



School of Engineering

Calculation On Piston, Rod, Crankshaft assemblies

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1.Introduction

Piston, connecting rod, crankshaft assemblies are an arrangement wherein a concave portion on a support end of a connecting rod is slidably engaged with a convex portion formed in a lower part of a piston, and a retainer ring member supported on a skirt portion of the piston is placed in contact with the lower part of the concave rod end. (United States Patent No. 5115725, 1992) In the write-up, author clearly demonstrates his own developed knowledge and understanding and that author carry out his own analysis about the calculation on piston, rod, crankshaft assemblies. By making few Goodman diagram, the mean stress and range of stress of the gudgeon pin, fatigue safety factors of the three critical rod- sections on the connecting rod, the bolt stiffness and the cap-rod stiffness and even the crankshaft they all can be analysed. The calculations on the standard project engine are all carried out with the working safety factors.

2. The mean stress and range of stresses for the gudgeon pin

As we know, gudgeon pin is one of the important parts of the engine, which plays a part in the economy performance, reliability about the engine. The great gas explosive force that the piston receives during operation is almost completely transmitted to the gudgeon pin, causing the gudgeon pin to undergo longitudinal bending deformation and lateral bending out of round deformation, resulting in a large stress concentration in the pin seat. And here are some data about the gudgeon pin. Outer circle dimension = 20 mm, inner circle dimension = 12 mm, $Y = 10$ mm, F_{net} max down on 5700Rpm and F_{net} max up around TDC exhaust on 6500-7000Rpm. Assuming the $F_{net}/2 = 6000$ N and $F_{inertial}$ pin upwards at TDC max Rpm = 1500N.

$$\text{Then } I = \frac{\pi}{64}(20^4 - 12^4)mm^4 = 6836mm^4$$

$$\text{Bending moment at it} = -6000 \times 15 - 1500 \times 10 = -10500N \cdot mm$$

$$\text{Bending stress at it} = -10500 \times \frac{10}{6838} = -154N/mm^2 \text{ in compression.}$$

$$\text{Shear net force on the right of it} = -1500-6000 = -7500 \text{ N}$$

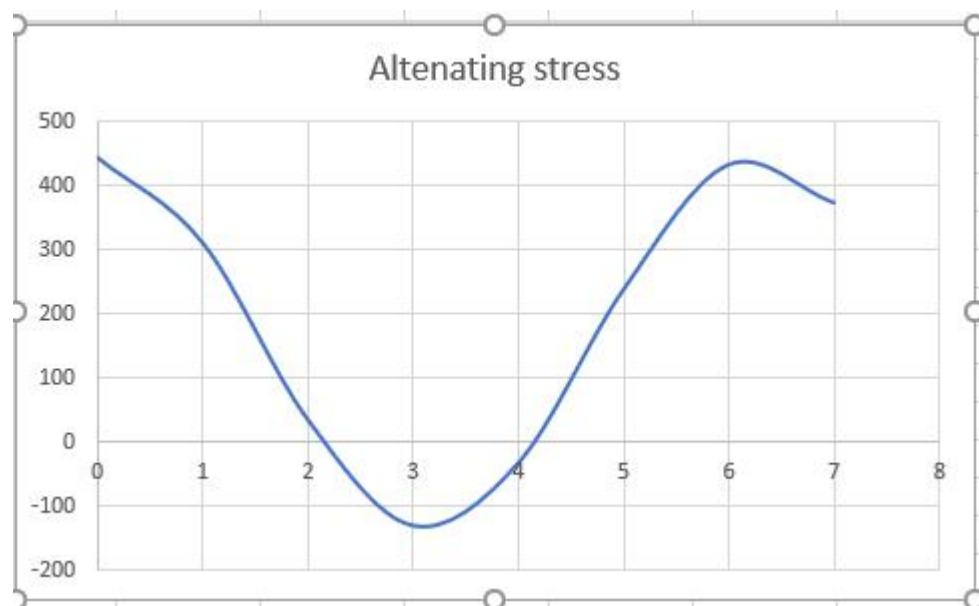
$$\text{Then shear stress at it} = -7500/\frac{\pi}{4}(20^2 - 12^2) = -171 \text{ N/mm}^2$$

