MATHS EXAM ACMA4006

Q one

ai) $y = 8x^3 - 2x^2 - x + 1$

$$\frac{dy}{dx} = 24x^2 - 4x - 1$$

aii) $y = -3(x-5)^2$

$y = -3((x-5)(x-5))$
$y = -3(x^2 - 5x - 5x + 25)$
$y = -3x^2 + 30x - 75$

$$\frac{dy}{dx} = -6x + 30$$

aiii) $y = \frac{7}{x^3} + 4e^{3x}$

$y = 7x^{-3} + 4e^{3x}$

$$\frac{dy}{dx} = -21x^{-4} + 12e^{3x}$$

aiv) $y = 3\sin 2x - 2\cos 2x + \tan\frac{x}{3}$

$$\frac{dy}{dx} = 6\cos 2x + 4\sin 2x + \frac{1}{3}\sec^2\frac{x}{3}$$

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av) $y = e^{2x} \tan 3x$ Product rule

$u \quad v$

$\frac{du}{dx} = 2e^{2x} \qquad \frac{dv}{dx} = 3\sec^2 3x$

$$\frac{dy}{dx} = (\tan 3x)(2e^{2x}) + (e^{2x})(3\sec^2 3x)$$

avi) $y = \frac{\sin 5x}{x^3+3x+4}$ $-u$ $-v$ Quotient rule

~~$\frac{dy}{dx}$~~ $\frac{du}{dx} = 5\cos 5x \qquad \frac{dv}{dx} = 3x^2+3$

$$\frac{dy}{dx} = \frac{(x^3+3x+4)(5\cos 5x) - (\sin 5x)(3x^2+3)}{(x^3+3x+4)^2}$$

avii) $y = \sqrt{3x^3+2x+5}$

$y = (3x^3+2x+5)^{\frac{1}{2}}$ Chain rule

$$\frac{dy}{dx} = 0.5(3x^3+2x+5)^{\frac{-1}{2}}(9x^2+2)$$

b) $y = \sin x$ 3'd derivative

$\frac{dy}{dx} = \cos x$

$\frac{d^2y}{dx^2} = -\sin x$

$$\frac{d^3y}{dx^3} = -\cos x$$

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c) $Z = 5x^3y^2 + 3x^2 + 4y^4$

