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200
1822 - 2022

T-test

Man Geni Addysg Uwch yng Nghymru
Where Higher Education Began in Wales

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Topics covered

- The t Test for Independent Samples
- The Critical Value of t : t_{crit}
- Requirements for Using the t Test for Independent Samples
- The t Test for Dependent (Matched) Samples
- The ANOVA test
- Post hoc Analyses-The Scheffé Method

What is a *t*-test(Student's T-test)?

- statistical test that is used to compare the means of two groups.
- It is often used in hypothesis testing
- determines whether a process or treatment actually has an effect on the population of interest, or whether two groups are different from one another.



William sealy gosset

Example

Scenario:

- A pharmaceutical company has developed a new drug to reduce blood pressure. They want to determine if their new drug is more effective than the existing widely-used drug in the market.

Study design:

- recruit 100 participants with high blood pressure
- participants are randomly divided into two groups:

Group A: Receives the new drug.

Group B: Receives the existing drug (standard treatment).

Measurement:

- Baseline for each participant: Blood pressure readings taken before the treatment
- after a month of treatment.

Question:

- Is there a significant difference in the mean reduction of blood pressure between the two groups (new drug vs. existing drug)?

One-tailed or two-tailed t test?

- refer to the directionality of the statistical test

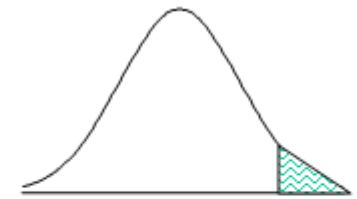
One-tailed t-test:

- used when interested to identify sample mean is significantly greater than or less than a specified value, rather than just different.
- It examines the hypothesis in only one direction.
- It's more sensitive to changes in one direction.

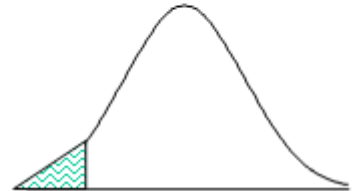
Example:

Testing if a new drug reduces blood pressure more than the existing drug.

The one-tailed test would look for evidence that the new drug performs better but doesn't consider if it performs worse.

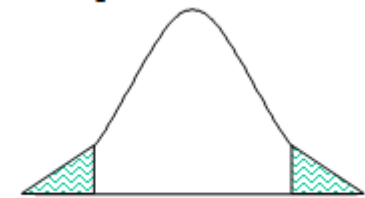


Positive one-tailed test



Negative one-tailed test

One-tailed or two-tailed t test?



Two-tailed test

- refer to the directionality of the statistical test

two-tailed t-test:

- used when interested in determining whether the sample mean is different from a specified value, without specifying a particular direction.
- It examines the hypothesis in both directions.
- It's represented by both the lower and upper tails of the distribution.

Example:

A two-tailed test would assess whether the new drug's effect on blood pressure is different from the existing drug, regardless of whether it's better or worse.

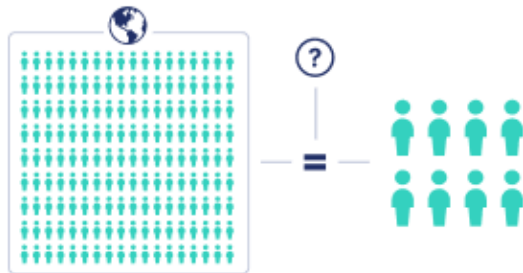
Different types of t- test

Type I

one-sample t-test : single group differs from a known value

- compare the mean of a single sample to a known population mean.

One-sample t test



Investigate whether there's a difference between a group and a standard value or whether a subgroup belongs to a population.

Example for one sample t test

Scenario:

Mean blood pressure of a group of patients differs significantly from the national average blood pressure of 120 mmHg.

- **Null Hypothesis (H_0):** The mean blood pressure of the group of patients is equal to the national average blood pressure ($\mu=120$ mmHg).
- **Alternative Hypothesis (H_1):** The mean blood pressure of the group of patients is different from the national average blood pressure ($\mu \neq 120$ mmHg).

Patient ID Blood Pressure (mmHg)

1	122
2	118
3	120
4	115
5	124
6	119
7	121
8	116
9	118
10	123.....

mean

121.32

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{n}}}$$

1. $\bar{x}$ is the sample average
2. μ is the population average
3. S is the sample standard deviation(s)
4. Number of observations(n)

Patient ID Blood Pressure (mmHg)

1	122
2	118
3	120
4	115
5	124
6	119
7	121
8	116
9	118
10	123

mean

121.32

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$t = \frac{121.32 - 120}{\frac{3.49}{5}}$$

$$t = \frac{1.32}{0.699} = 1.887$$

Std. dev.=3.49

SQRT(25)= 5

$$t = 1.887$$

This is your **calculated t-value**.

What does **calculated t-value** mean?

- Compare it with what is known as the critical value of t (t_{crit}).
- Find the t-table value(from the table).
- You need two values to find this:
- The **alpha level**: Level of significance (standard level is 0.05)
- The **degrees of freedom: $n-1$**

where n is the number of items in the sample

$$t_{crit} = 1.711 \text{ (from the table)}$$

This is your one-tailed critical t-value.