To calculate fatigue at it

Tensile $\sigma_{max} = + 424N/mm^2$

Compression $\sigma_{min} = - 154N/mm^2$

The range of the stress = $\sigma_{max} - \sigma_{min} = 578N/mm^2$



The Goodman diagram about the stress as the graph above show

σ_{max} needs to be combined max stress

$$\tau_{max} = \frac{20000-1000}{\frac{\pi}{4}(40^2-12^2)} = 94.5N/mm^2$$

$$\sigma_{max} \text{ combined} = \sqrt{424^2 + (2 \times 94.5)^2} = 464N/mm^2$$

$$\sigma_{min} \text{ combined} = - 171N/mm^2$$

$$\text{Range of stress} = 635N/mm^2$$

$$\text{And the mean stress} = (\sigma_{max} + \sigma_{min})/2 = 146.5N/mm^2$$

The theory of fatigue damage accumulation believes that when the stress on a part is higher than the fatigue limit, each load cycle causes a certain amount of damage to the part, and this damage can be accumulated; when the damage accumulates to a critical value, the part will occur Fatigue failure. The more important fatigue damage accumulation theories include linear and non-linear fatigue damage accumulation theory. Linear fatigue damage accumulation theory holds that the fatigue damage generated by each cycle load is independent of each other. Total damage is a linear accumulation of each fatigue damage.

As we know the $UTS = 1000 \text{ N/mm}^2$

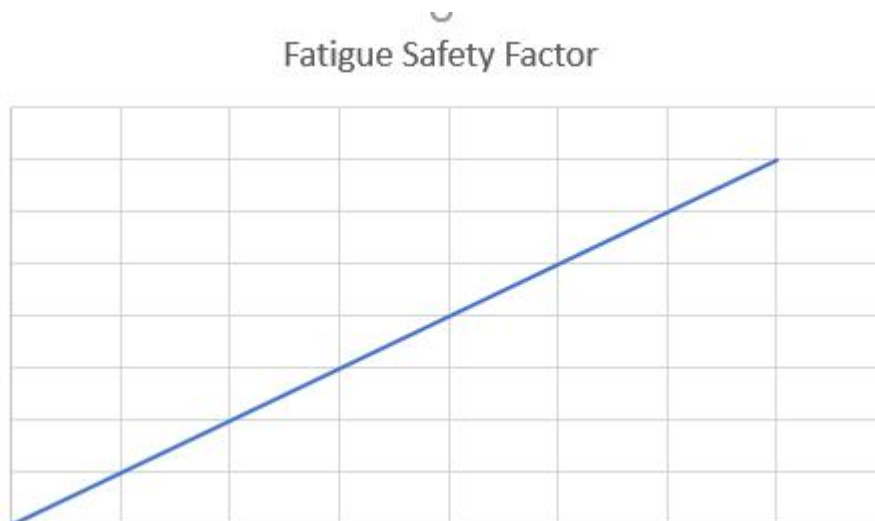
6000RPM number of cycles 4 strokes per second

$= 6000 / (2 \times 60) = 50$ complete cycles per second.

Aim is to get 10^7 cycles, it will last forever,

let N = number of hours to get to 10^7 cycles.

$N = 55$



Last forever if fatigue safety factor = 1 of steel.

Under variable stress, the ultimate stress of the part can neither take the strength limit of the material nor the yield limit of the material, and it should be the fatigue limit

