

Maths Assessment 2

Peter Down

a) $y = e^x \cos x$
 $\frac{dy}{dx} = \underline{\underline{-e^x \sin x \cos x}}$

b) $y = \cos x \sin x$
 $\frac{dy}{dx} = \underline{\underline{\cos^2 x - \sin^2 x}}$

c) $y = \frac{\cos x}{\sin x}$ $u = \cos x$ $v = \sin x$
 $\frac{du}{dx} = -\sin x$ $\frac{dv}{dx} = \cos x$

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$$\begin{aligned} \frac{dy}{dx} &= \frac{u \frac{dv}{dx} - v \frac{du}{dx}}{v^2} \\ &= \frac{\cos x (\cos x) - \sin x (-\sin x)}{\sin^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\sin^2 x} \\ &= \frac{1}{\sin^2 x} \\ &= \underline{\underline{\csc^2 x}} \end{aligned}$$

d) $y = \frac{3x^2}{\cos x}$ $u = 3x^2$ $v = \cos x$
 $\frac{du}{dx} = 6x$ $\frac{dv}{dx} = -\sin x$

$$\begin{aligned} \frac{dy}{dx} &= \frac{u \frac{dv}{dx} - v \frac{du}{dx}}{v^2} \\ &= \frac{3x^2 (-\sin x) - \cos x (6x)}{\cos^2 x} \\ &= \frac{-3x^2 \sin x - 6x \cos x}{\cos^2 x} \\ &= \underline{\underline{-\frac{3x(2 \cos x + x \sin x)}{\cos^2 x}}} \end{aligned}$$

$$e) z = (3x+2y)(4x-5y) \\ = 12x^2 - 15xy + 8xy - 10y^2$$

$$\frac{dz}{dx} = 24x - 15y + 8y \\ = \underline{\underline{24x - 7y}}$$

$$\frac{dz}{dy} = \underline{\underline{-7x - 10y}}$$

$$f) z = \sin(3x+2y)$$

$$u = \sin(3x+2y)$$

$$\frac{dz}{dy} = \underline{\underline{-7x - 10y}}$$

$$f) z = \sin(3x+2y)$$

$$u = \sin(3x+2y) \\ \frac{du}{dx} = 3\cos(3x+2y)$$

$$v = xy \\ \frac{dv}{dx} = y$$

$$\frac{du}{dy} = 2\cos(3x+2y)$$

$$\frac{dv}{dy} = x$$

$$\therefore \frac{dz}{dx} = \frac{xy(3\cos(3x+2y)) - \sin(3x+2y)(y)}{(xy)^2} \\ = \underline{\underline{y(3x\cos(3x+2y) - \sin(3x+2y))}} \\ (xy)^2$$

$$\therefore \frac{dz}{dy} = \frac{xy(2\cos(3x+2y)) - \sin(3x+2y)(x)}{(xy)^2} \\ = \underline{\underline{x(2y\cos(3x+2y) - \sin(3x+2y))}} \\ (xy)^2$$

$$(2a) \quad y = 2x^2 + 4x + 3, \quad x=2, x=5$$

$$\begin{aligned} \int_2^5 2x^2 + 4x + 3 \, dx &= \left[\frac{2}{3}x^3 + 2x^2 + 3x \right]_2^5 \\ &= \left(\frac{2(5)^3}{3} + 2(5)^2 + 3(5) \right) - \left(\frac{2(2)^3}{3} + 2(2)^2 + 3(2) \right) \\ &= \left(\frac{250}{3} + 50 + 15 \right) - \left(\frac{16}{3} + 8 + 6 \right) \\ &= \left(\frac{445}{3} \right) - \left(\frac{58}{3} \right) \\ &= \underline{\underline{129}} \end{aligned}$$

$$b) \quad a = (6, 8, 0), \quad b = (12, 4, 0)$$

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$$\begin{aligned} a \cdot b &= 6(12) + 8(4) + 0(0) \\ &= \underline{\underline{144}} \end{aligned}$$

$$\begin{aligned} a \times b &= (8(0) - 0(4), 0(12) - 6(0), 6(4) - 8(12)) \\ &= \underline{\underline{(0, 0, -42)}} \end{aligned}$$

$$\begin{aligned} 3) \quad a) \quad (3+j5) - (5-j4) - (-2-j3) &= 3-5+2+j5+j4+j3 \\ &= \underline{\underline{j12}} \end{aligned}$$

$$\begin{aligned} b) \quad (5+j7) + (3-j4) - (6-j3) &= 5+3-6+j7-j4-j3 \\ &= \underline{\underline{2}} \end{aligned}$$

$$\begin{aligned} c) \quad (4-j3)(4+j3) &= 16+j12-j12-j^2 9 \\ &= 16-j^2 9 \\ &= \underline{\underline{25}} \end{aligned}$$

$$\begin{aligned}
 d) (4+j7)(4-j7) &= 16 - j^2 28 + j^2 28 - j^2 49 \\
 &= 16 - j^2 49 \\
 &= \underline{\underline{65}}
 \end{aligned}$$

$$\begin{aligned}
 e) \frac{4-j5}{2-j} &= \frac{4-j5}{2-j} \times \frac{2+j}{2+j} \\
 &= \frac{(4-j5)(2+j)}{4+1} \\
 &= \frac{8+j4-j10-j^2 5}{5} \\
 &= \frac{13-j6}{5}
 \end{aligned}$$

$$\begin{aligned}
 f) \frac{(2+j3)(1-j2)}{3+j4} &= \frac{2-j4+j3-j^2 6}{3+j4} \\
 &= \frac{8-j}{3+j4} \times \frac{3-j4}{3-j4} \\
 &= \frac{(8-j)(3-j4)}{(3+j4)(3-j4)} \\
 &= \frac{24-j32-j3+j^2 4}{9-j12+j12-j^2 16} \\
 &= \frac{20-j35}{25}
 \end{aligned}$$

$$④ \quad A = \begin{pmatrix} 4 & -4 \\ -7 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$a) \begin{pmatrix} 4 & -4 \\ -7 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 7 & -3 \\ -8 & 3 \end{pmatrix}}}$$

$$b) \begin{pmatrix} 4 & -4 \\ -7 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & -5 \\ -6 & -1 \end{pmatrix}}}$$

$$c) \begin{pmatrix} 4 & -4 \\ -7 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 4(3) + (-4)(-1) & 4(1) + (-4)(2) \\ -7(3) + (1)(-1) & -7(1) + (1)(2) \end{pmatrix}$$

~~$$\begin{pmatrix} 12 + 4 & 4 - 8 \\ -21 - 1 & -7 + 2 \end{pmatrix}$$~~

$$= \underline{\underline{\begin{pmatrix} 16 & -4 \\ -22 & -5 \end{pmatrix}}}$$

$$d) |A| = (4 \times 1) - (-7 \times 4)$$

$$= 4 - 28$$

$$= \underline{\underline{-24}}$$

$$e) \text{adj } A = \underline{\underline{\begin{pmatrix} 1 & 4 \\ 7 & 4 \end{pmatrix}}}$$

$$f) A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{-24} \begin{pmatrix} 1 & 4 \\ 7 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{24} & -\frac{1}{6} \\ -\frac{7}{24} & -\frac{1}{6} \end{pmatrix}$$