



12 Pages

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Complete, fold and seal
(see note 2 below)

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1	Student Number	WIL18847411
2	Title of Examination Paper	DSP + COMMUNICATION
3	Module Code	GOEE 5003
4	Date and Time of Examination	23 MAY 2023 1:30pm
5	Examination Venue	TYCOCH
6	Seat Number	F3
7	Number of Answer Books handed in	1

[illegible]

(Gweler yr ochr arall am y fersiwn Cymraeg)

Q 1

i) $y[n] = x[2n] + 8$

$$y[n] = \{H\{x(n)\}\} = x(2n) + 8$$

$$y_1(n) = a_1 x_1(2n) + 8a_1 \quad \checkmark$$

$$y_2(n) = a_2 x_2(2n) + 8a_2 \quad \checkmark$$

Q1

15/15

~~weighted inputs~~
weighted outputs = $a_1 x_1(2n) + a_2 x_2(2n) + 8a_1 + 8a_2$ ①

weighted inputs y_3 =

$$y_3(n) = \{H\{a_1 x_1(n) + a_2 x_2(n)\}\} = a_1 x_1(2n) + a_2 x_2(2n) + 16$$
 ②

Are ① + ② equal = No $\therefore$ system is Non Linear

ii) The system $y[n] = x[2n] + 8$ is Non Causal due to the system contains a future value of $x(n)$

iii) $y[n] = x[2n] + 8$

$$y(n-k) = x(2n-k) + 8$$

$$= x(2n-2k) + 8$$

delay output ①

$$y(n,k) = \{x(2n-k)\} = x(2n-k) + 8$$
 ②

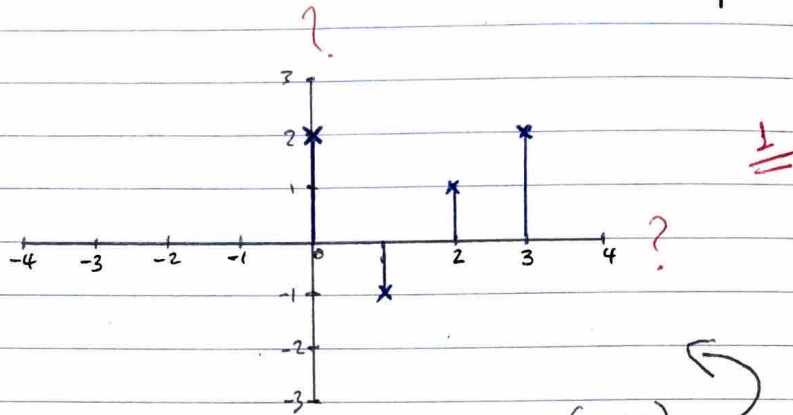
is ① + ② equal? No $\therefore$ system is Time Variant

Q2. $h[n] = [2, -1, 1, 2]$
 $x[n] = [1, 2, 3, 2]$

$$\frac{4}{4} = \frac{-1}{7}$$

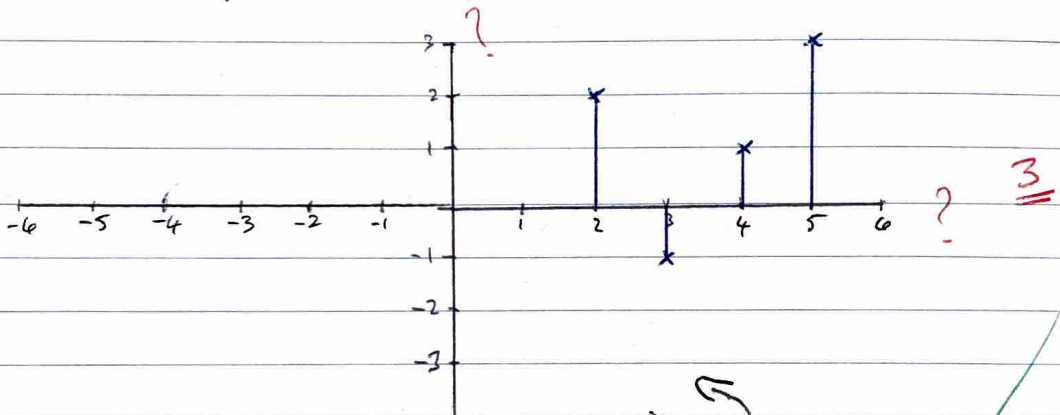
Q2
11/15
Q2a
1/2

a)

