}_s - \dot{Z}_{r1} + b\dot{\theta} - d_r\dot{\varphi}) + bK_{sr}(Z_s - Z_{r2} + b\theta + c_r\dot{\varphi}) \\ & + bC_{r2}(\dot{Z}_s - \dot{Z}_{r2} + b\dot{\theta} + c_r\dot{\varphi}) - aK_{sf}(Z_s - Z_{f1} - a\theta - d_f\dot{\varphi}) \\ & - aC_{f1}(\dot{Z}_s - \dot{Z}_{f1} - a\dot{\theta} - d_f\dot{\varphi}) - aK_{sf}(Z_s - Z_{f2} - a\theta + d_f\dot{\varphi}) \\ & - aC_{f1}(\dot{Z}_s - \dot{Z}_{f1} - a\dot{\theta} + d_f\dot{\varphi}) + ma_x h_p] \end{aligned} \quad (3-11)$$

$$\begin{aligned} \ddot{\varphi} = \frac{-1}{I_r} [& c_f \cdot K_{sf}(Z_s - Z_{f2} - a\theta + c_f\dot{\varphi}) + c_f \cdot C_{f2}(\dot{Z}_s - \dot{Z}_{f2} - a\dot{\theta} - c_f\dot{\varphi}) + c_r \\ & \cdot K_{sr}(Z_s - Z_{r2} + b\theta + c_r\dot{\varphi}) + c_r \cdot C_{r2}(\dot{Z}_s - \dot{Z}_{r2} + b\dot{\theta} + c_r\dot{\varphi}) - d_f \\ & \cdot K_{sf}(Z_s - Z_{f1} - a\theta - d_f\dot{\varphi}) - d_f \cdot C_{f1}(\dot{Z}_s - \dot{Z}_{f1} - a\dot{\theta} - d_f\dot{\varphi}) - d_r \\ & \cdot K_{sr}(Z_s - Z_{r1} + b\theta - d_r\dot{\varphi}) - d_r \cdot C_{r1}(\dot{Z}_s - \dot{Z}_{r1} + b\dot{\theta} - d_r\dot{\varphi}) - c_f \cdot K_{ARf}(c_f\dot{\varphi}) \\ & - c_r \cdot K_{ARr}(c_r\dot{\varphi})] + d_f \cdot K_{ARf}(-d_f\dot{\varphi}) + d_r \cdot K_{ARr}(-d_r\dot{\varphi}) + ma_y h_r] \end{aligned} \quad (3-12)$$

Figure 3-14 shows the overall 7DOF Simulink model. It comprises 27 subsystems; three (Sprung mass, Sprung Pitch and Sprung Roll) refer to the movement of the sprung mass; four unsprung masses are each contained in their subsystems and are labelled according to their position. The computation of damping rates is located below the unsprung masses. The upper four-by-four subsystems compute equations necessary for the previously mentioned subsystems.

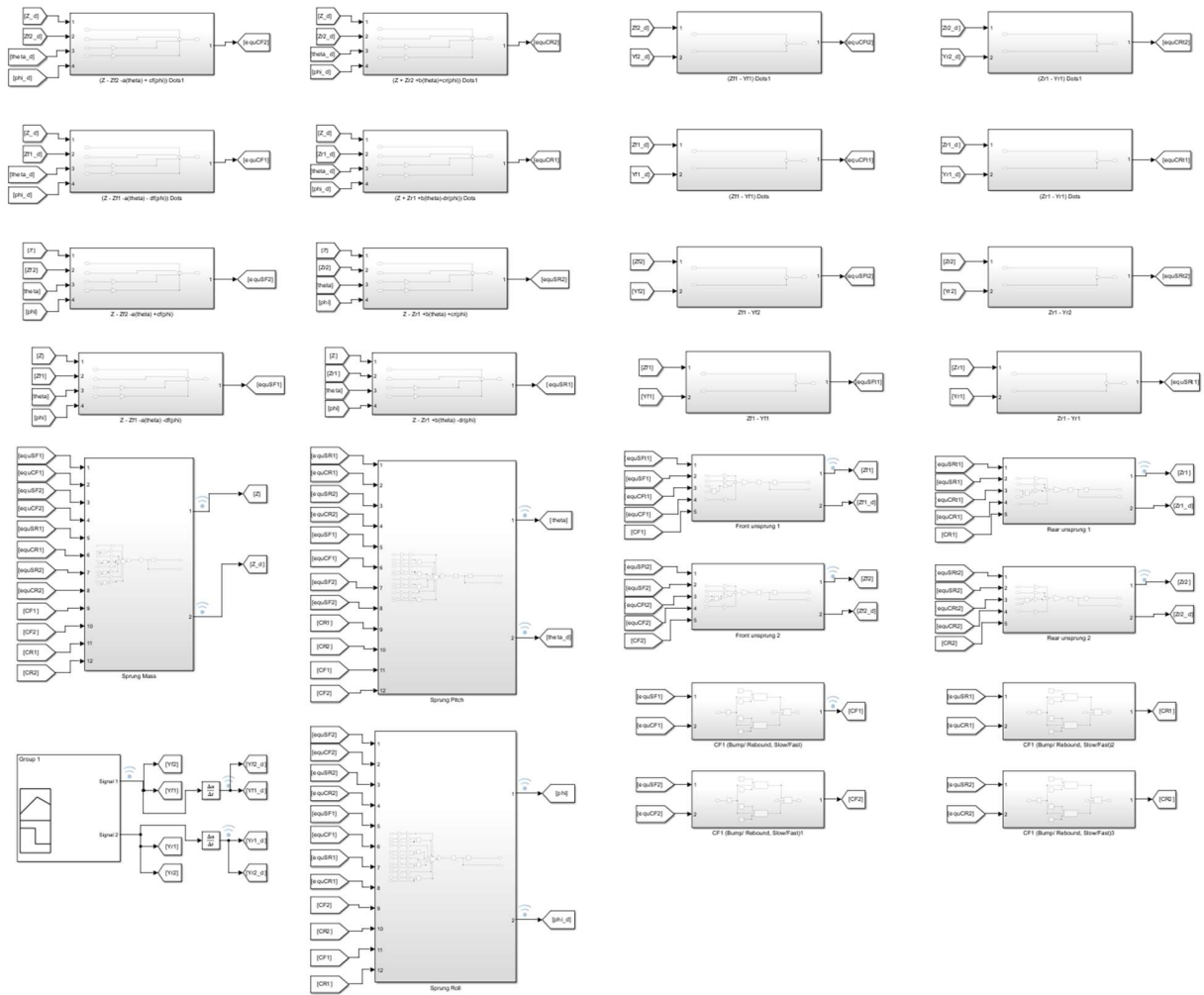


Figure 3-14: Overall 7DOF MSD Simulink Model

Figure 3-15 shows the 16 subsystems used to compute various equations.

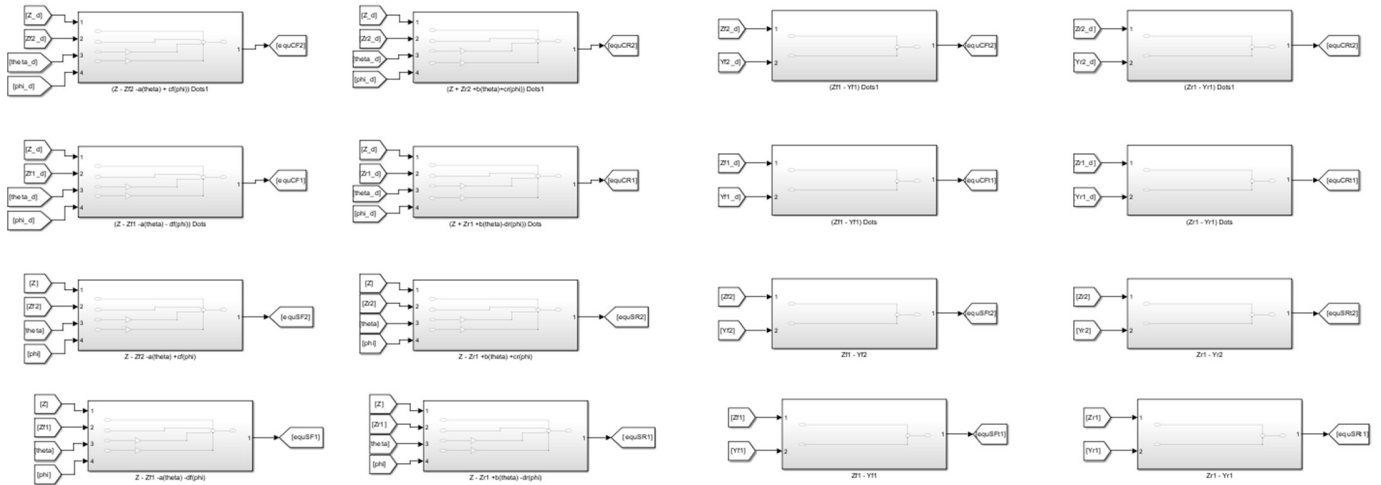


Figure 3-15: Subsystems used to compute various equations.

