

SECTION A.

x	f	xf	$f(x-\bar{x})^2$
3	3	9	28.091
4	17	68	72.141
5	29	145	32.58
6	46	276	0.166
7	33	231	29.159
8	18	144	67.745
9	4	36	34.574
Σ	150	909	264.456

$$\bar{x} = \frac{909}{150} = 6.06$$

$$\sigma = \sqrt{\frac{264.456}{150}} = 1.33$$

b) (i) $\mu = 200$ $\sigma = 40$

$$Z = \frac{x - \mu}{\sigma} = \frac{245 - 200}{40} = 1.125$$

from tables = 0.3697

$$\therefore P(>245) = 0.5 - 0.3697 = 0.1303$$

(ii) Area = 0.5 - 0.1 = 0.4

from tables $Z = 1.282$

$$Z = \frac{X - 200}{40}$$

$$1.282 \times 40 + 200 = X$$

$$X = 251.28$$

3

2

2

7

3

2

3

2

2

2

4 18 25

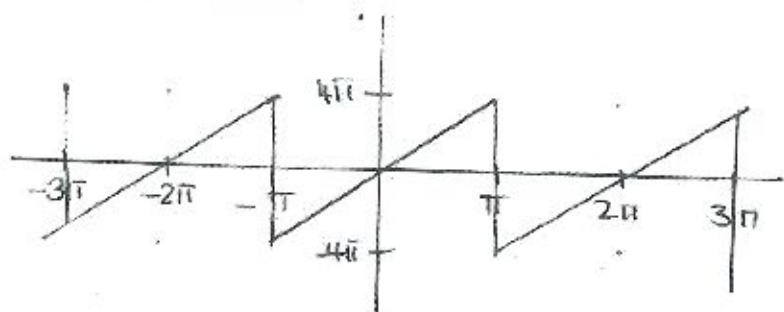
Exam Solutions/Mark Scheme

SECTION B				Q1 total
1.	$\frac{dy}{dt} + 4y = 10 \quad y(0) = 3.$ $(s\bar{y} - y_0) + 4\bar{y} = \frac{10}{s}$ since $y(0) = 3$ $s\bar{y} - 3 + 4\bar{y} = \frac{10}{s}$ $\bar{y}(s+4) = \frac{10}{s} + 3$ $\bar{y} = \frac{10 + 3s}{s(s+4)}$ $\frac{10+3s}{s(s+4)} = \frac{A}{s} + \frac{B}{(s+4)}$ $10 + 3s = A(s+4) + B(s)$ put $s = -4$ $10 + 3(-4) = A(0) + B(-4)$ $-2 = -4B$ $B = 1/2$ put $s = 0$ $10 = A(4) + B(0)$ $A = 5/2$ $\mathcal{L}^{-1} \left\{ \frac{5}{2s} + \frac{1}{2(s+4)} \right\} = \frac{5}{2} + \frac{1}{2} e^{-4t}$			
		13		
		2		
		1		
		3		
		3		
		1		
		4		
		4		
		2, 2		25

			Qu total
2.	$R=30 \quad L=0.6 \quad E=30$		
	$L \frac{dI}{dt} + RI = E$		
	$0.6 \frac{dI}{dt} + 30I = 30$	3	
	$\frac{dI}{dt} + 50I = 50$	2	
	$\int P dt = \int 50 dt = 50t$	3	
	$\therefore IF = e^{50t}$	3	
	$I e^{50t} = \int e^{50t} \cdot 50 dt$	3	
	$I e^{50t} = e^{50t} + C$	4	
	When $I=0 \quad t=0$.		
	$0 = e^0 + C$	2	
	$C = -1$	2	
	$\therefore I e^{50t} = e^{50t} - 1$	-	
	or $I = 1 - e^{-50t}$	3	25

3

$$f(x) = 4x$$



ODD FUNCTION

$$\therefore a_0 = 0$$

$$\text{and } a_n = 0.$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} 4x \sin nx \, dx$$

Find $\int 4x \sin nx \, dx$ by parts:

$$\text{Let } u = 4x \quad \frac{dv}{dx} = \sin nx$$

$$\frac{du}{dx} = 4 \quad v = -\frac{1}{n} \cos nx$$

$$\int 4x \sin nx = (4x) \left(-\frac{1}{n} \cos nx\right)$$

$$- \int -\frac{1}{n} \cos nx (4) \, dx$$

$$= -\frac{4x}{n} \cos nx + \int \frac{4}{n} \cos nx \, dx$$

$$= -\frac{4x}{n} \cos nx + \frac{4}{n^2} \sin nx$$

$$\therefore b_n = \frac{2}{\pi} \left[-\frac{4x}{n} \cos nx + \frac{4}{n^2} \sin nx \right]_0^{\pi}$$