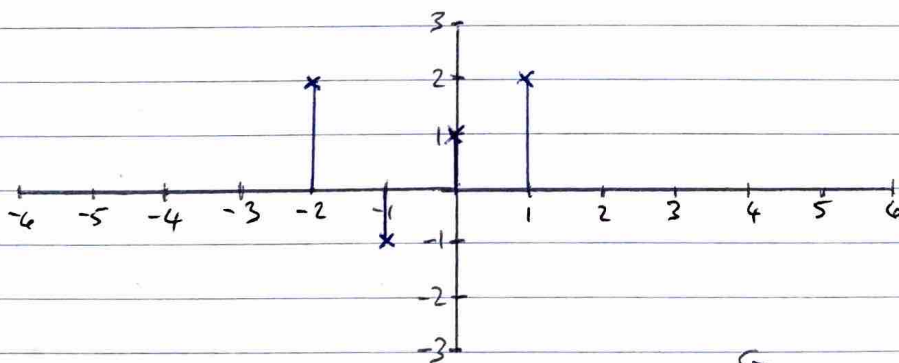


Impulse response of the filter ($h[n]$)

b)



Impulse response of filter ($h[n-2]$)



Impulse response of filter ($h[n-2]$)

Q2b
3/4

c)

n	0	1	2	3	4	5	6
h[n]	2	-1	1	2			
x[n]	1	-2	3	2			
x[0]h[n]	2	-1	1	2			
x[1]h[n-1]	Delay	-4	2	-2	-4		
x[2]h[n-2]	Delay	Delay	6	-3	3	6	
x[3]h[n-3]	Delay	Delay	Delay	4	-2	2	4
y[n]	2	-5	9	1	-3	8	4
	✓	✓	✓	✓	✓	✓	✓

Q2C

7/9

$$y[n] = \sum_{k=-\infty}^{k=\infty} x[k]h[n-k]$$

? ✓

Q3

$$a) \quad x(n) = 25 \sin(0.167\pi n) u(n)$$

$$\Downarrow$$

$$X(z) = \frac{z \sin(\alpha)}{z^2 - 2z \cos(\alpha) + 1} \quad \checkmark$$

$$X(z) = \frac{z \cdot 25 \sin(0.167\pi)}{z^2 - 2z \cdot 25 \cos(0.167\pi) + 1} \quad \checkmark$$

$$X(z) = \frac{z(12.52)}{z^2 - 2z(21.64) + 1} \quad \underline{4}$$

$$b) \quad y[n] + 1.2y[n-1] + 0.3y[n-2] - x[n-1] + x[n-3] = 0$$

Z Transform =

~~Y(z) + 1.2z^{-1}Y(z) + 0.3z^{-2}Y(z) - z^{-1}X(z) + z^{-3}X(z) = 0~~

$$Y(z) + 1.2z^{-1}Y(z) + 0.3z^{-2}Y(z) - z^{-1}X(z) + z^{-3}X(z) = 0$$

factorise =

$$Y(z) + 1.2z^{-1}Y(z) + 0.3z^{-2}Y(z) - \cancel{z^{-1}X(z)} + z^{-3}X(z) = 0 \quad \checkmark \quad \checkmark \quad \checkmark$$

$$Y(z) + 1.2z^{-1}Y(z) + 0.3z^{-2}Y(z) + \cancel{z^{-3}X(z)} = \cancel{z^{-1}X(z)} \quad \checkmark \quad \checkmark \quad \checkmark$$

$$Y(z) + 1.2z^{-1}Y(z) + 0.3z^{-2}Y(z) = z^{-1}X(z) - z^{-3}X(z)$$

$$= 1 + 1.2z^{-1} + 0.3z^{-2} \quad Y(z) = z^{-1}X(z) - z^{-3}X(z)$$

$$\text{Transfer Function: } H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} - z^{-3}}{1 + 1.2z^{-1} + 0.3z^{-2}} \quad \checkmark$$

Q3

25/30

Q3a

4/6

Q3b

12/12

$$c) \quad y[n] - \left(\frac{1}{2}\right) y[n-1] + \left(\frac{1}{16}\right) y[n-2] - x[n] = 0$$

Q3c

z Transform =

9/12

$$Y(z) - \frac{1}{2} z^{-1} Y(z) + \frac{1}{16} z^{-2} Y(z) - X(z) = 0$$

factorise =

$$Y(z) - \frac{1}{2} z^{-1} Y(z) + \frac{1}{16} z^{-2} Y(z) - \cancel{X(z)} = 0$$

