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ULTRASONIC ATTENUATION COEFFICIENT DETERMINATION AND PROBABILITY OF SIZING ANALYSIS

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M.Sc Automotive Engineering
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Abstract

In this coursework I have performed two different tasks.

(1) The first task is to determine attenuation coefficient (α) by method of time of flight using ultrasonic NDT method. I have two different types of material samples, one of them is mild steel rectangular block and second is brake disc. The initiative task is to measure thickness of these two samples using vernier callipers. Silverwing D-Scan 705 high performance ultrasonic flaw detector is used for scanning the material. To calculate the attenuation coefficient, two consecutive values of 2 echo rates have been used. The attenuation coefficient is determined by averaging the values obtained by performing some experiments on both the samples.

(2) The second task I have 21 flaw true defect size in the wall thickness and measurements for these 21 defects which are measured by 10 different operators using ultrasonic thickness gauging procedure. In this part of assignment, through linear regression and statistical analysis method I assessed the sizing accuracy of individual technician and equipment. I analyse the measurements of all individual technician by plotting the error normal distribution graph using mean of error and standard deviation of error and compare this result to estimate that which operator or technician measured near the true values of defect.

PART – 1: ULTRASONIC ATTENUATION COEFFICIENT

1. Apparatus

1 ultrasonic zero-degree 5MHz probe

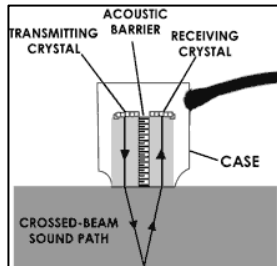


Figure 1: Zero-degree contact type dual element transducer ultrasonic probe

Ultrasonic zero-degree probe which frequency is 5 MHz is shown in figure 1. This probe is contact dual element transducer. As the name suggests, it is used in direct contact with sample surface. A dual element transducer contains two different elements in single housing. In single housing, one element transduces, and another element receives the ultrasonic wave. This dual element transducer has no delay. This probe transduces longitudinal or compression wave propagation in the material.

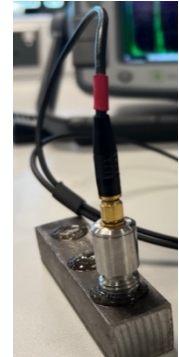


Figure 2: Probe used in experiment

Ultrasonic flaw detector

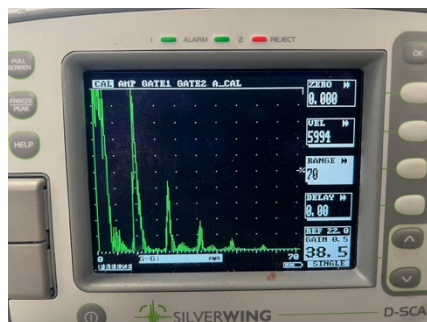


Figure 3: Silverwing D-Scan 705 ultrasonic flaw detector

A silverwing D-Scan 705 flaw detector is performed as narrow band amplifier. The features of this detector like time gain and delay are helpful to get correct measurement and ultimately correct calculation. This detector displays the output of the backwall signal in A-Scan type display.

Schematic set up of apparatus for experiment

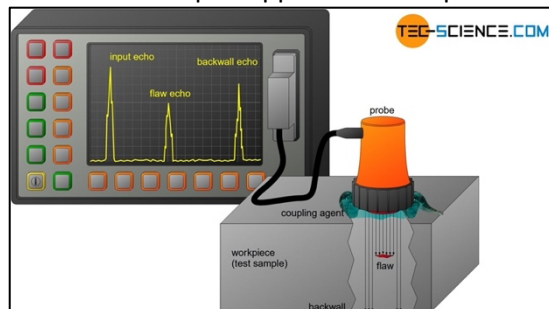


Figure 4: Schematic set up diagram of the apparatus for the experiment

The schematic set up is showing in figure 4. In this experiment, as a coupling agent I use gel which is work as ultrasonic gel. The function of this coupling agent is to prevent the probe contact surface from friction and wearing. This coupling agent is not considered as delay. Experiment setup in lab is shown in the figure 5 and 6.