Figure 3-16 shows the four subsystems representing the unsprung masses in Equations 3-6 to 3-9. Each subsystem has six inputs and produces two outputs. Figure 3-17 shows the inner workings of the front unsprung 1 subsystem. Inputs 1 and 2 relate to the distance needed to compute spring and tyre spring forces, respectively. Inputs 3 and 4 are the velocity of 1 and 2 and are used for damping force calculations. Input 5 is the damping coefficient. The Cf1 gain allows for a static damping coefficient if necessary. Input 6 is the roll angle used to calculate the anti-roll bar force on the wheel. Vertical displacement and velocity are output.

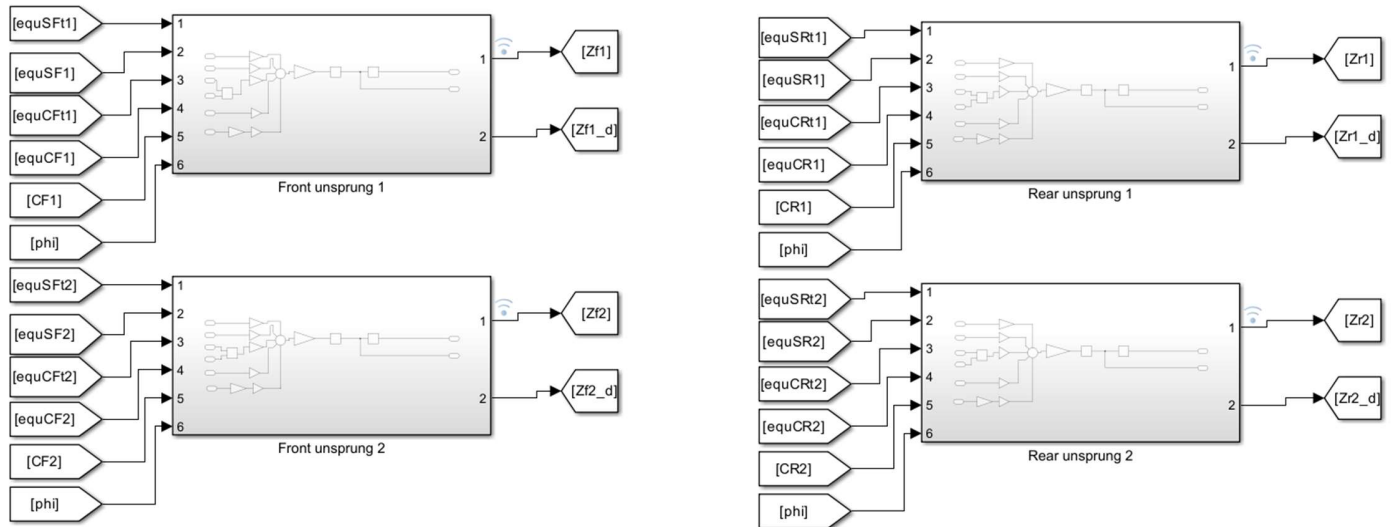


Figure 3-16: Four subsystems representing the unsprung masses.

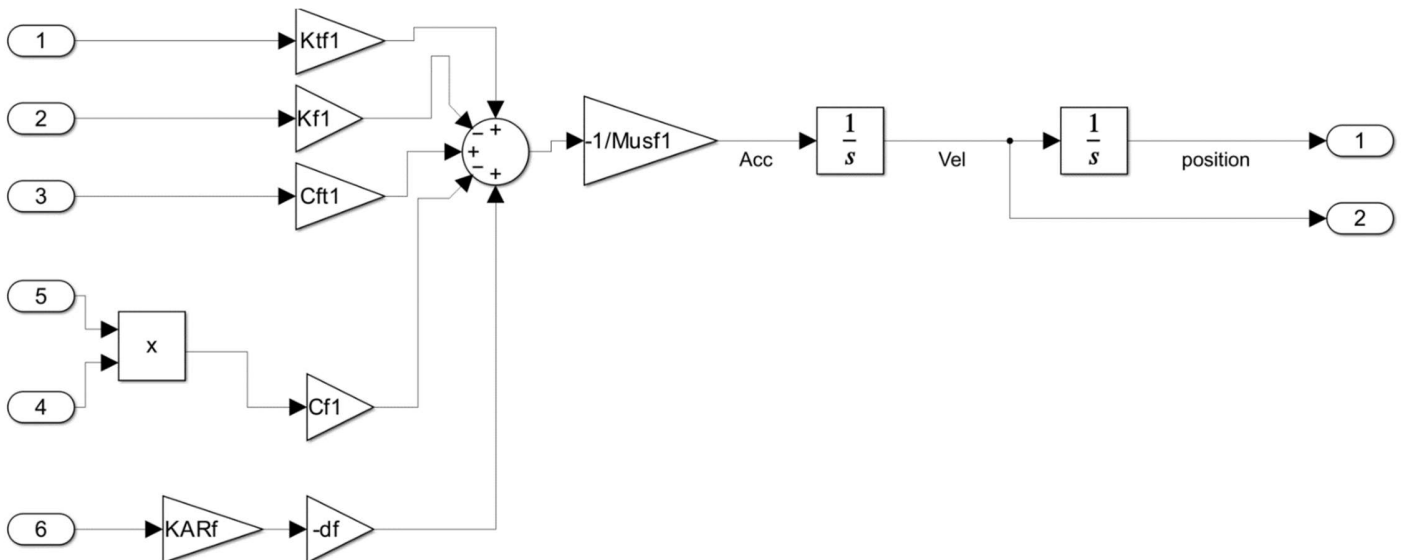


Figure 3-17: Front unsprung 1 subsystem

Figure 3-18 shows the four subsystems used to determine damping coefficients. Figure 3-19 shows the inner workings. Spring velocity (input) is used to differentiate between the fast vs slow range, and spring displacement determines whether the wheel is undergoing bump or rebound.

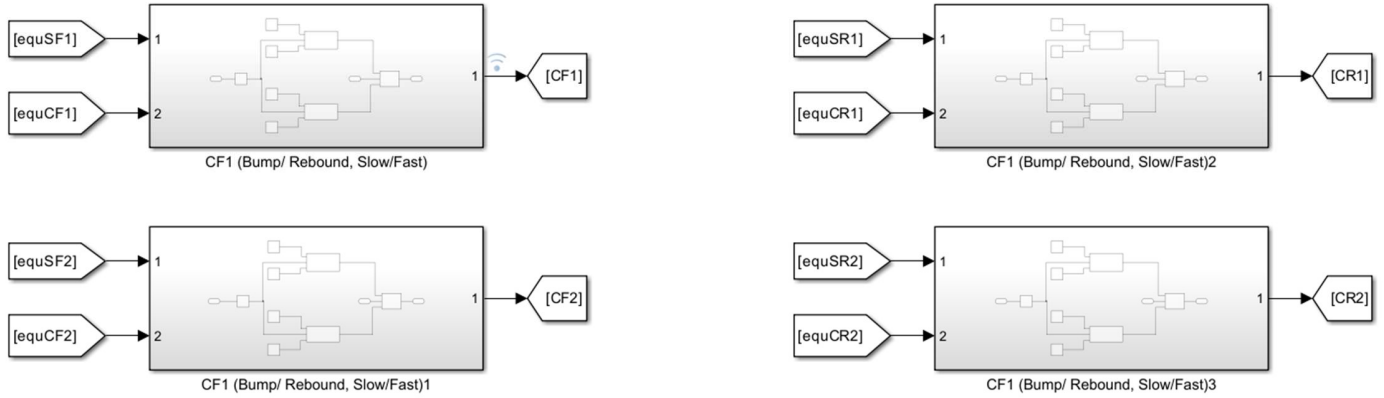


Figure 3-18: Four subsystems used to determine damping coefficients.