3. Standard unmodified fatigue safety factors.

(Oslo, 2009) Fatigue life is governed by several variables that are highly uncertain. The safety factor on fatigue life is used in a deterministic way to account for the estimated fatigue damage uncertainty. High uncertainties lead to high fatigue safety factors, and vice versa. Evaluation of the uncertainties in the variables governing fatigue design provides a grip on what the safety factor should be. According to the data about the connecting rod, Total area at A is 164mm^2 . Then the nominal compressive stress at A = $\frac{40000}{\text{total area}} = \frac{40000}{164} = -244\text{N/mm}^2$

$$\text{At A } \sigma_{\max \text{ nominal}} = 61 \text{ N/mm}^2$$

$$\sigma_{\min \text{ nominal}} = -244 \text{ N/mm}^2$$

$$K_T = 1.5$$

$$\sigma_{\text{mean nominal}} = \frac{-244 + 61}{2} = -91.5 \text{ N/mm}^2$$

$$\text{Range nominal} = -(-244 - 61) = 305 \text{ N/mm}^2$$

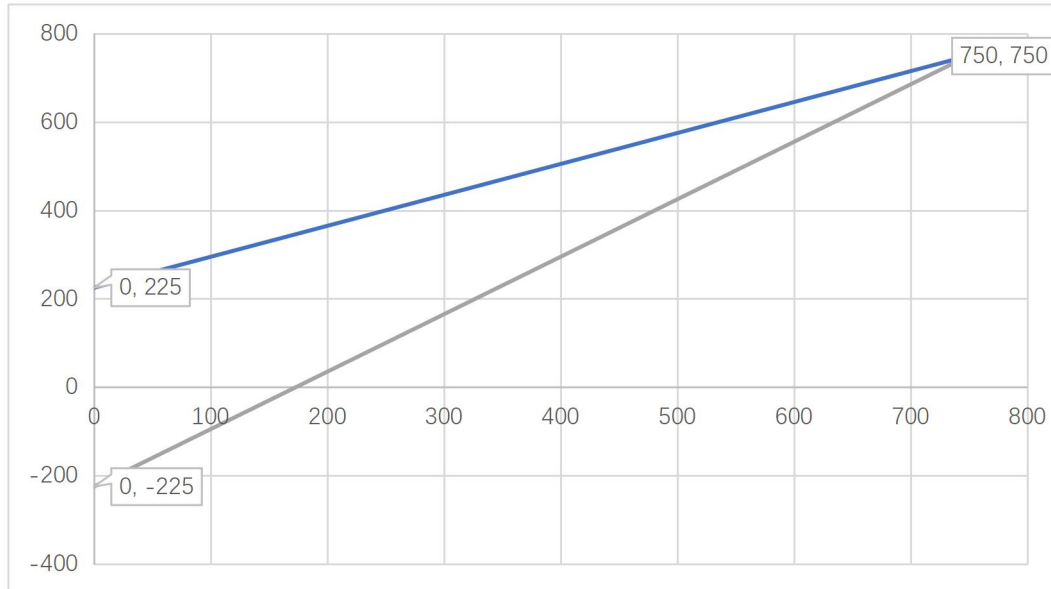
Fatigue limit refers to the maximum stress value when no stress occurs after infinite stress cycles. It is also called the endurance limit. The fatigue limit of a material is an inherent property of the material. It varies with the characteristics of the cycle, the form of deformation of the test piece, and the environment in which the material is located, and it needs to be determined by fatigue tests.

$$\text{UTS} = 750 \text{ N/mm}^2$$

$$\text{Endurance limit} = 0.45 \times 750 = 337.5 \text{ N/mm}^2 \text{ and final}$$

$$\text{endurance limit} = \frac{337.5}{1.5} \text{ N/mm}^2 = 225 \text{ N/mm}^2$$

Here is the Goodman diagram about the connecting rod at A, the x axis is mean stress and the Y axis is alt stress.



$$2 \times 225 \times \left(\frac{750 + 91.5}{750} \right) = 505 \text{ N/mm}^2$$

Actual Range = 305 N/mm^2 then the fatigue safety factor can be determined $= \frac{\text{ALLOWABLE RANGE}}{\text{ACTUAL NOMINAL RANGE}} = \frac{505}{305} = 1.6$