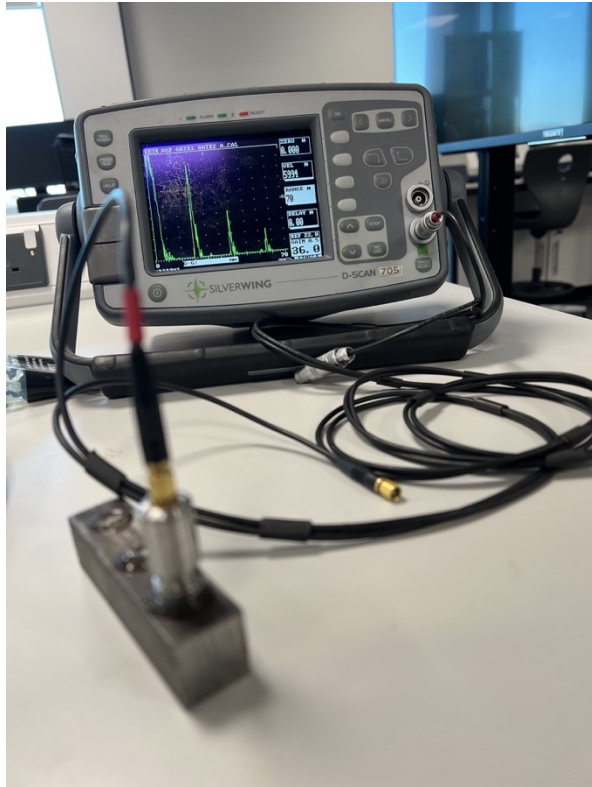


Figure 5: Apparatus connections for experiment for block



Figure 6: Apparatus connections for experiment for disc brake

2. Introduction, method, and procedure

When the sound wave travel through the medium or material, its intensity decreased with respect to distance. The main two reason for diminishing the intensity of wave is scattering and absorption of sound wave energy into material. This phenomenon of diminishing energy into material or medium is called Attenuation. The attenuation is generally proportional to the square of the sound wave frequency.

The amplitude change of the decaying compression wave can be given by below equation.

$$A = A_0 e^{-\alpha z} \dots\dots\dots(1)$$

Where;

A_0 : is initial (unattenuated) amplitude of sound wave

α : attenuation coefficient (Np/m)

z : travelled distance in the material

Np (Neper) is logarithmic dimensionless quantity, and it can be converted into the decibels by dividing it by 0.1151.

The decibel is logarithmic unit that describes ratio of two measurements. The difference between two measurements X_1 and X_2 is describe in decibels as

$$\Delta X \text{ (dB)} = 10 \log \left(\frac{X_2}{X_1} \right) \dots\dots\dots(2)$$

Since the intensity of sound wave is proportional to the square of pressure amplitude, so from equation (1) and (2) and distance travel by sound t , the attenuation coefficient is described as

$$\alpha \text{ (dB)} = \frac{10}{t} \log \frac{P_2^2}{P_1^2} = \frac{20}{t} \log \frac{P_2}{P_1} = \frac{20}{t} \log \frac{V_2}{V_1} \dots\dots\dots(3)$$

α = the attenuation coefficient between two measurements

V_2 and V_1 = are the two different transducer output voltage (or readings)

This calculation produces the attenuation coefficient in dB/unit time. Then knowing the velocity of the sound wave, it can be converted into dB/length. But the detector used in this experiment has converted time to the distance so here no need to convert twice.

In this experiment of measuring the ultrasonic attenuation coefficient, I first measure thickness of the mild steel rectangular block, which average thickness is measured 18.9 mm using vernier calliper. I took 4 measurements for measuring the thickness of the block as shown in the figure 7. Then I measured the thickness of the disc brake rotor, which average thickness through the rotor is 12.3 mm as shown in the figure 8. At last, I measured the thickness of the brake hub which average thickness is measured 0.83 mm as shown in figure 9.