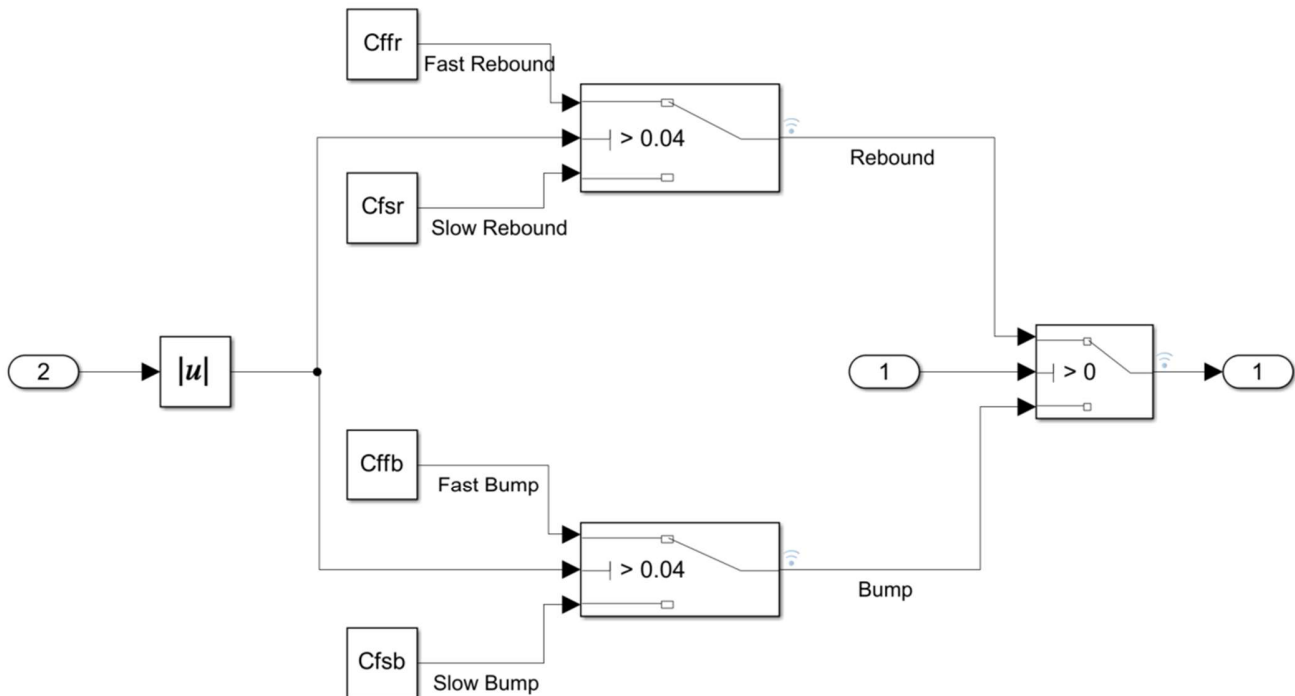


Figure 3-19: Cf1 Subsystem

Figure 3-20 shows the three subsystems used to determine the sprung mass's vertical displacement, pitch, and roll. The input signal representing the ground is in the bottom left.

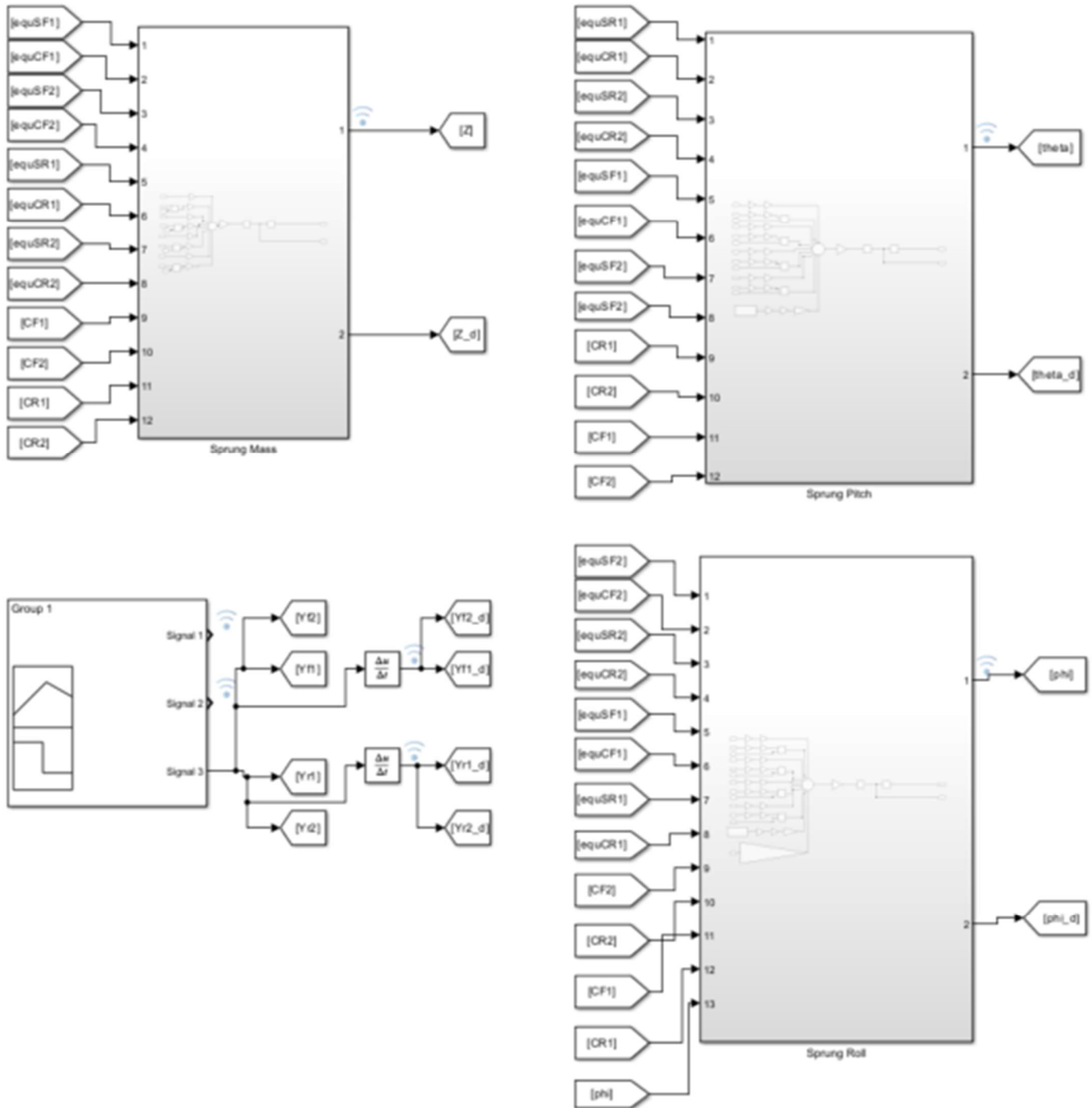


Figure 3-20: Displacement, Pitch and Roll subsystems and Signal Generator

Figure 3-21 shows the subsystem used to determine vertical displacement, as shown in Equation 3-10.

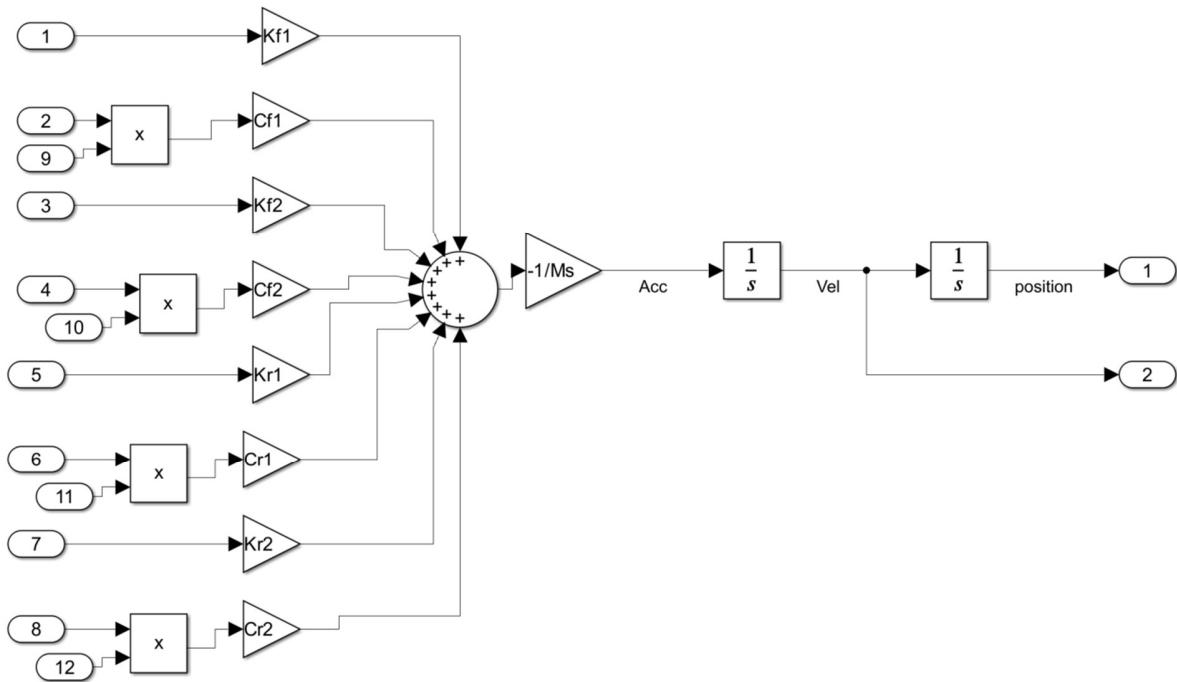


Figure 3-21: Unsprung Mass Vertical Displacement

Figure 3-22 shows the subsystem used to determine unsprung mass pitch, as shown in Equation 3-11. Pitch angle and velocity are the outputs. The signal builder is used to add longitudinal accelerations.