Conclusion

- Calculated t-value = 1.887
- Critical t-value for a one-tailed test at $\alpha=0.05$ and $df=24$ is approximately 1.711

$$1.887 > 1.711$$

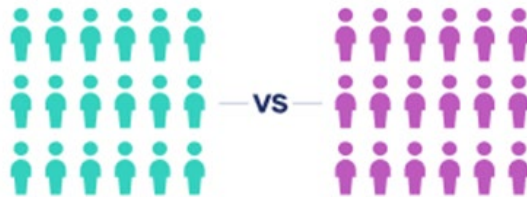
- Therefore, based on this comparison, **we reject the Null hypothesis.**
- **Conclusion:** Mean blood pressure of the sampled patients significantly differs from the national average blood pressure of 120 mmHg.



Type II

- Independent sample t-test(two-sample t-test):
 - two groups differ from each other

Independent-samples t test



Investigate whether there's a difference between two groups (between-subjects).

Example for independent Sample t test(2 paired)

Scenario:

Impact of pH value in Soil on Seed Germination: A Study on Plant Species X.

- Experiment involves two groups
- Eight pots each filled with soil adjusted to distinct pH levels.
- Group 1 = **pH 5.5**
- Group 2 = **pH 7.0**.
- **Plant Species X** planted in each pot across both groups **50 seeds in each pot**

Research Problem:

Does the pH of soil affects seed germination of a specific plant species?



Hypothesis

- H_0 = There is no significant difference between the germination of seeds in two soils of different pH
- H_1 = there is significant difference between the germination of seeds in two soils of different pH

If the value for t **exceeds** the critical value ($P = 0.05$), then you can **reject** the null hypothesis.

Pot	Group 1 (pH5.5)	Group 2 (pH7.0)
1	38	39
2	41	45
3	43	41
4	39	46
5	37	48
6	38	39
7	41	46
8	36	44
Mean (Σ)	39.1	43.5

Find the Calculated t-value(unequal variance)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}\right)}}$$

Where

t = calculated t value

n = Number of items in samples

SD_1^2, SD_2^2 = Variance(square of standard deviation) of the two populations

$\bar{X}_1, \bar{X}_2$ = Mean of the two sample(group 1 and group 2)

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\left(\frac{SD_1^2}{n_1} + \frac{SD_2^2}{n_2}\right)}}$$

Mean of group 1:39.1

Mean of group 2:43.5

Variance of group 1: 5.55

Variance of group 2: 11.7

n_1 and $n_2 = 8$

$t = -2.97$

Degrees of freedom (df)

Independent sample t test,

$$df = n_1 + n_2 - 2 = 8 + 8 - 2 = 14$$

The value of t_{crit} is determined by df and by selected level of significance (5% or 0.05).

Conclusion

- Calculated t-value = 2.97
- Critical t-value for a one-tailed test at $\alpha=0.05$ and $df=14$ is approximately 1.761

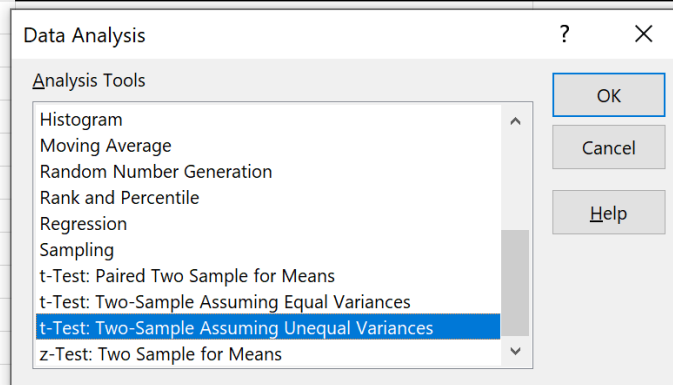
2.97 > 1.761 (t value **exceeds** the critical value)

Therefore, based on this comparison, **we reject the Null hypothesis.**

Conclusion: Significant difference between two groups, pH **does** affect the germination rate for this plant.

T-test in Excel

	A	B	C	D	E	F	G	H
1	Pot	Group 1 (pH7.0) Group 2 (pH7.0)						
2		1	38	39				
3		2	41	45		t-Test: Two-Sample Assuming Equal Variances		
4		3	43	41				
5		4	39	46			Variable 1	Variable 2
6		5	37	48		Mean	39.125	43.5
7		6	38	39		Variance	5.553571429	11.71429
8		7	41	46		Observations	8	8
9		8	36	44		Pooled Variance	8.633928571	
10						Hypothesized Mean Difference	0	
11	Mean	39.125	43.5			df	14	
12	std.dev.	2.356602	3.422613872			t Stat	-2.977856923	
13	square of std.dev.	5.553571	11.71428571			P(T<=t) one-tail	0.004989965	
14	n	8				t Critical one-tail	1.761310136	
15						P(T<=t) two-tail	0.00997993	
16		2.158482				t Critical two-tail	2.144786688	
17	Final	1.469177	0.694196429					
18			1.464285714					
19			2.158482143					
20		0.005578	1.469177369					
21								
22			-2.977856923					
23								
24								
25								
26								
27								
28								