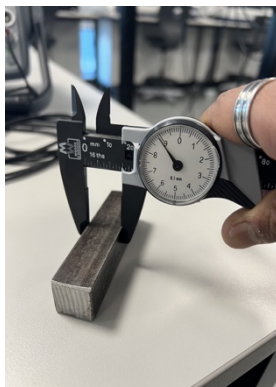


Figure 7: Thickness measurement of mild steel block



Figure 8: Thickness measurement of brake rotor



Figure 9: Thickness measurement of brake hub

After taking thickness measurements, set the velocity of the flaw detector to 5994 m/s which is almost same to the real value of 5960 m/s. The range in the flaw detector is set to the 70, that means screen can display the backwall signals until wave will travel distance of 70 mm. Because within this range I got minimum 3 echoes in full screen and 3 echoes are sufficient to calculate the α and here the probe used for the experiment has no delay so the delay in the detector also set to the zero. The gain value is set up depends on the sample and the output signal on the detector screen. Gain which is also called amplification factor is the extent to which a device increase the strength of the signal. So observer can observe the output signal accurately. But, eventually, the attenuation coefficient is ratio of 2 output signal readings on same screen. So it is not concern what the gain or amplification factor is set to be.

3. Measurements and calculations

The measurement that I required for the calculation of α (Attenuation coefficient) is height of the output signal of the detector. To get accurate value of echo height the gain is set to value 36 for mild steel block. And for the brake rotor and brake hub the gain values are set to 42 and 38.5 respectively as shown in the figure 10, 11 and 12.

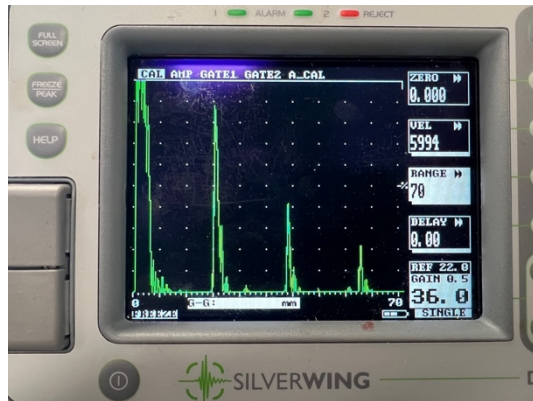


Figure 10: Output signal for mild steel rectangular block

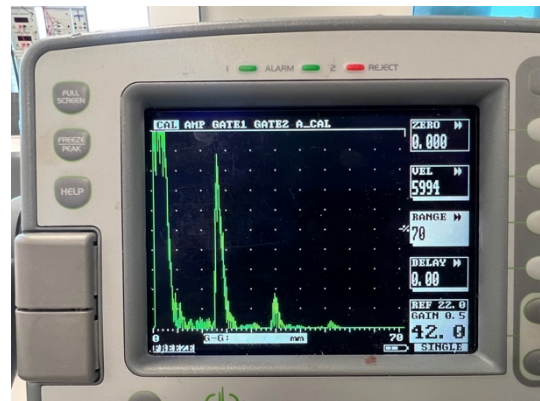


Figure 11: Output signal for disc brake rotor

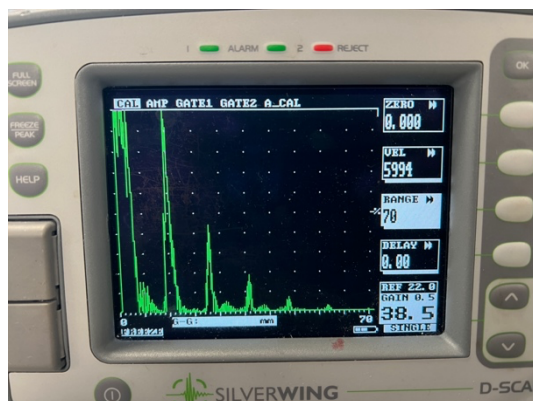


Figure 12: Output signal for disc brake hub

For measure the value of the signal, the peak position of each echo is measured. To find position of the echo peak, count the dot. If the position of the echo is between two dots, then it should be estimating the position of the peak. For instance, as shown in figure 10 that is one of the measurements of the rectangular block, the first echo position is estimated by 8.9. A height of the first echo covers the 8 full dot and does not completely reach to the 9th dot. So here I estimate the nearest value of the position.

The number of echoes which are clearly seen by naked eyes for block, brake rotor and brake hub are 3, 2 and 4 respectively. The measurement of echo peak position for block, brake rotor and brake hub 3 are shown in below tables 1, 2 and 3 respectively.