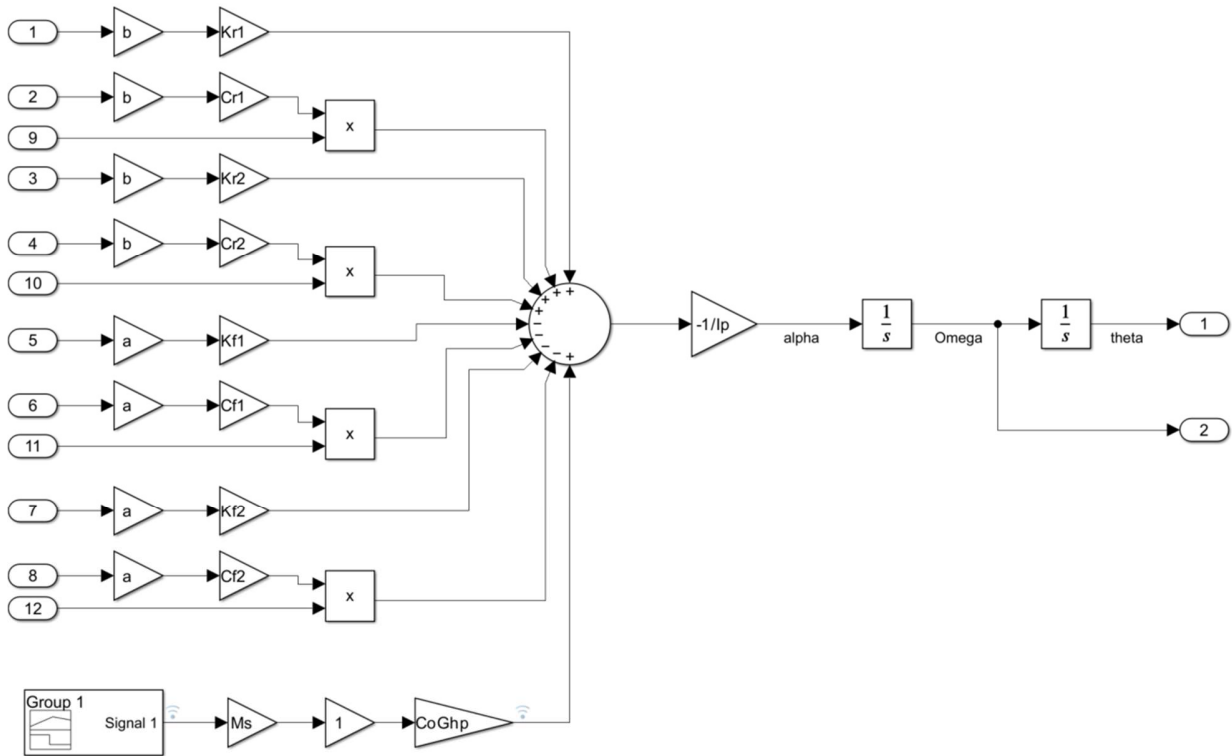


Figure 3-22: Unsprung Mass Pitch

Figure 3-23 shows the subsystem used to determine unsprung mass roll, as shown in Equation 3-12. Roll angle and velocity are the outputs with the signal builder used to add lateral accelerations. Input 13 adds anti-roll bar simulation.

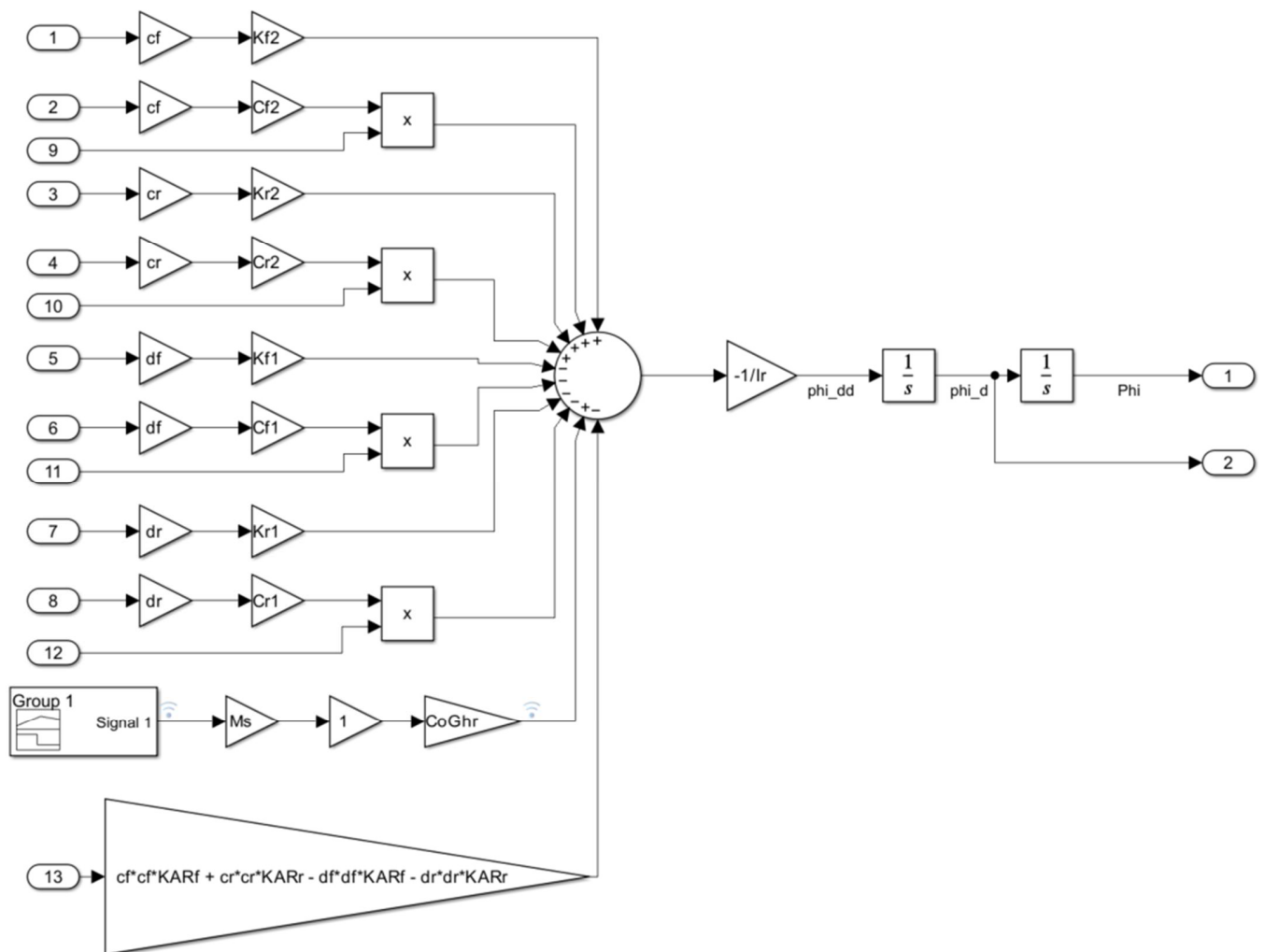


Figure 3-23: Subsystem used to determine unsprung mass roll

3.3.2. Validation

Validation of the student's model was conducted using work done on the design of a gain-scheduling type nonlinear controller for a full-vehicle active suspension system in Turkey (ONAT, 2007). The MATLAB code used to contain variable parameters and a graph of the 5cm incident bump is in Appendix B. Figures 3-24, and 3-25 show the results from the 2007 study, and Figures 3-26 and 3-27 show the results of vertical and pitch acceleration vs time for the student's model.

Similar trends are noted between the two models, with pitch accelerations having different sign conventions. Any discrepancies can be attributed to a slight difference in input variables.