Total area at B is 279 mm^2 , the total force through area above B is 12000 N. Then the maximum nominal stress can be calculated as

$$\sigma_{\text{max nominal}} = \frac{12000}{279} = 43 \text{ N/mm}^2$$

and the maximum compressive force at B =

$40000 - 2000 = 38000 \text{ N}$, Assuming NBT downforce is reduced by similar amount to inertial force going up, Then the minimum nominal stress can be calculated as

$$\sigma_{\text{mamin nominal}} = \frac{-38000}{279} = -136 \text{ N/mm}^2$$

$$K_T = 1.5$$

$$\sigma_{\text{mean nominal}} = \frac{-136 + 43}{2} = -46.5 \text{ N/mm}^2$$

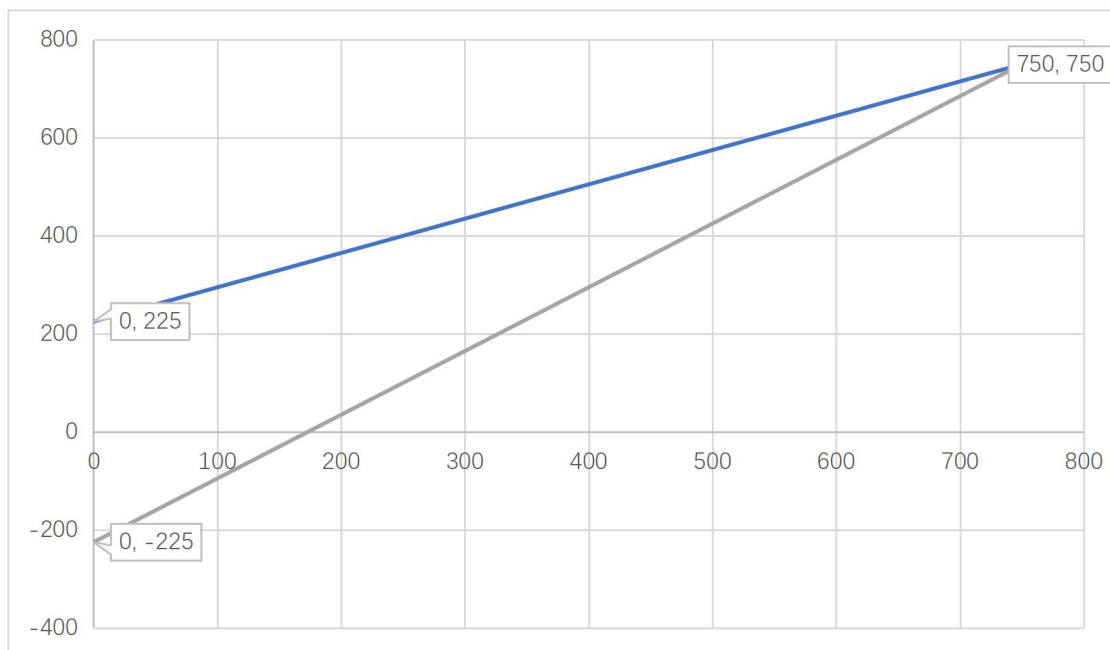
$$\text{Range nominal} = -(-136 - 43) = 179 \text{ N/mm}^2$$

$$UTS = 750 \text{ N/mm}^2$$

Endurance limit = $0.45 \times 750 = 337.5 \text{ N/mm}^2$ and final

$$\text{endurance limit} = \frac{337.5}{1.5} \text{ N/mm}^2 = 225 \text{ N/mm}^2$$

Then the Goodman diagram at B can be demonstrated as the graph show, the x axil is mean stress and the Y axil is alt stress.



$$2 \times 225 \times \left(\frac{750 + 46.5}{750} \right) = 477.9 \text{ N/mm}^2$$

Actual Range = 179 N/mm^2 then the fatigue safety factor can be determined $= \frac{\text{ALLOWABLE RANGE}}{\text{ACTUAL NOMINAL RANGE}} = \frac{477.9}{179} = 2.67$