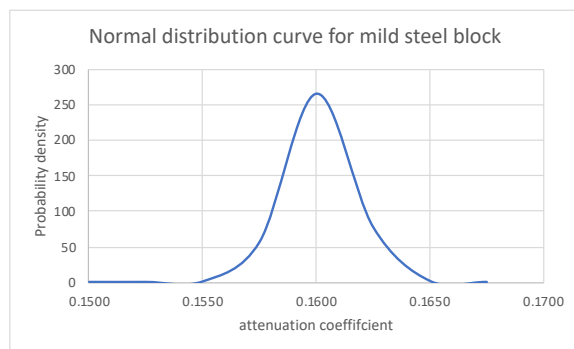
4. Calculations

thickness	18.9 mm	20/2t	20/4t			
gain	36.0000	0.5291	0.2646	α By ratio of echoes		
position	E1	E2	E3	e2/e1	e3/e2	e3/e1
1.0000	8.8000	4.4000	2.2000	0.1593	0.1593	0.1593
2.0000	9.2000	4.6000	2.5000	0.1593	0.1401	0.1497
3.0000	8.9000	4.4000	2.2000	0.1619	0.1593	0.1606
		Mean		0.1601	0.1529	0.1565
		Std. Dev.		0.0015	0.0111	0.0059

Table 1: Echo position measurement values for mild steel block

Table 1 data is for mild steel block, which positions are different from each other. Position 1 is near the left-hand corner of the block, position 2 is at the middle of the block, and position 3 is near right hand side corner of the block. Measurements at corner are taken far from the edge of the block. For mild steel block, I got 3 echoes for every position. Here, for ratio E2/E1 and E3/E2, the distance travels by sound wave are 2 times thickness of the block. And for ratio of echo E3/E1, the distance will be 4 times thickness.

Now calculate α dB/mm for all measurements from equation (3). Let's consider α that is calculated from ratio of echo E2 and E1. The standard deviation for this ratio minimum out of 3 ratios. if I calculate mean of the α dB/mm for the mild steel block for E2/E1 ratio, it is 0.1601 and std. deviation is 0.0015. Here negative values indicate that intensity of the sound wave is diminishing or decrease the intensity. So here I consider only positive number of attenuation coefficient.



Graph 1: Normal distribution curve for block

From normal distribution, the probability of α dB/mm can be found is between 0.1572 and 0.1631 is 95%. This range is very narrow and if I consider 2 digits value then the only one value of α can be found that is 0.16 dB/mm. That means in 1 mm penetration of sound wave in block, 0.16 dB energy will diminish. So after first backwall, 6.048 dB intensity will decrease from 5MHz of US wave.

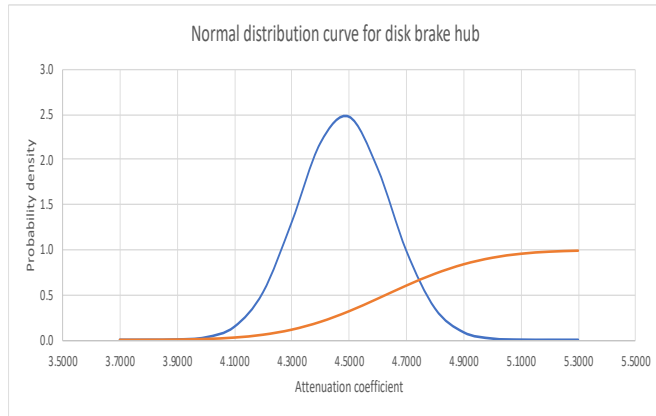
thickness	12.3 mm	20/2t	
		0.81	α
position	E1	E2	E2/E1
1	8.8	1.95	0.53
2	6.9	1.4	0.56
3	8.1	1.7	0.55
4	7.1	1.5	0.55
		α mean	0.55

Table 2: Echo position measurement values for disc brake rotor

Table 2 is for the disc brake rotor. For rotor I got only 2 echoes for each position. The average value of α is 0.55 dB/mm for brake rotor.

