

Exam 1 (2019-20) Solutions/Mark Scheme

				Qu total
1a)	(i) $y = -2x^3 - 3x^{-2} + 4x^{1/2}$ $\frac{dy}{dx} = 6x^2 + 6x^{-3} + 2x^{-1/2}$ (ii) $y = (3-7x)^5$ Let $u = 3-7x$ so $y = u^5$ $\frac{du}{dx} = -7$ $\frac{dy}{du} = 5u^4$ $\frac{dy}{dx} = -7 \times 5u^4$ $= -35(3-7x)^4$ (iii) $y = \frac{4x-3}{2+5x}$ Let $u = 4x-3$ $v = 2+5x$ $\frac{du}{dx} = 4$ $\frac{dv}{dx} = 5$ $\frac{dy}{dx} = \frac{(2+5x)(4) - (4x-3)(5)}{(2+5x)^2}$ $= \frac{8+20x-20x+15}{(2+5x)^2}$ $= \frac{23}{(2+5x)^2}$ (iv) $y = 3x^2 \sin x$ Let $u = 3x^2$ $v = \sin x$ $\frac{du}{dx} = 6x$ $\frac{dv}{dx} = \cos x$ $\frac{dy}{dx} = (3x^2)(\cos x) + (\sin x)(6x)$ $= 3x^2 \cos x + 6x \sin x$			

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1b)	(i) $y = 3x^3 - 8x^2 - 4x + 8$ $\frac{dy}{dx} = 9x^2 - 16x - 4$ for turning point $\frac{dy}{dx} = 0$ $0 = 9x^2 - 16x - 4$ $x = \frac{16 \pm \sqrt{(16)^2 - 4(9)(-4)}}{2(9)}$ $= \frac{16 \pm \sqrt{256 + 144}}{18}$ $= 2 \text{ or } -0.22$ If $x = 2, y = -8$ If $x = -0.22, y = 8.46$ (ii) $\frac{d^2y}{dx^2} = 18x - 16$ where $x = 2, \frac{d^2y}{dx^2} = +ve$ so min where $x = -0.22, \frac{d^2y}{dx^2} = -ve$ so max (iii) Sketch.			

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2a(i)	$S = 4t^{1/2} + 4t^{-3} - 5t^3$ $\frac{dS}{dt} = 2t^{-1/2} - 12t^{-4} - 15t^2$			
(ii)	$S = (4+3t)^5$ Let $u = 4+3t$ so $S = u^5$ $\frac{du}{dt} = 3 \quad \frac{dS}{du} = 5u^4$ $\frac{dS}{dt} = 3 \times 5u^4$ $= 15(4+3t)^4$			
(iii)	$S = \frac{3t-2}{4t+5}$ Let $u = 3t-2$ $v = 4t+5$ $\frac{du}{dt} = 3 \quad \frac{dv}{dt} = 4$ $\frac{dS}{dt} = \frac{(4t+5)(3) - (3t-2)(4)}{(4t+5)^2}$ $= \frac{12t+15-12t+8}{(4t+5)^2}$ $= \frac{23}{(4t+5)^2}$			
(iv)	$S = t^3 \cos t$ Let $u = t^3$ $v = \cos t$ $\frac{du}{dt} = 3t^2 \quad \frac{dv}{dt} = -\sin t$ $\frac{dS}{dt} = (t^3)(-\sin t) + (\cos t)(3t^2)$ $= -t^3 \sin t + 3t^2 \cos t$			

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2b)	(i) $S = 3t^3 - 10t^2 + 8t + 11$ $V = \frac{ds}{dt} = 9t^2 - 20t + 8$ When $t = 2$ $V = 9(2)^2 - 20(2) + 8$ $= 4 \text{ m/s}$ (ii) $0 = 9t^2 - 20t + 8$ $t = \frac{20 \pm \sqrt{(-20)^2 - 4(9)(8)}}{2(9)}$ $= \frac{20 \pm \sqrt{400 - 288}}{18}$ $= 1.7 \text{ or } 0.52$ (iii) $a = \frac{d^2s}{dt^2} = 18t - 20$ When $a = 12$ $12 = 18t - 20$ $32 = 18t$ $t = 1.78 \text{ s}$			