According to the data, the total area at C is 144 mm^2 , the total force through area above C is 7000 N. Then the maximum nominal stress can be calculated as,

$$\sigma_{\text{max nominal}} = \frac{7000}{144} = 48.61 \text{ N/mm}^2$$

$$\sigma_{\text{min nominal}} = 0 \text{ N/mm}^2$$

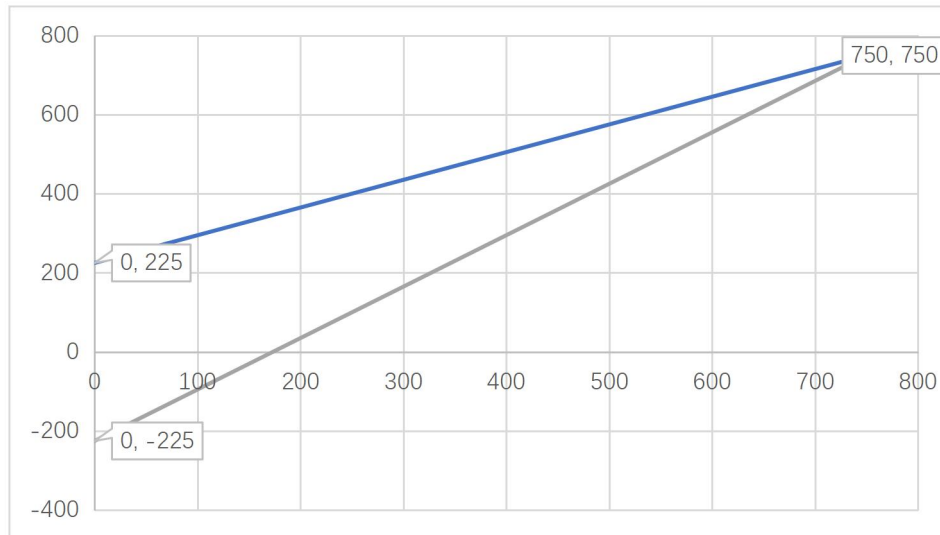
$$\sigma_{\text{mean nominal}} = \frac{48.61}{2} = 24.305 \text{ N/mm}^2$$

$$\text{Range nominal} = 48.61 \text{ N/mm}^2$$

$$\text{UTS} = 750 \text{ N/mm}^2$$

Endurance limit = $0.45 \times 750 = 337.5 \text{ N/mm}^2$ and final

$$\text{endurance limit} = \frac{337.5}{1.5} \text{ N/mm}^2 = 225 \text{ N/mm}^2$$



$$2 \times 225 \times \left(\frac{750 - 24.305}{750} \right) = 435.42 \text{ N/mm}^2$$

Actual Range = 48.61 N/mm^2 then the fatigue safety factor

can be determined $= \frac{\text{ALLOWABLE RANGE}}{\text{ACTUAL NOMINAL RANGE}} = \frac{435.42}{48.61} = 8.96$

4. Calculation on con-rod

1) The ratio of the bolt stiffness to the cap-rod stiffness.

Here are some data of young's modulus about the cap-rod and bolt and area of them. Author let the bolt stiffness be K_B and the cap-rod stiffness be K_{ROD} , L is the compress length of cap-rod. After measuring the size about the graph on the assignment, the total length about the area A is 80mm and the dimension of the area B is 10mm.

$$K_{ROD} = \frac{E_{ROD} \cdot A_{ROD}}{L}$$

$$K_B = \frac{E_{BOLT} \cdot A_{BOT}}{L}$$

$$\text{Then } \frac{K_{ROD}}{K_{BOLT}} = \frac{E_{ROD} \cdot A_{ROD}}{E_{BOLT} \cdot A_{BOLT}}$$

$$E_{ROD} = 170 \times 10^3$$

$$E_{BOLT} = 210 \times 10^3$$

$$A_{BOLT} = \pi \times 10^2 \text{mm}^2$$

$$\frac{K_{ROD}}{K_{BOLT}} = \frac{E_{ROD} \cdot A_{ROD}}{E_{BOLT} \cdot A_{BOLT}} = \frac{170 \times 10^3 \times 400}{210 \times 10^3 \times 314} = 1.03$$

2) The stiffness of the standard bolt for the stretch region up to the elastic limit