$\quad \quad \quad + X(z) \quad \quad + X(z)$

$$Y(z) - \frac{1}{2} z^{-1} Y(z) + \frac{1}{16} z^{-2} Y(z) = X(z)$$

$$= \left| -\frac{1}{2} z^{-1} + \frac{1}{16} z^{-2} \right| Y(z) = X(z)$$

$$\text{Transfer Function} = H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2} z^{-1} + \frac{1}{16} z^{-2}}$$

$$\text{Frequency Domain} = H(e^{j\omega}) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{1 - \frac{1}{2} e^{-j\omega} + \frac{1}{16} e^{-j2\omega}} \quad \checkmark$$

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2} \cos(\omega) - \frac{1}{16} \sin(\omega)} \quad \times$$

$$|H\omega| = \sqrt{\frac{1}{1 - \left(\frac{1}{2} \cos(\omega)\right)^2 - \left(\frac{1}{16} \sin(\omega)\right)^2}} \quad \times \quad \checkmark$$

Q4

a)

3 tap
low pass

cut off = 1000

sample = 10000

Q4a

10/10

① Normalised cutoff frequency $\omega_c = 2\pi f_c T_s$

$$2 \times \frac{1000}{10000} = 0.2\pi$$

② Filter Length = $2M+1=3 \therefore M=1$

$$h(k) = \begin{cases} \frac{\omega_c}{\pi} & k=0 \\ \frac{\sin(\omega_c \cdot k)}{\pi \cdot k} & -M \leq k \leq M \end{cases}$$

$$h(k) = \begin{cases} \frac{0.2\pi}{\pi} & k=0 \\ \frac{\sin(0.2\pi \cdot 1)}{\pi \cdot k} & -M \leq k \leq M \end{cases} = \begin{cases} 0.2 \\ 0.187 \end{cases}$$

$$④ h_0 = 0.2$$

$$\left. \begin{aligned} h_{-1} &= 0.187 \\ h_1 &= 0.187 \end{aligned} \right\} \text{using symmetry, as } h_1 = h_{-1}$$

⑤ Coefficients of filter

$$b_0 = h(k=1) = 0.187$$

$$b_1 = h(0) = 0.2$$

$$b_2 = h(k+1) = 0.187$$

b) 5 tap
bandpass

$$L \text{ cutoff} = 1000$$

$$H \text{ cutoff} = 1250$$

$$\text{sample} = 5000$$

Q4b

15/15

① Normalized cutoff frequency $\omega_c = 2\pi f_c T_s$

$$\omega_L = 2 \times \frac{1000}{5000} = 0.4\pi$$

$$\omega_H = 2 \times \frac{1250}{5000} = 0.5\pi$$

② Filter Length $= 2M + 1 = 5 \therefore M = 2$

$$\textcircled{3} \quad h(k) = \begin{cases} \frac{\omega_H - \omega_L}{\pi} & k=0 \\ \frac{\sin(\omega_H \cdot k)}{\pi \cdot k} - \frac{\sin(\omega_L \cdot k)}{\pi \cdot k} & -M \leq k \leq M \end{cases}$$

$$h(k) = \begin{cases} \frac{0.5\pi - 0.4\pi}{\pi} & k=0 \\ \frac{\sin(0.5\pi \times 1)}{\pi \times 1} - \frac{\sin(0.4\pi \times 1)}{\pi \times 1} & k=1 \\ \frac{\sin(0.5\pi \times 2)}{\pi \times 2} - \frac{\sin(0.4\pi \times 2)}{\pi \times 2} & k=2 \end{cases}$$

PTO

$$\begin{aligned}
 & \textcircled{4} \quad \left. \begin{aligned} h_0 &= 0.1 \\ h_{-1} &= 0.01558 \\ h_1 &= 0.01558 \end{aligned} \right\} \text{using symmetry, as } h_1 = h_{-1} \\
 & \quad \left. \begin{aligned} h_{-2} &= -0.0935 \\ h_2 &= -0.0935 \end{aligned} \right\} \text{using symmetry, as } h_2 = h_{-2}
 \end{aligned}$$

⑤ Coefficients for filter

$$\begin{aligned}
 b_0 &= h(k-2) = -0.0935 \\
 b_1 &= h(k-1) = 0.01558 \\
 b_2 &= h(k) = 0.1 \\
 b_3 &= h(k+1) = 0.01558 \\
 b_4 &= h(k+2) = -0.0935
 \end{aligned}$$

c) Time Division Multiplexing TDM

When a signal is sent in blocks in a set time period which is

Q4c

0/15