thickness	0.83 mm	20/2t	20/4t	20/6t						
		12.04	6.02	4.016	α By ratio of echoes					
position	E1	E2	E3	E4	e2/e1	e3/e2	e4/e3	e3/e1	e4/e1	e4/e2
1.0000	9.4	4.1	1.7	0.6	4.3415	4.6064	5.4494	4.4740	4.7991	5.0279
2.0000	8.2	3.6	1.5	0.5	4.3074	4.5809	5.7484	4.4441	4.8789	5.1647
3.0000	8.0	3.8	1.7	0.7	3.8953	4.2089	4.6428	4.0521	4.2490	4.4258
4.0000	10.0	4.4	1.8	0.7	4.2958	4.5335	5.0852	4.4146	4.6382	4.8094
			Mean		4.2100	4.4824	5.2315	4.3462	4.6413	4.8569
			Std. Dev.		0.1825	0.1601	0.4131	0.1711	0.2425	0.2793

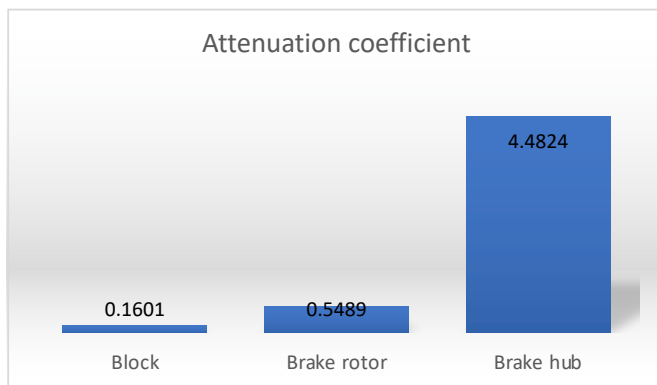
Table 3: Echo position measurement values for disc brake hub



Graph 2: Normal distribution curve for brake hub

Table 3 is for brake hub. If I consider E3/E2 for calculation of α because the standard deviation for this ratio is minimum out of others. For this ratio of echoes E3/E2, the mean and std dev values are 4.4824 and 0.1601 respectively. From normal distribution, the probability of α can be found between 4.1702 and 4.7946 is 95%. If I consider range of α in between 4.3223 and 4.6425 is 68% (1SD). For sake of comparison between α of these 3 different samples let's consider α of brake rotor is 4.4824 dB/mm.

5. Conclusion



From the comparison of value of α for block, brake rotor and brake hub, the α brake hub is significantly higher than the α block and α brake rotor. As mentioned before that attenuation is dissipation of energy into the medium. So, if attenuation coefficient value is as higher as the energy will dissipate more into the material. α of block is minimum out of 3 sample so I can say that sound wave will penetrate in the

mild steel without losing much intensity. Even in the brake rotor, energy will penetrate without losing less energy compared to brake hub. The α for brake hub is maximum out of 3 samples. So, energy will diminish more into the hub as compared to the block and brake rotor. This is due to grain structure of hub, method of manufacturing and heat treatment applied on the hub during the manufacturing. I can say that for the brake hub, the material would have been less homogeneous as compared to rotor. The sound wave would have scattered more into the hub.

PART – 2: WALL DEFECT SIZING ACCURACY ANALYSIS

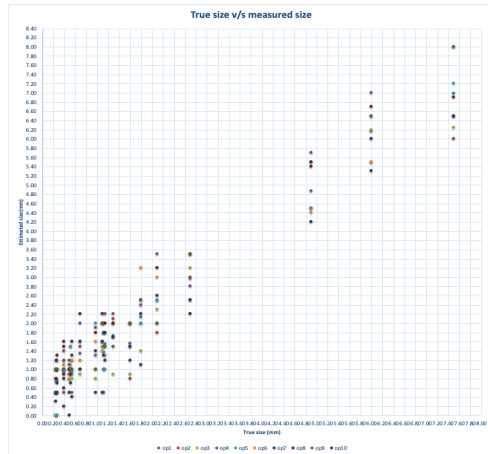
1. Introduction and Method

The Non-Destructive Testing (NDT) is the testing and analysing method to evaluate the material, component, structure, or system in the industry without fracturing or breaking the material. NDT is use for detection of flaw in the material or components. The detection of the flaw and the sizing accuracy is described by the Probability of detection. POD is a means of describing how well an inspection can be detected the required data. The probability of detection will lie somewhere between 0 and 1. 0 means no probability of detection of flaw and 1 state no chance of detecting no flaw. By assigning POD values to a procedure, it is possible to compare the effectiveness of different inspection techniques. Alternatively, POD values can be used as a target which procedures have to be shown to be able to achieve.