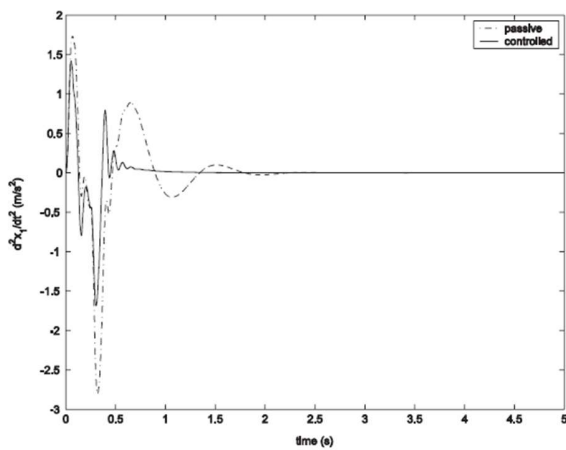


Figure 3-24: Vertical acceleration of the vehicle body for the road profile with a 5cm bump (ONAT, 2007)

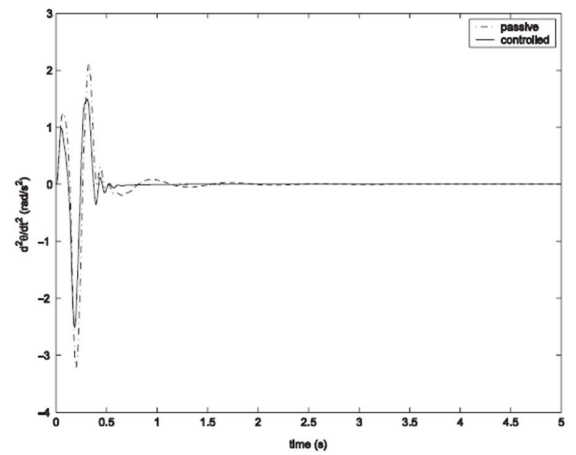


Figure 3-25: Pitch acceleration of the vehicle body for the road profile with a 5cm bump. (ONAT, 2007)

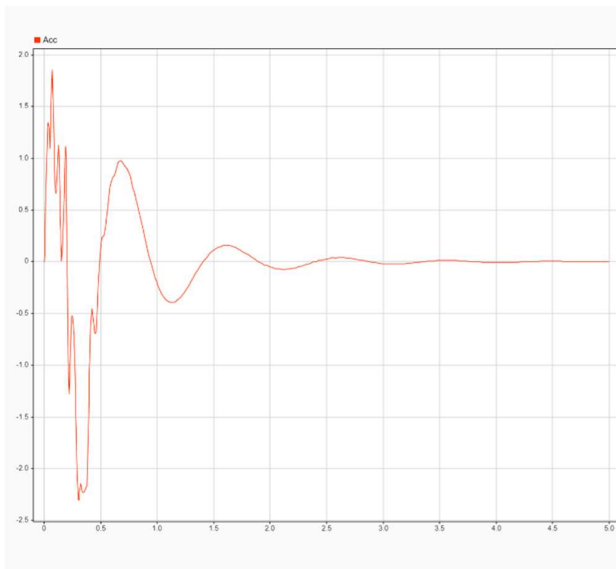


Figure 3-26: Student model vertical acceleration of the vehicle body for the road profile with a 5cm bump

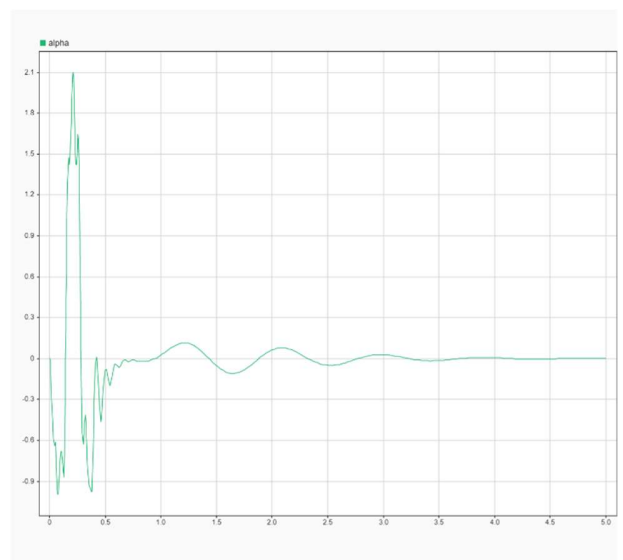


Figure 3-27: Student model pitch acceleration of the vehicle body for the road profile with a 5cm bump

4. Investigations

This report section outlines the three investigations applied to the student-built model. Table 4-1 shows the vehicle parameters used throughout unless otherwise stated. A tyre spring coefficient of 210,000 was estimated (AVON, 2023). The base MATLAB code used can be seen in Appendix C.

Table 4-1: Vehicle Parameters

Vehicle Mass (kg)	M_T	514
Front Corner Unsprung Mass (kg)	M_{usf}	22
Rear Corner Unsprung Mass (kg)	M_{usr}	22
Vehicle Sprung Mass (kg)	M_s	426
Centre of Gravity Height (m)	CoG	0.277
Weight Split F:R (%)	-	45:55
Wheelbase (m)	W	2.443
Front Track (m)	T_f	1.62
Rear Track (m)	T_r	1.57
Front Tyre Rolling Radius (m)	r_f	0.2667
Rear Tyre Rolling Radius (m)	r_r	0.2794
Front Motion Ratio	MR_f	1
Rear Motion Ratio	MR_r	1
Front Default Spring Rate (N/mm)	K_{sf}	18
Rear Default Spring Rate (N/mm)	K_{sr}	16
Front Roll Centre Height (mm)	h_f	0.9
Rear Roll Centre Height (mm)	h_r	59
Front Slow Bump	N/m/s	1660
Front Fast Bump	N/m/s	511
Front Slow Rebound	N/m/s	1915
Front Fast Rebound	N/m/s	511
Rear Slow Bump	N/m/s	1477
Rear Fast Bump	N/m/s	537
Rear Slow Rebound	N/m/s	1612
Rear Fast Rebound	N/m/s	537

4.1. BPS MoTeC Data Comparison

Table 4-2 shows the setups used when comparing the Simulink and BPS models. For this investigation, a double lane change was done using the Formula Ford car in BPS; the related data was recorded and accessed via MoTeC 1.0. Recorded lateral and longitudinal accelerations were applied to the Simulink model as a signal input (Figures 3-22 and 3-23). The different roll responses are compared in the following section.

Table 4-2: Setups Used for BPS Investigation

Springs Investigation		Setup 1	Setup 2	Setup 3
Front Spring Rate (N/mm)	K_{sf}	14	18	23
Rear Spring Rate (N/mm)	K_{sr}	20	16	12
Front Anti Roll Bar Rate (N/mm)	K_{arf}	0	0	0
Rear Anti Roll Bar Rate (N/mm)	K_{arr}	0	0	0
Sprung Mass Ride Frequency	Hz	1.932	1.932	1.958
Front Unsprung Mass Ride Frequency	Hz	16.068	16.211	16.387
Rear Unsprung Mass Ride Frequency	Hz	16.281	16.139	15.996
Front to Rear	%	-1.31%	0.44%	2.45%

4.2. Anti-Roll Bar Effects

Table 4-3 shows the setups used when conducting the ARB investigation. For this investigation, the double lane change Setup 2 was used. The recorded lateral and longitudinal accelerations with no ARBs were applied to the Simulink model, and ARB spring rates were varied. The different roll responses are discussed in the following section.

Table 4-3: Setups used when conducting the ARB investigation.

ARB Investigation		Setup 4	Setup 5	Setup 6
Front Spring Rate (N/mm)	K_{sf}	18	18	18
Rear Spring Rate (N/mm)	K_{sr}	16	16	16
Front Anti Roll Bar Rate (N/mm)	K_{arf}	8	15	22
Rear Anti Roll Bar Rate (N/mm)	K_{arr}	24	16	8

4.3. Damper Investigation - Response 5 cm level change

Table 4-4 shows the setups used when conducting the Damper investigation. A 5cm bump and spring rates from Setup 2 were used for this investigation. Underdamped, critically damped, overdamped and the base damping setup are evaluated by comparing the effect on sprung and unsprung mass vertical displacement.

Table 4-4: Setups used when conducting the Damper investigation

Damper Investigation		Setup 7	Setup 8	Setup 9	Setup 10
Front Spring Rate (N/mm)	K_{sf}	18	18	18	18
Rear Spring Rate (N/mm)	K_{sr}	16	16	16	16
Front Anti Roll Bar Rate (N/mm)	K_{arf}	0	0	0	0
Rear Anti Roll Bar Rate (N/mm)	K_{arr}	0	0	0	0
Damping Characteristics	ζ	0.7	1	1.2	*
Front Damping Coefficient ((N/mm/s)	C_f	2129.4	3042	3650.4	*
Rear Damping Coefficient (N/mm/s)	C_r	2007.6	2868	3441.6	*

5. Results and Discussion

This section evaluates the results of the previously mentioned investigations.