The elastic limit refers to the metal material's external force to a certain limit. If the external force is removed, its deformation disappears and returns to its original state. The elastic limit refers to the ability of the metal material to resist the external force of this limit. If it continues to be used Increased tensile force will cause the object to deform plastically until it breaks. K_B can be determined as the same way above that $K_{BOLT} = E_{BOLT} \cdot A_{BOLT}$, To make the E_{BOLT} to the elastic limit. As we know stiffness is constant up to yield, stiffness after yield is much less and is possible to get close to limit 0. Then

$K_{BOLT\ max} = E_{BOLT\ max} \cdot A_{BOLT} = 65940000$ or the K_{BOLT} get close to the limit 0.

3) The maximum value of the alternating load on the bolt

$$K_{BOLT} = K_{BOLT\ static} - K_B \delta_X$$

UP=DOWN, Then $K_{BOLT\ static} - K_B \delta_X = 5000 +$

$K_{ROD-CAP\ static} - K_{ROD-CAP} \delta_X,$

Additional bolt stretch $\delta_X = \frac{5000}{K_B + K_{ROD\ CAP}}$

Extra force in bolt

$$= \delta_X K_B = \frac{5000 K_B}{K_B + K_{ROD\ CAP}} = \frac{5000}{1 + E_{ROD\ CAP} A_{ROD\ CAP} / E_{BOLT} A_{RBOLT}} = 2463 N / mm^2$$

4) The required bolt tightening torque

When tightened to 90% yield from the static bolt force, the required bolt tightening torque can be calculated. For a typical thread, if tightened to 90% yield, Axial stress is 60% yield.

UTS = 1000 N/mm²

yield ~ 900 N/mm²

Axial stress ~ 594 N/mm²

$$\text{Force} = \frac{17.75^2 \times 594}{4} = 26242 N$$

5) Mean stress and range of stress in the critical part

To calculate tightening torque, there are 2 areas of friction. The coefficient of friction about effective thread and beneath head is 0.18 and 0.12.

Circumferential friction force at threads

$$= 0.18 \times 26242 = 4723N$$

Resisting torque at tread

$$= \frac{7.5}{2} \times 4723N \cdot mm = 17.7N \cdot m$$

$$\text{Circumferential friction force} = \mu_{HEAD} \times 26242 = 3149N$$

Average seating face dimension is 10.5mm.

Then the torque needed to overcome head friction $= \mu_{HEAD} \times 26242 = 3149N$ The effective fatigue safety factor of the standard bolts $10.5mm \times 3149N = 16.6N \cdot m$

Total torque when tightened $= 16.6N \cdot m + 17.7N \cdot m = 34.3N \cdot m$ Based on the Ford reusable bolt,

Range $\approx 78N/mm^2$ comes from extra force

$$\sigma_{min} = 90\% \text{ of yield} = 0.9 \times 990 = 891N/mm^2$$

$$\sigma_{max} = \sigma_{min} + RANGE = 891 + 78 = 969N/mm^2$$

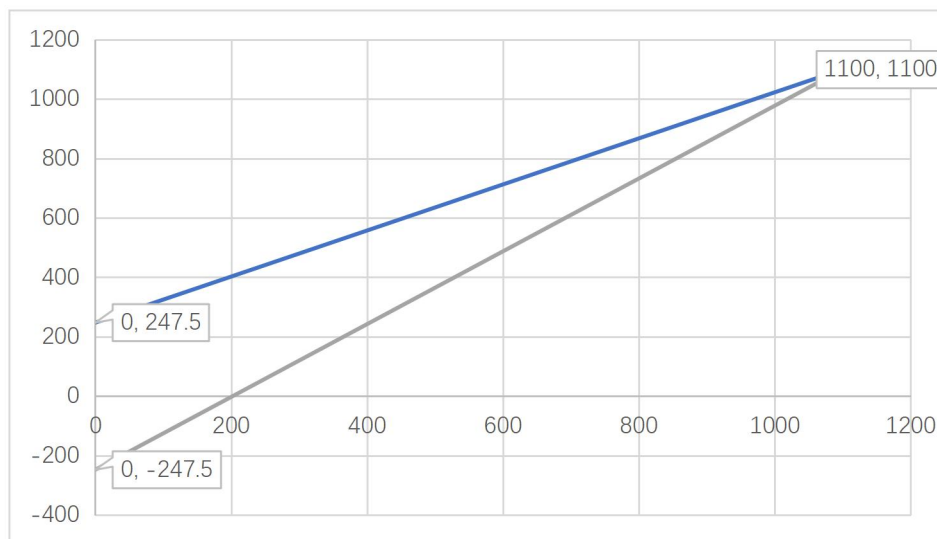
$$\sigma_{mean} = \frac{\sigma_{min} + \sigma_{max}}{2} = \frac{969 + 891}{2} = 930 N/mm^2$$