$\frac{\partial Z}{\partial x} = 15x^2y^2 + 6x$

$x$, $y$

$\frac{\partial^2 Z}{\partial x^2} = 30xy^2 + 6$

$\frac{\partial^2 Z}{\partial y \partial x} = 30x^2y$

$\frac{\partial Z}{\partial y} = 10x^3y + 16y^3$

$y$, $x$

$\frac{\partial^2 Z}{\partial y^2} = 10x^3 + 48y^2$

$\frac{\partial^2 Z}{\partial x \partial y} = 30x^2y$

Q Two

ai) $\frac{2x+3}{6} = \frac{x}{3} + \frac{5x-1}{6}$ multiply by 6

$2x + 3 = 2x + 5x - 1$

$4 = 5x$

$\frac{4}{5} = x$

$0.8 = x$

aii) $2\ln(x+6) = \ln(x+3) + \ln(x+7)$

$\ln(x+6)^2 = \ln(x+3)(x+7)$

$(x+6)(x+6) = (x+3)(x+7)$

$x^2 + 12x + 36 = x^2 + 10x + 21$

$2x = -15$

$x = \frac{-15}{2}$

$\boxed{x = -7.5}$

aiii) $3^{x+5} = 2^x \times 6$

$(x+5)\log 3 = x\log 2 + \log 6$

$0.48x + 2.39 = 0.30x + 0.78$

$0.18x = -1.61$

$x = \frac{-1.61}{0.18}$

$\boxed{x = -8.94}$

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b) $y = x^2 + 2x + 2$ ①

$y = 2x^2 + 11x + 10$ ②

② - ①

$0 = x^2 + 9x + 8$

$(x + 8)(x + 1)$

$x + 8 = 0$ $\quad$ $x + 1 = 0$

$\boxed{x = -8}$ $\quad$ $\boxed{x = -1}$

$-8 \rightarrow$ ① $\quad$ $-1 \rightarrow$ ①

$\boxed{y = 50}$ $\quad$ $\boxed{y = 1}$

c) $9x^2 - 4x - 5 = 0$ $\quad$ $a = 9 \quad b = -4 \quad c = -5$

$x = \dfrac{-b \pm \sqrt{b^2 - 4(a)(c)}}{2a}$

$x = \dfrac{4 \pm \sqrt{16 - 4(9)(-5)}}{18}$

$x = \dfrac{4 \pm 14}{18}$

$x = \dfrac{4 + 14}{18}$ $\quad$ $x = \dfrac{4 - 14}{18}$

$\boxed{x = 1.00}$ $\quad$ $\boxed{x = -0.56}$

di) $x^2 + 11x + 30 = 0$

$(x + 6)(x + 5)$

$x + 6 = 0$ $\quad$ $x + 5 = 0$

$\boxed{x = -6}$ $\quad$ $\boxed{x = -5}$

dii) $x^2 + 13x + 22 = 0$

$(x + 11)(x + 2)$

$x + 11 = 0 \quad x + 2 = 0$

$\boxed{x = -11} \quad \boxed{x = -2}$

diii) $x^2 - 100 = 0$ $\quad \sqrt{100} = 10$

$(x + 10)(x - 10)$

$x + 10 = 0 \quad x - 10 = 0$

$\boxed{x = -10} \quad \boxed{x = 10}$

div) $2x^2 + 7x + 5 = 0$

$(2x + 5)(x + 1)$

$2x + 5 = 0 \quad x + 1 = 0$

$x = \frac{-5}{2} \quad \boxed{x = -1}$

$\boxed{x = -2.5}$

Q Three

ai) $\sin x = 0.75$

$x = \sin^{-1} 0.75$

$\boxed{x = 48.59°}$

S | A
T | C

$x = 180 - 48.59$

$\boxed{x = 131.41°}$

aii) $-5\cos x = 2$

$x = \cos^{-1}\left(\frac{2}{-5}\right)$ Ignore minus

$x = 66.42$ Not an answer

S | A
T | C

$x = 180 - 66.42$

$\boxed{x = 113.58°}$

$x = 180 + 66.42$

$\boxed{x = 246.42°}$

aiii) $8\tan x = 1$

S | A
T | C

$x = \tan^{-1}\frac{1}{8}$

$\boxed{x = 7.13°}$

$x = 180 + 7.13$

$\boxed{x = 187.13°}$

aiv) $\tan^2 x - 14\tan x + 24 = 0$

$(\tan x - 2)(\tan x - 12)$

$\tan x - 2 = 0$ $\tan x - 12 = 0$

S | A
T | C

$x = \tan^{-1} 2$ $x = \tan^{-1} 12$

$\boxed{x = 63.43°}$ $\boxed{x = 85.24°}$

$x = 180 + 63.43$ $x = 180 + 85.24$

$\boxed{x = 243.43°}$ $\boxed{x = 265.24°}$

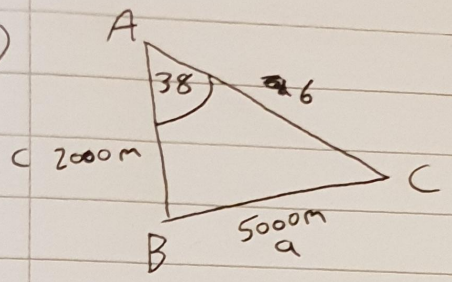
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b)




$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{5000}{\sin 38} = \frac{AC}{\sin B} = \frac{2000}{\sin C}$$

$$\frac{5000}{\sin 38} = \frac{2000}{\sin C}$$

$$(\sin C)(5000) = (\sin 38)(2000)$$

$$C = \sin^{-1}\left(\frac{(\sin 38)(2000)}{5000}\right)$$

$$C = 14.26°$$

$$B = 180 - 38 - 14.26$$

$$B = 127.74°$$

$$\frac{5000}{\sin 38} = \frac{AC}{\sin 127.74}$$

$$\sin 127.74\left(\frac{5000}{\sin 38}\right) = AC$$

$$6422m = AC$$