5.1. BPS MoTeC Data Comparison

Figures 5-1 to 5-3 show the results from the test involving parameters from Tables 4-2. Tuning the distance from the centre of mass to the roll centre was necessary to obtain Simulink results with similar amplitudes to the recorded BPS data. The Simulink model shows the same response as BPS but tends to lag by 0.2 seconds.

Model complexity is a crucial reason for discrepancies; rfactor-based car models are typically more complex than lumped mass Simulink models. BPS considers more detailed physics and dynamics, such as aerodynamics, tire behaviour, and suspension geometry. Simulink add-ons can determine roll centre changes based on initial suspension geometry (Goel, 2021). Utilising this would improve accuracy but requires more data and work. Limitations in inputs are another difference. The Simulink model lacks inputs linking steering angle, throttle position, and dynamic tyre forces, which would improve model accuracy.

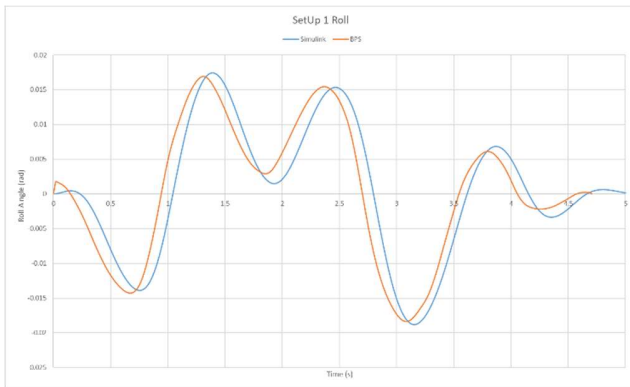


Figure 5-1: Set Up 1 Roll

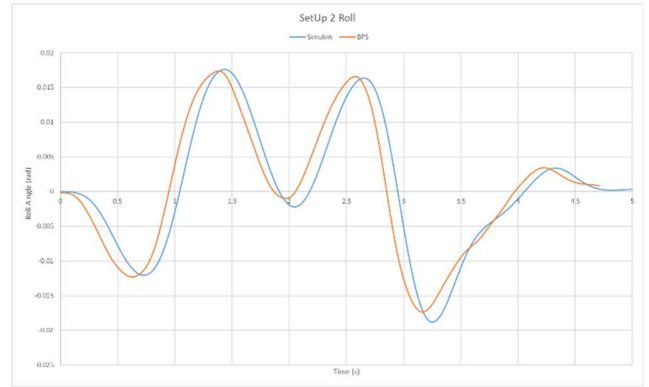


Figure 5-2: Set Up 2 Roll

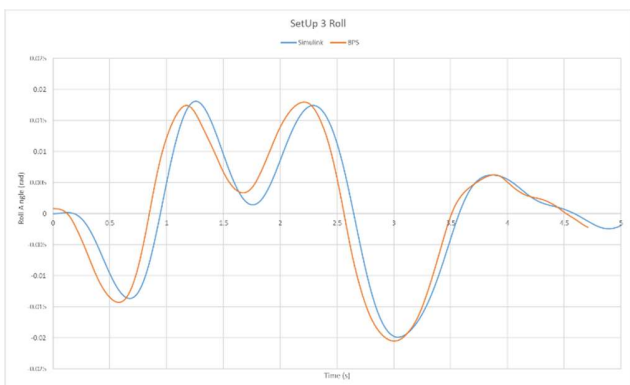


Figure 5-3: Set Up 3 Roll

5.2. Anti-Roll Bar Effects

An investigation into the effect of anti-roll bars on the student-built model was conducted as stipulated in section 4.2. Despite the introduction of anti-roll bars reducing vehicle roll, Figure 5-4 shows that varying front vs rear ARB rates has little effect on the roll characteristics of the unsprung mass. This was due to the method of modelling assuming the vehicle body to be infinitely stiff. In this model, the sum of ARB stiffness is more important than the location and distribution.

Figure 5-5 shows that variations in the ARB stiffness influence individual wheel movement. The yellow line represents set up 2 with no ARB, the green line is 8 N/mm, the red line is 15 N/mm, and the blue line is 22 N/mm. As expected, as ARB stiffness increases, local unsprung mass displacement decreases.

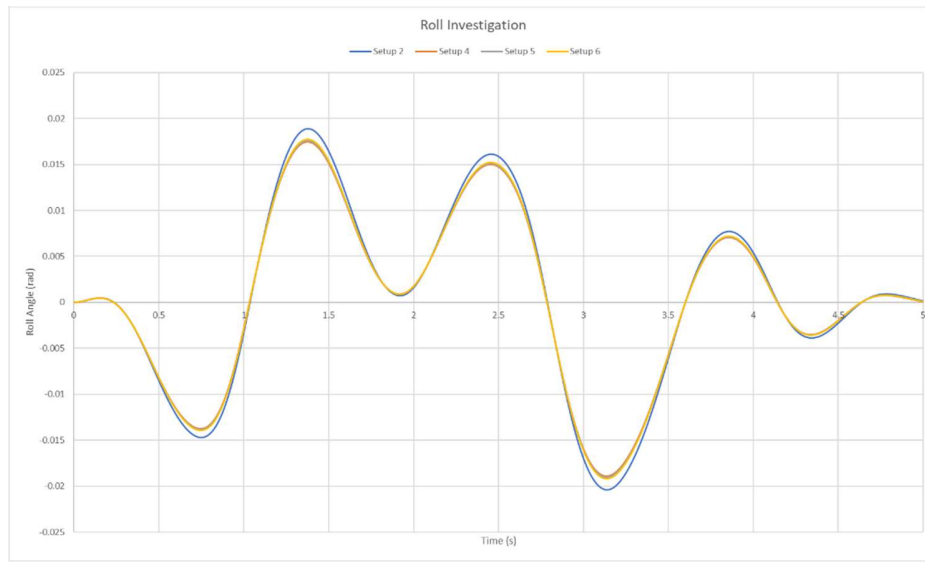


Figure 5-4: Roll Investigation

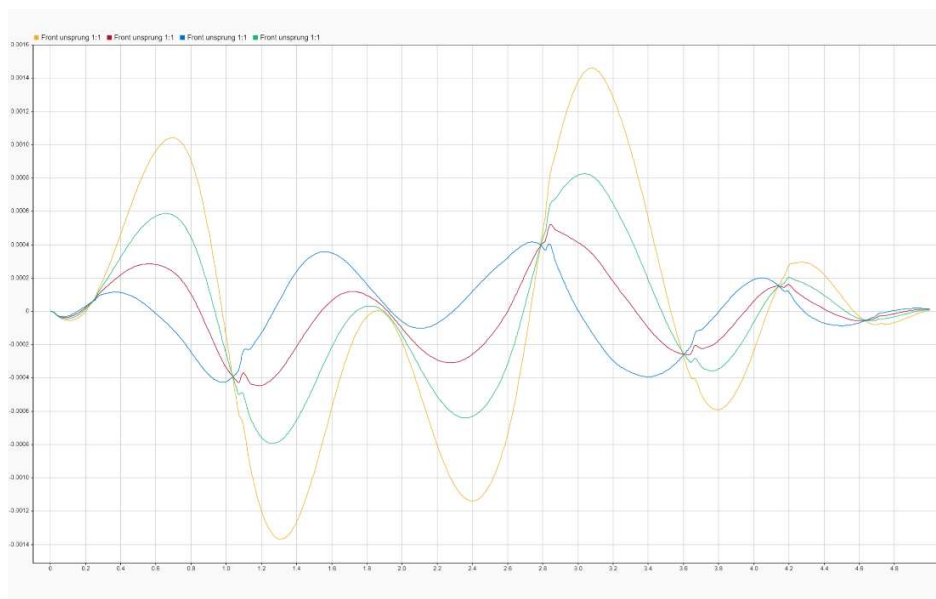


Figure 5-5: Front Unsprung Mass deflection for varying ARB stiffness

5.3. Damper Investigation - Response 5 cm bump

Figure 5-6 shows the results of a damper investigation performed as stipulated in section 4.3. The dotted orange line represents a 5cm bump being applied to the rear unsprung masses. The yellow line is the result of set up ten, where the vehicle is damped per the input damping characteristics. This utilises four subsystems shown in Figure 3-18. The green line shows slight under-damping. Both yellow and green lines result in the mass going from bump to rebound and then stabilising. The blue line shows critical damping, and the red line is overdamped.