6) Effective fatigue safety factor of the standard bolts

According to the result above, $KT=2$, at the thread root,

$UTS \approx 1100 N/mm^2$, polished endurance limit = 0.45 UTS

$495 N/mm^2$ final endurance limit at thread root = $247.5 N/mm^2$



$$\text{Allowable range} = 2 \times EL \times \left(1 - \frac{MEAN}{UTS}\right)$$

$$= 2 \times 247.5 \times \left(1 - \frac{930}{1100}\right) = 76.5 N/mm^2$$

$$\text{Fatigue safety factor} = \frac{\text{Allowable range}}{\text{Actual range}} \approx \frac{76}{78} \approx 0.98$$

7) The modified bolt

The connecting rod bolt is a key fastener. It requires enough fatigue strength and anti-loosening performance. It must be strictly required when designing and manufacturing. The central idea of the optimization of the design of the connecting rod bolt is: Require the bolt to meet the certain performance conditions to maximize its reliability, or make the structure or component meet the best performance index, and its working reliability is not lower than a certain level.

Assuming the rod as the same above,

8) Bolt designs sketch

5. Fatigue safety factors for the standard engine crankshaft

The crankshaft is the most important component in the engine. It receives the force from the connecting rod and converts it into torque that is output through the crankshaft and drives other accessories on the engine to work. The crankshaft is subject to the combined effects of centrifugal force of the rotating mass, gas inertia force with periodic changes, and reciprocating inertia force, so that the crankshaft is subjected to bending and torsional loads. Therefore, the crankshaft is required to have enough strength and rigidity, and the surface of the journal

needs to be wear-resistant, uniform in work and good in balance.

According to the data from class. Through A, B.

$$I = \frac{bD^3}{12} mm^4 = \frac{42.5 \times 20^3}{12} mm^4 = 28333 mm^4$$

To get bending moment at A, Force = 15000N direction is up,

Bending moment at A is $15000 \times 12 = 180000 N \cdot mm$

TDC exhaust: inertial force up on crankpin

$$M_{rec} R w^2 \left(1 + \frac{R}{L}\right) + M_{rot} R w^2 \approx 12000 N$$

6000N is the engine pushing down at centre of main journal.

At B, there is only one force 15000N and its direction is up.

The distance from B to the centre of main bearing is 60mm

Bending moment at B = $15000 \times 60 = 900000 N \cdot mm$

Bending stress at B $= \frac{M}{I} Y = \frac{900000}{28333} \times 10 \approx 316 N/mm^2$ Shear

force = 15000N and direction is up, shear stress of B =

$$\frac{15000}{\pi \times \frac{60^2}{4}} N/mm^2 \approx 5.31 N/mm^2$$

$$\sigma_{res} = \sigma_{max} = \sqrt{316^2 + 2 \times 12^2} \approx 316.46 N/mm^2$$

$$\sigma_{min} \approx \frac{-12000}{30000} \times 316 \approx -126.4 N/mm^2$$

$$\text{Range of the stress} = 442.86 \text{ N/mm}^2$$

$$\sigma_{\text{mean}} = 95.03 \text{ N/mm}^2$$

$$\text{Final endurance at B } \frac{0.45 \times 750}{2} \sim 169 \text{ N/mm}^2$$