Type III

- **Paired samples t-test (Dependent t-test):**

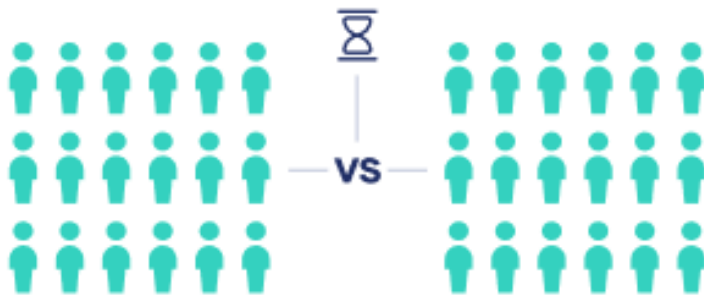
- paired or matched observations, such as before-and-after measurements on the same group of individuals.

Example:

Evaluating if there's a significant difference in blood pressure before and after a diet within the same group of patients.

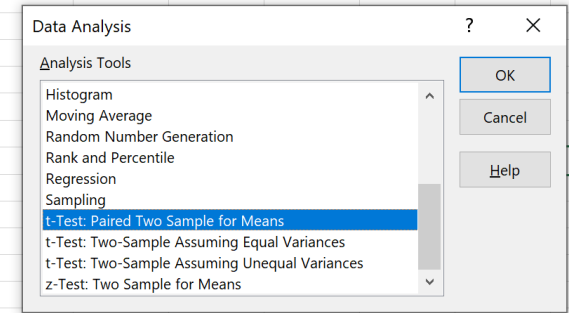
Paired-samples t test

Investigate whether there's a difference within a group between two points in time (within-subjects).



Student	Exam 1	Exam 2 Score
	Score	
Bob	63	69
Nina	65	65
Tim	56	62
Kate	100	91
Alonzo	88	78
Jose	83	87
Nikhil	77	79
Julia	92	88
Tohru	90	85
Michael	84	92
Jean	68	69
Indra	74	81
Susan	87	84
Allen	64	75
Paul	71	84
Edwina	88	82

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Student	Exam 1 Sco	Exam 2 Sco	Difference											
2	Bob	63	69	6											
3	Nina	65	65	0											
4	Tim	56	62	6											
5	Kate	100	91	-9											
6	Alonzo	88	78	-10											
7	Jose	83	87	4			t-Test: Paired Two Sample for Means								
8	Nikhil	77	79	2											
9	Julia	92	88	-4				Variable 1	Variable 2						
10	Tohru	90	85	-5			Mean	78.125	79.4375						
11	Michael	84	92	8			Variance	160.3833	83.72917						
12	Jean	68	69	1			Observations	16	16						
13	Indra	74	81	7			Pearson Correlation	0.841729							
14	Susan	87	84	-3			Hypothesized Mean Difference	0							
15	Allen	64	75	11			df	15							
16	Paul	71	84	13			t Stat	-0.74978							
17	Edwina	88	82	-6			P(T<=t) one-tail	0.232494							
18							t Critical one-tail	1.75305							
19							P(T<=t) two-tail	0.464987							
20							t Critical two-tail	2.13145							
21															
22															
23															



Sumarize:The 5 Stages of Analysis

1. State your hypotheses
 - H_0 (the null hypothesis)
 - H_A or H_1 (the alternative hypothesis)
2. Select α , your significance level
 - ***0.05 is usually used*** but 0.01 can be used in some cases
3. Calculate t
4. Find t_{crit}
5. Decide whether or not to reject the null hypothesis

Exercise

- Suppose we have two Field, A and B, and we need to measure the yield over each Field.

Field A	Field B
25	14
14	11
23	13
21	9
24	15
17	12
19	9
20	11
15	8
22	13
16	12
21	14

Recap

When to use a t test

- A t test can only be used when comparing the means of two groups (a.k.a. pairwise comparison).
- If you want to compare more than two groups, or if you want to do multiple pairwise comparisons, use an ANOVA test or a post-hoc test.

Recap

- **One-sample, two-sample, or paired t test?**
- If the groups come from a single population (e.g., measuring before and after an experimental treatment), perform a **paired t test**. This is a within-subjects design.
- If the groups come from two different populations (e.g., two different species, or people from two separate cities), perform a **two-sample t test** (a.k.a. **independent t test**). This is a between-subjects design.
- If there is one group being compared against a standard value (e.g., comparing the acidity of a liquid to a neutral pH of 7), perform a **one-sample t test**.

Summary

- Topics covered
 - The t Test for Independent Samples
 - The Critical Value of t : t_{crit}
 - Requirements for Using the t Test for Independent Samples
 - The t Test for Dependent (Matched) Samples

References:

Chapter 11 of *Statistics for the Terrified*.