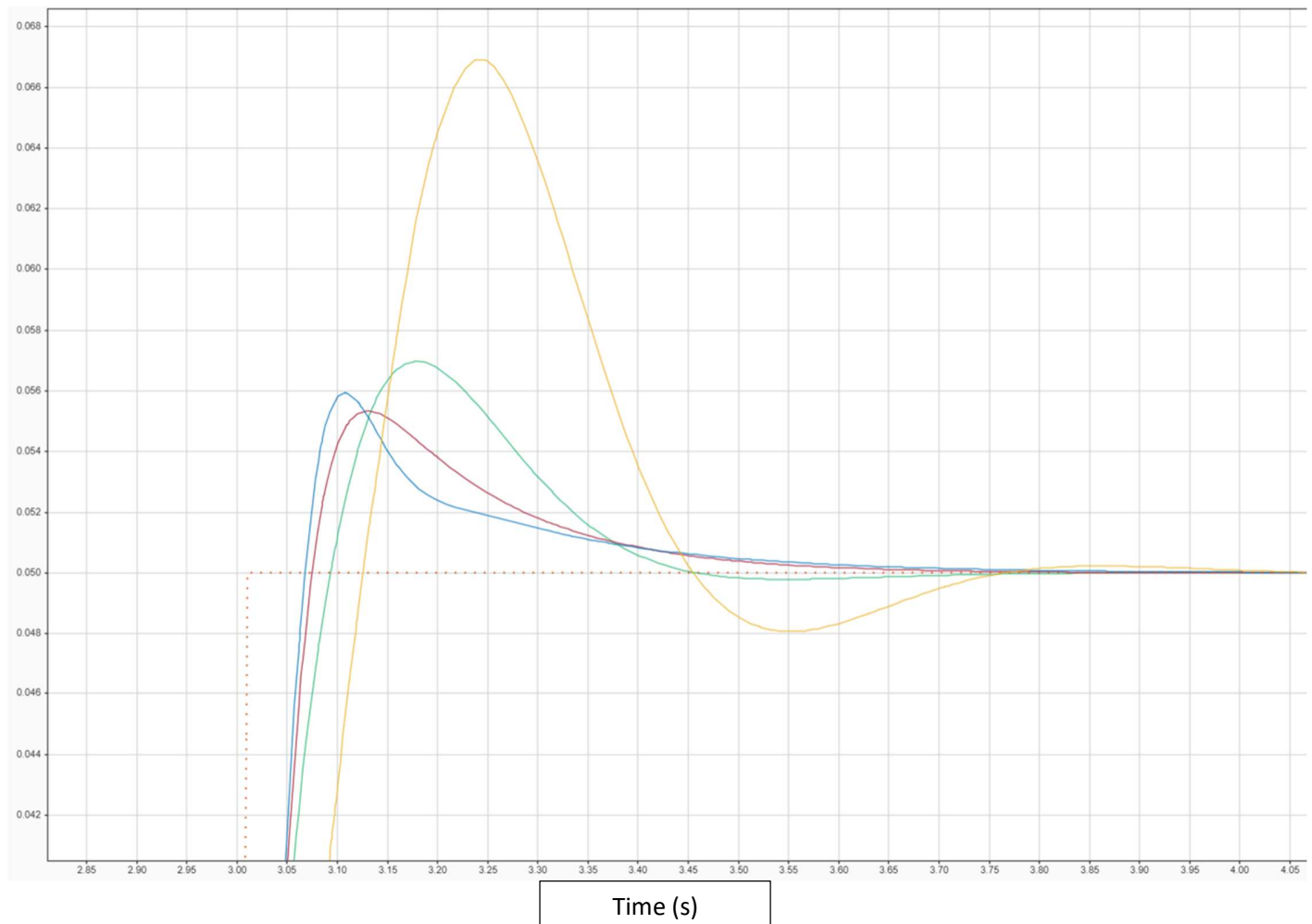


Figure 5-6: Results of Damping Investigation

6. Conclusion

The aim and objectives of the assignment have been completed. While lumped mass models of a vehicle can be useful for analysing vehicle performance and control, they are not detailed enough to accurately depict the nonlinear behaviour of the suspension system or the intricacies of the vehicle's structure. Being able to virtual prototype is advantageous, but it is crucial to understand the level of detail and accuracy and use models appropriately.

A time delay was noted when comparing the Simulink and BPS model responses. Accounting for this, a designer may alter vehicle characteristics and evaluate the magnitude of changes, understanding that the time is likely not entirely accurate. The ARB investigation shows the need for an ADAMS-type model when evaluating anything requiring chassis flex. One can evaluate the overall effects of changing damping, but it is likely to be somewhat inaccurate. Conducting tests in BPS to gather data on the effect of ARBs to validate and calibrate the Simulink model is recommended. The damping investigation shows expected results based on hand calculations. Further validation is recommended.

The Simulink model can be used as a starting point for vehicle setup with the ability to look at the effects of different ride frequencies and damping coefficients. Once general parameters are found, a more detailed model should be used for further fine-tuning.

7. References

- AVON. (2023, January 20). *Formula Student Downloads*. Retrieved from Avon Tyres: <https://www.avontyres.com/en-gb/tyre-care/motorsport-technical-data/technical-data-resources/formula-student/>
- BPS. (2022). *SINGLE-SEATER SIMULATOR*. Retrieved from baseperformance: <https://www.baseperformance.net/>
- Goel, N. (2021, May 19). *Roll Center Calculator App with MATLAB App Designer*. Retrieved from MathWorks: <https://blogs.mathworks.com/student-lounge/2021/05/19/roll-center-calculator-app-with-matlab-app-designer/>
- Hancox, J. (2022, February 07). *Is rFactor 2 Worth it in 2022?* Retrieved from OVERTAKE: <https://www.overtake.gg/games/rfactor-2/rfactor-2-review-is-it-worth-it-in-2022/>
- IMAGE SPACE INCORPORATED. (2019). *isiMotor Software Engine Licensing*. Retrieved from Image Space Incorporated: <https://imagespaceinc.com/#engine>
- ISO. (2011). *ISO 3888-2 : Passenger cars — Test track for a severe lane-change manoeuvre — Part 2: Obstacle avoidance*. International Standard, Switzerland. Retrieved January 22, 2023, from <https://cdn.standards.iteh.ai/samples/57253/5e8cdcc7c2fe4c0c8fe6f1de8beb5dba/ISO-3888-2-2011.pdf>
- Jazae, R. N. (2014). *Vehicle Dynamics: Theory and Application*. Bundoora: Springer.
- Kim, C., & Ro, P. I. (2002). An Accurate Full Car Ride Model Using Model Reducing Techniques. *Journal of Mechanical Design*, 697-705. doi:10.1115/1.1503065
- MathWorks. (2022). *Double Lane Change Reference Application*. Retrieved January 06, 2023, from MathWorks Support: <https://uk.mathworks.com/help/vdynblks/ug/double-lane-change-reference-application.html>
- Na, S., Jang, J., Yoo, W., & Kim, K. (2015). Dynamic vehicle model for handling. *Advances in Mechanical Engineering*, 7(II). doi:10.1177/1687814015618126
- ONAT, C. (2007). LPV Model Based Gain-scheduling Controller for a Full Vehicle Active Suspension System. *Journal of Vibration and Control*, 1630-1666.
- Sasena, M., & Abhinav, S. (2018, November 13). *Building Real-Time Driver-in-the-Loop Simulators*. Retrieved from MathWorks Videos and Webinars: <https://uk.mathworks.com/videos/building-real-time-driver-in-the-loop-simulators-1542313377351.html>
- Sastry et al. (2017). Analysis of MR Damper for Quarter and Half Car Suspension Systems of a Roadway Vehicle. *International Journal of Vehicle Structures & Systems*, 17-22.
- Skill-Lync. (2020, July 29). *A Guide to Help FEA Engineers Get Started With Vehicle Dynamics - Part 1*. Retrieved January 22, 2023, from SKILL LINK Mechanical: <https://skill-lync.com/blogs/guide-fea-vehicle-dynamics-part-1>
- Yang, S., Li, S., & Lu, Y. (2013, December). An overview on vehicle dynamics. *International Journal of Dynamics and Control*, 385-387. doi:10.1007/s40435-013-0032-y
- Zbarsky, E. S. (2023). *Mass-Spring-Damper Systems*. Retrieved from MathWorks File Exchange: <https://github.com/MathWorks-Teaching-Resources/Mass-Spring-Damper-Systems/releases/tag/v1.0.0>

8. Appendix

8.1. Appendix A

```
clear
ms = 450; % Half of the Hyundai i20
mr = 45; % Unsprung rear mass
mf = 45; % Unsprung front mass
ksf = 30000; % Front spring force
csf = 750.00; %Front damping coefficient
ktf = 200000; %Front tyre spring coefficients
ct = 750; % Tyre damping coefficients
ktr = 200000;
ksr = 25000;
csr = 750.00;
a = 1.11; % Estimated distance from COM to rear axel
b = 1.415; % Estimated distance from COM to front axel
h = 0.75; % Estimated distance from COM to ax
I = 575; %Estimated pitch moment of inertia
```