Exam Solutions/Mark Scheme

			Qu total
$b_n = \frac{2}{\pi} \left[-\frac{4\pi}{n} \cos(n\pi) + \frac{4}{n} \sin(n\pi) \right]$ $- \frac{2}{\pi} \left[-\frac{4(0)}{n} \cos 0 + \frac{4}{n} \sin 0 \right]$	2		
$b_n = -\frac{8}{n} \cos(n\pi)$	1		
$n=1, b_1 = -\frac{8}{(1)} \cos \pi = \frac{8}{(1)}$			
$n=2, b_2 = -\frac{8}{(2)} \cos(2\pi) = -\frac{8}{(2)}$			
$n=3, b_3 = -\frac{8}{(3)} \cos(3\pi) = \frac{8}{(3)}$			
$b_4 = -\frac{8}{(4)} \text{ etc.}$	2		
$f(x) = \frac{8}{(1)} \sin x - \frac{8}{(2)} \sin 2x + \frac{8}{(3)} \sin 3x$ $- \frac{8}{(4)} \sin 4x - \dots$	2		25

Exam Solutions/Mark Scheme

			Qu total
4	$L \frac{dI}{dt} + RI + \frac{1}{C} \int I dt = E(t)$ $L I' + RI' + \frac{1}{C} I = E'(t)$ Since $E(t) = 20 \sin 10t$ $\frac{dE}{dt} = 200 \cos 10t$ $20 I'' + 50 I' + \frac{1}{0.1} I = 200 \cos 10t$ $2 I'' + 2.5 I' + 0.5 I = 10 \cos 10t$ Find C.F.: char Eqn $m^2 + 2.5m + 0.5 = 0$ $m = \frac{-2.5 \pm \sqrt{2.5^2 - 4(1)(0.5)}}{2(1)}$ $= \frac{-2.5 \pm \sqrt{4.25}}{2}$ $= -0.219 \text{ or } -2.281$ real and different (case 1) $I_c = A e^{-0.219t} + B e^{-2.281t}$ $I_p = A \sin 10t + B \cos 10t$ $I_p' = 10A \cos 10t - 10B \sin 10t$ $I_p'' = -100A \sin 10t - 100B \cos 10t$ Sub: $(-100A \sin 10t - 100B \cos 10t) + 2.5(10A \cos 10t - 10B \sin 10t) + 0.5(A \sin 10t + B \cos 10t) = 10 \cos 10t$ $-100A \sin 10t - 100B \cos 10t + 25A \cos 10t - 25B \sin 10t + 0.5A \sin 10t + 0.5B \cos 10t = 10 \cos 10t$ Take series: $-100A - 25B + 0.5A = 0$ $-99.5A - 25B = 0$ $A = \frac{25B}{-99.5} \rightarrow A = -0.251B$	2 3 2 2 1 1 1 1 1	

Exam Solutions/Mark Scheme

			Qu total
Take cosines:			
$-100B + 25A + 0.5B = 10$			
$-99.5B + 25(0.251B) = 10$			
$-105.775B = 10$			
$B = \frac{10}{-105.775} = -0.0945$	1		
$\therefore A = -0.251(-0.0945)$			
$= 0.0237$	1		
G.S: $I = Ae^{-0.219t} + Be^{-2.281t}$			
$+ 0.0237 \sin 10t - 0.0945 \cos 10t$			
$I(0) = 0$			
$0 = Ae^0 + Be^0 + 0.0237 \sin 0 - 0.0945 \cos 0$	1		
$0 = A + B - 0.0945$ (1)			
$\frac{dI}{dt} = -0.219Ae^{-0.219t} - 2.281Be^{-2.281t}$			
$+ 0.237 \cos 10t + 0.945 \sin 10t$	2		
$I'(0) = 0$			
$0 = -0.219Ae^0 - 2.281Be^0 + 0.237 \cos 0$	1		
$+ 0.945 \sin 0$			
$0 = -0.219A - 2.281B + 0.237$ (2)	1		
(1) $\times 0.219$			
$-0.219A + 0.281B = 0.237$			
$0.219A + 0.219B = 0.0207$			
$2.062B = 0.216$			
$B = \frac{0.216}{2.062} = 0.105$	1		
$A + 0.105 = 0.0945$			
$A = -0.011$			
$\therefore I = -0.011e^{-0.219t}$			
$+ 0.105e^{-2.281t}$			
$+ 0.0237 \sin 10t - 0.0945 \cos 10t$	1		
			25