In this coursework, 21 flaws are found which are measured by 10 technicians with ultrasonic wall thickness method. Table (1) shows actual defect size as well as size measured by all technicians.

Defect	true size	Op1	Op2	Op3	Op4	Op5	Op6	Op7	Op8	Op9	Op10
A	0.25	0.50	0.50	0.00	1.00	0.00	0.50	0.50	0.80	1.00	0.30
B	0.27	0.73	1.20	0.50	1.00	0.00	1.00	0.50	0.80	1.00	1.20
C	0.28	0.76	1.00	1.20	1.00	0.00	0.50	0.50	1.30	1.00	0.70
D	0.40	0.91	0.50	1.00	1.20	1.00	1.00	0.60	1.50	1.00	0.80
E	0.40	0.98	1.20	1.00	1.40	1.00	1.10	0.20	1.60	1.00	0.90
F	0.50	0.79	0.90	1.00	1.10	1.00	0.80	0.00	1.00	1.20	0.50
G	0.52	1.01	1.50	1.10	0.90	1.50	1.00	1.00	0.90	1.00	0.70
H	0.55	0.96	1.20	0.80	0.50	0.90	1.20	1.30	1.60	1.00	0.40
I	0.70	1.35	1.50	0.90	2.00	1.60	1.20	1.60	2.20	1.00	1.00
J	0.98	1.31	1.00	0.80	1.90	2.00	1.60	0.50	1.80	1.00	1.40
K	1.12	1.37	0.50	1.00	1.00	1.80	1.80	1.80	2.00	1.50	1.30
L	1.15	1.56	1.80	1.30	2.20	1.00	1.50	1.80	2.00	1.50	1.20
M	1.10	1.60	2.00	1.40	2.00	1.60	1.50	0.50	2.20	2.00	1.60
N	1.30	1.73	2.10	0.90	2.20	2.00	2.00	1.70	2.00	1.50	1.70
O	1.60	1.56	0.80	0.90	2.00	2.00	1.50	1.20	2.00	2.00	1.50
P	1.80	2.14	2.00	1.40	2.40	2.00	3.20	2.20	2.50	2.50	1.10
Q	2.10	2.49	1.80	2.30	3.50	2.50	3.00	2.00	3.20	2.00	2.60
R	2.70	2.97	3.00	3.20	2.80	2.50	3.50	2.50	3.50	3.50	2.20
S	4.90	4.87	4.50	4.50	5.70	4.50	4.40	4.20	5.40	5.50	5.50
T	6.00	6.17	5.50	6.20	7.00	6.50	5.50	5.30	6.70	6.50	6.00
U	7.50	6.99	6.00	6.25	8.00	7.20	6.50	6.50	6.90	8.00	6.50

Table 2.1: True size of the flaw and measured size of the flaw by 10 technicians



Graph 2.1: Scattering plot of true size and measured size in mm

In this report I assess the sizing accuracy of the individual operator through linear regression and statistical analysis technique. The linear regression is a modelling technique for analysing data to make prediction. In simple linear regression, a bivariate model is built to predict a response variable from an explanatory variable. In this given data, the response variable is dependent variable is estimated size from operator and explanatory variable is true flow size.

The graph of true size of flaw v/s measured flaw size by all the technician is seen in the graph 2.1. here if the true size and the measured flaw size are same or say if aa the technician measured exact size of all the flaw, then all the points would fall exactly on a 45° line. But the graph

shows that there are some errors in the true flaw size and the measured size for all the technicians. Form the graph is also seen that between 0.2 mm to 2.6 mm, there are number of defects. There are only 3 major flaws in the sample which are more than 4 mm. The size between 3 mm to 4 mm, there is not any flaw can be found.

The first step to analysing the data is find the error of true size and the estimated size by all technicians. After the find the error, I find the mean of the error for all operators. The next step is to find standard deviation of error for individual operator. With help of std dev value we can find 95% probability of the error. The 95% probability value use to find upper and lower range of the error. That means $\text{Mean} \pm 2 \times \text{SD}$ (Standard Deviation) is the range. That is states that the probability of error lie between these values is 95%. That means values beyond these range can be found has only 5% chances. Now after knowing the range, mean and std dev, we can plot normal distribution curve and accumulated distribution curve for analysing.