8.2. Appendix B

```
clc
clear

%Masses
Ms = 1100; %sprung mass
Musf1 = 25; %unsprung front mass
Musf2 = 25;
Musr1 = 45;
Musr2 = 45;
Ip = 1848; %Pitch Moment of Inertia
Ir = 550; %Roll Moment of Inertia

%lengths
a = 1.2; %(1-Mdf)*wheelbase; %com distance to front axel
b = 1.4; %(1-Mdr)*wheelbase; %com distance to rear axel
cf = 0.65; %Tf/2;
df = 0.85; %Tf/2;
cr = 0.65; %Tr/2;
dr = 0.85; %Tr/2;
CoGh = 0.277; %height at which forces will be applied
rf = 0.2667; %front tyre rolling radius, what do we do with it?
rr = 0.2794; %rear tyre rolling radius, idk
mrf = 1; %front motion ratio
mrr = 1; %rear motion ratio
rcfh = 0.9*10^-3; %front roll center height, idk
rcrh = 59*10^-3; %rear roll center height, idk

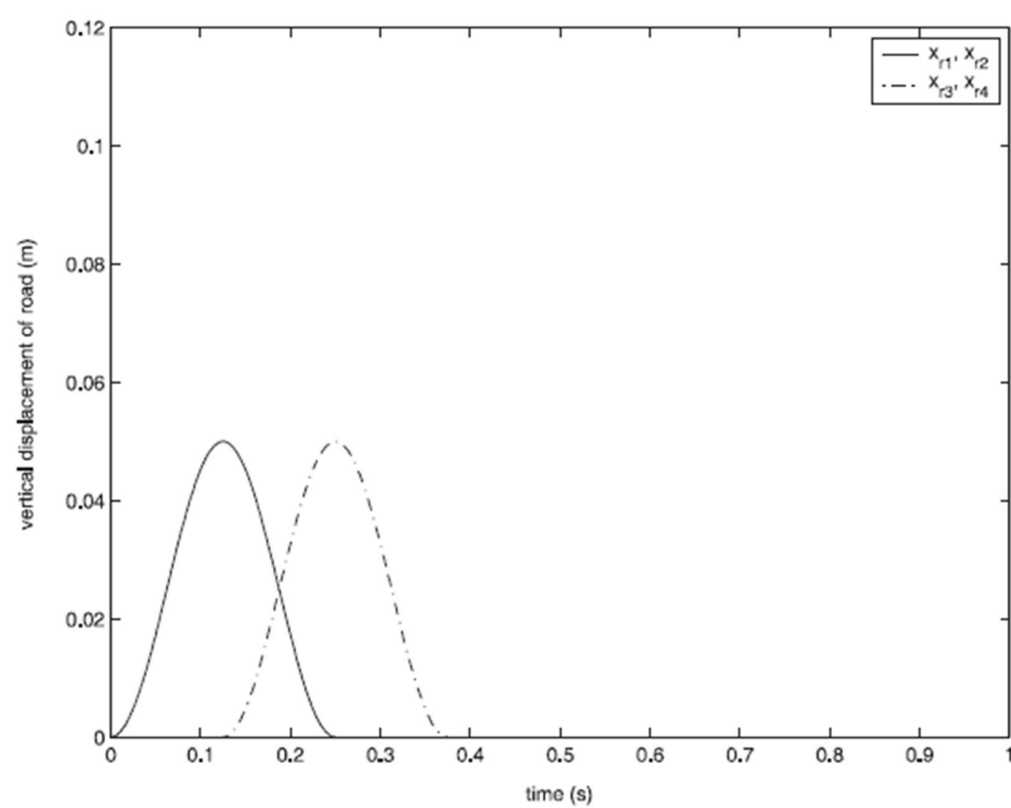
%Normal Springs
Kf1 = 15*10^3; %Front default spring rate
Kf2 = 15*10^3; %Front default spring rate
Kr1 = 17*10^3; %Rear default spring rate
Kr2 = 17*10^3; %Rear default spring rate

%Tyre Spring
Ktf1 = 250000;
Ktf2 = 250000;
Ktr1 = 250000;
Ktr2 = 250000;

%ARB as Springs
KARf = 0;
KARr = 0;

%Dampers
Cf1 = 1150;
Cf2 = 1150;
Cr1 = 1150;
Cr2 = 1150;

%Tyre Damper
Cft1 = 0;
Cft2 = 0;
Crt1 = 0;
Crt2 = 0;
```



8.3. Appendix C

```
clc
clear

%masses
Ms = 426; %sprung mass
Musf1 = 22; %unsprung front right mass
Musf2 = 22;
Musr1 = 22;
Musr2 = 22;
Mdf = 0.45; %mass distribution front
Mdr = 0.55; %mass distribution rear
Ip = 498.77; %Pitch Moment from BPS
Ir = 545.86; %Roll moment from BPS

%lengths
wheelbase = 2.443;
Tf = 1.62; %front track
Tr = 1.57; %rear track
a = (1-Mdf)*wheelbase; %com distance to front axel
b = (1-Mdr)*wheelbase; %com distance to rear axel
cf = Tf/2;
df = Tf/2;
cr = Tr/2;
dr = Tr/2;
CoGhr = 0.277-0.1; %height at which lateral/roll forces will be applied Not accurate!
CoGhp = 0.060; %height at which longitudinal/pitch forces will be applied
rf = 0.2667; %front tyre rolling radius, what do we do with it?
rr = 0.2794; %rear tyre rolling radius, idk
mrf = 1; %front motion ratio
mrr = 1; %rear motion ratio
rcfh = 0.9*10^-3; %front roll center height, idk
rcrh = 59*10^-3; %rear roll center height, idk

%Normal Springs
Kf1 = 18*10^3; %Front default spring rate and for set up 2
%Kf1 = 14*10^3; %Front spring rate Set up 1
%Kf1 = 23*10^3; %Front spring rate set up 3

Kf2 = 18*10^3; %Front default spring rate and for set up 2
%Kf2 = 14*10^3; %Front spring rate Set up 1
%Kf2 = 23*10^3; %Front spring rate set up 3

Kr1 = 16*10^3; %Rear default spring rate and for set up 2
%Kr1 = 20*10^3; %Front spring rate Set up 1
%Kr1 = 12*10^3; %Front spring rate set up 3

Kr2 = 16*10^3; %Rear default spring rate and for set up 2
%Kr2 = 20*10^3; %Front spring rate Set up 1
%Kr2 = 12*10^3; %Front spring rate set up 3

%Tyre Spring
Ktf1 = 212000; %AVON data 14140
Ktf2 = 2120000;
Ktr1 = 212000;
Ktr2 = 212000;
```

```

%ARB as Springs
KARf = 0;
KARr = 0;

%Dampers
Cf1 = 1;
Cf2 = 1;
Cr1 = 1;
Cr2 = 1;
Cfsb = 1660; % Slow bump for front (N/m/s)
Cffb = 511; % Fast bump for front (N/m/s)
Cfsr = 1915; % Slow rebound for front (N/m/s)
Cffr = 511; % Fast rebound for front (N/m/s)
Crsb = 1477; % Slow bump for rear (N/m/s)
Crfb = 537; % Fast bump for front (N/m/s)
Crsr = 1612; % Slow rebound for front (N/m/s)
Crfr = 537; % Fast rebound for front (N/m/s)
Cth = 0.25; % Damper velocity threshold fast vs slow (m/s)

%Tyre Damper
Cft1 = 100;
Cft2 = 100;
Crt1 = 100;
Crt2 = 100;

% MoTeC data SetUp1
% load('dlc1.mat','G_Force_Lat');
% time = 0:0.0195:4.707;
% ay = G_Force_Lat.Value;
%
% load('dlc1.mat','G_Force_Long')
% ax = G_Force_Long.Value;
%
% load('dlc1.mat','Gyro_Roll_Angle')
% Roll_ang = Gyro_Roll_Angle.Value;
% plot(Roll_ang)

%MoTeC data SetUp2
% load('dlc2.mat','G_Force_Lat');
% time = 0:0.0195:5.215;
% ay = G_Force_Lat.Value;
%
% load('dlc2.mat','G_Force_Long')
% ax = G_Force_Long.Value;
%
% load('dlc2.mat','Gyro_Roll_Angle')
% Roll_ang = Gyro_Roll_Angle.Value;
% plot(Roll_ang)

%MoTeC data SetUp3
% load('dlc3.mat','G_Force_Lat');
% time = 0:0.0195:5.9375;
% ay = G_Force_Lat.Value;
%

```

```
% load('dlc3.mat', 'G_Force_Long')
% ax = G_Force_Long.Value;
%
% load('dlc3.mat', 'Gyro_Roll_Angle')
% Roll_ang = Gyro_Roll_Angle.Value;
% plot(Roll_ang)
```