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3a(i)	$\int (6 - 4 \sin 3x) dx = 6x + \frac{4}{3} \cos 3x + C$		
(ii)	$\int_2^4 (4 - 3x + 5x^2) dx$ $= \left[4x - \frac{3x^2}{2} + \frac{5x^3}{3} \right]_2^4$ $= \left[4(4) - \frac{3(4)^2}{2} + \frac{5(4)^3}{3} \right] - \left[4(2) - \frac{3(2)^2}{2} + \frac{5(2)^3}{3} \right]$ $= 98\frac{2}{3} - 15\frac{1}{3}$ $= 83\frac{1}{3}$		
(iii)	$\int_0^{\pi/6} (4 \cos 2x + \sin 4x) dx$ $= \left[2 \sin 2x - \frac{1}{4} \cos 4x \right]_0^{\pi/6}$ $= \left[2 \sin(\pi/3) - \frac{1}{4} \cos(2\pi/3) \right] - \left[2 \sin 0 - \frac{1}{4} \cos 0 \right]$ $= 1.8571 - (-0.25)$ $= 2.107$		
b (i)	$\int_0^2 5x^3 dx = \left[\frac{5x^4}{4} \right]_0^2 = \left[\frac{5(2)^4}{4} \right] - 0$ $= 20$		
(ii)	$\int_0^2 \pi (5x^3)^2 dx = \pi \int_0^2 25x^6 dx$ $= \pi \left[\frac{25x^7}{7} \right]_0^2$ $= \pi \left[\frac{25(2)^7}{7} \right] - 0 = 457\pi$ or 1436		

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4a.	$\int (2x+3)2\cos x \, dx$ $\text{Let } u=2x+3 \quad \frac{dv}{dx}=2\cos x$ $\frac{du}{dx}=2 \quad v=2\sin x$ $\int (2x+3)2\cos x \, dx = (2x+3)(2\sin x) - \int 2\sin x \cdot 2 \, dx$ $= (4x+6)\sin x + 4\cos x + C$			
4b.	$\int (5\cos 2x - 6\sin 3x) \, dx$ $= \frac{5}{2}\sin 2x + 2\cos 3x + C$			
(ii)	$\int_3^6 (2-3x+6x^2) \, dx$ $= \left[2x - \frac{3x^2}{2} + 2x^3 \right]_3^6$ $= \left[2(6) - \frac{3(6)^2}{2} + 2(6)^3 \right] - \left[2(3) - \frac{3(3)^2}{2} + 2(3)^3 \right]$ $= 390 - 46.5 = 343.5$			
(iii)	$\int_0^4 (4e^{2x} + 4x^{-3}) \, dx$ $= \left[2e^{2x} - 2x^{-2} \right]_0^4$ $= \left[2e^8 - 2(4)^{-2} \right] - \left[2e^0 \right]$ $= 5961.79 - 2$ $= 5959.79$			

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5a)	$\frac{1}{2t(t-1)} = \frac{A}{2t} + \frac{B}{t-1}$ $1 = A(t-1) + B(2t)$ put $t=1$ $1 = A(0) + B(2)$ $1 = 2B \rightarrow B = \frac{1}{2}$ put $t=0$ $1 = A(-1) + B(0)$ $A = -1$ $\int_3^4 \left(\frac{1}{2(t-1)} - \frac{1}{2t} \right) dt$ $= \left[0.5 \ln(t-1) - 0.5 \ln t \right]_3^4$ $= [0.5 \ln 3 - 0.5 \ln 4] - [0.5 \ln 2 - 0.5 \ln 3]$ $= -0.1438 - (-0.2027)$ $= 0.0589$																
b)	<table><tr><td>x</td><td>3</td><td>3.25</td><td>3.5</td><td>3.75</td><td>4</td></tr><tr><td>y</td><td>0.08333</td><td>0.06838</td><td>0.05714</td><td>0.04848</td><td>0.04167</td></tr></table> $h = \frac{4-3}{4} = 0.25$ $\int_3^4 \approx \frac{1}{3}(0.25) \left[(0.08333 + 0.04167) + 4(0.06838 + 0.04848) + 2(0.05714) \right]$ $= \frac{1}{3}(0.25)(0.70672) = 0.0589$ Error = 0%					x	3	3.25	3.5	3.75	4	y	0.08333	0.06838	0.05714	0.04848	0.04167
x	3	3.25	3.5	3.75	4												
y	0.08333	0.06838	0.05714	0.04848	0.04167												