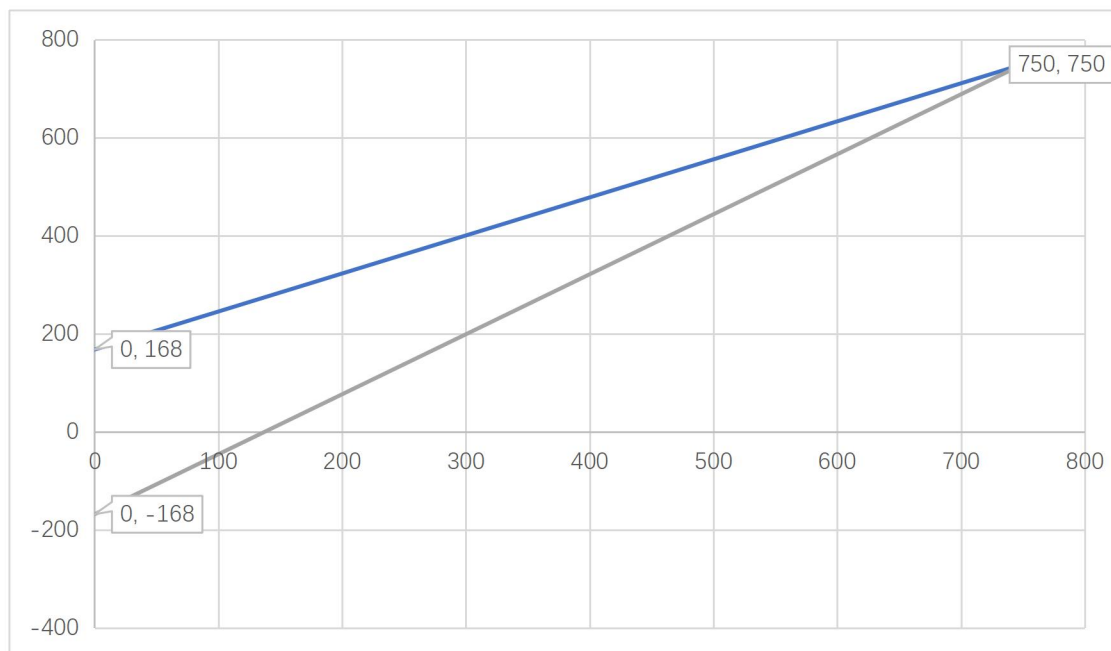
$$\text{Then the } \sigma_{\text{max}} \text{ at fillet} = 316.46 \text{ N/mm}^2$$

$$\text{Then the } \sigma_{\text{min}} \text{ at fillet} = -126.4 \text{ N/mm}^2$$

$$\text{Mean} = \frac{316.46 - 126.4}{2} - 800 = -704.97 \text{ N/mm}^2$$

$$\text{Polished } 0.45 \times 750 \quad \text{KT} = 2$$

$$\text{Final endurance limit } \frac{0.45 \times 750}{2} = 168 \text{ N/mm}^2$$



$$\text{Allowable range} = 2 \times 168 \times \frac{750 + 704.97}{750}$$

$$= 651.8 \text{ N/mm}^2$$

$$\begin{aligned} \text{Fatigue safety factor} &= \frac{\text{Allowable range}}{\text{Actual range}} = \frac{651.8}{442.86} \\ &= 1.47 \text{ N/mm}^2 \end{aligned}$$

6. Conclusion

In this report, many different fatigue safety factors such as gudgeon pin, the three critical rod sections, bolt, and the crankshaft are all calculated. By doing this assignment, author builds up his confidence of how to accurately and simply develop analytical techniques to allow design of the piston, rod, crankshaft assemblies, in which author also apply many calculations successfully to demonstrate ability to determine different fatigue safety factors.

7. Reference

Horiuchi. (1992). *United States Patent No. 5115725*.

Oslo. (2009, 02 24). Fatigue Safety Factors for Deepwater
Risers . *ASME 2002 21st International Conference on
Offshore Mechanics and Arctic Engineering* , pp. 349-355.