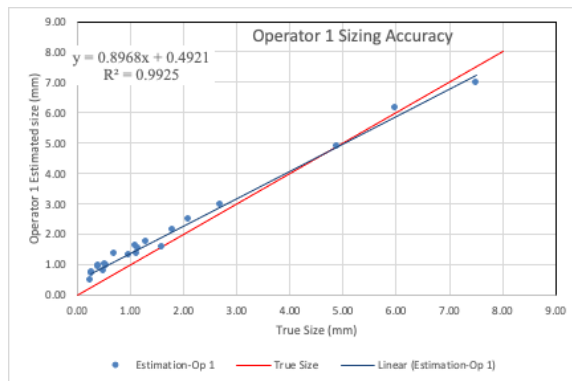
From the true size and the estimation of flaw graph can be plotted for linear regression. Linear regression is expressed by the equation $y = mx + c$ where m is the slop of the line and c is the Y-interface. The R^2 value of trendline is the known as the correlation index or coefficient of determination, is value between 0 and 1. It express the proportion of variance in the dependent variable explained by the independent variable. An R^2 value 0 means that knowing x does not help to predict y. As the R^2 value increase towards 1 the more accurate the function fits the data. A mean of error, standard deviation, 95% probability, slop, Y-interface, and value of R^2 for all individual operator estimation is shown in the table 2.1.

Operator	Error Mean	Std Dev	95% Probability	m	c	R^2
Op 1	-0.3146	0.2521	0.4917	0.8968	0.4921	0.9925
Op 2	-0.2086	0.6598	1.2866	0.7513	0.6364	0.9149
Op 3	-0.0729	0.5015	0.9779	0.8549	0.3225	0.942
Op 4	-0.6990	0.3874	0.7555	1.0021	0.6954	0.9622
Op 5	-0.3086	0.4495	0.8765	0.9229	0.4411	0.9476
Op 6	-0.3895	0.5273	1.0283	0.8056	0.724	0.9485
Op 7	-0.0133	0.5380	1.0491	0.8229	0.318	0.9381
Op 8	-0.7514	0.4159	0.8110	0.8727	0.9704	0.963
Op 9	-0.5038	0.2482	0.4840	0.9945	0.5132	0.9839
Op 10	-0.1419	0.4449	0.8676	0.8911	0.3292	0.9519

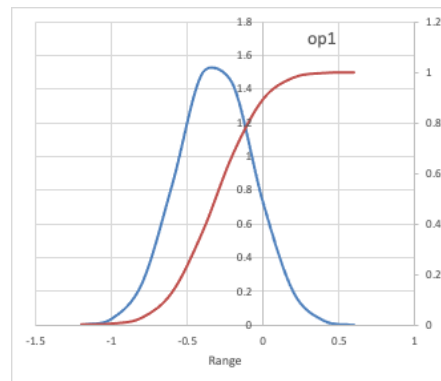
Table 2.2: Mean error, std dev, 95% probability, slop, y-interface, and R^2 value for individual operator

2. Results

Operator 1



Graph 2.2: Operator 1 linear regression



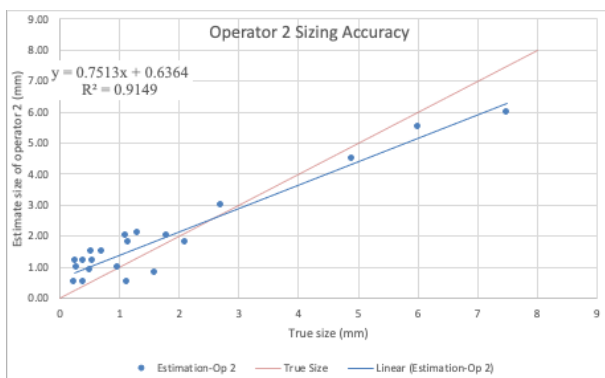
Graph 2.3: Operator 1 normal distribution

As shown in the graph 2.2, the values of the operator 1 is quit near to the linear trendline. That means the estimation which operator 1 has given is more accurate and trustworthy to predict the flaw size. Also, the determination coefficient value for operator 1 trendline has higher value which is almost near to the 1. But on the other hand, the slope of the trendline for operator 1 is not parallel the true size line. As here can be seen that the measurement of the op1 is overestimated.

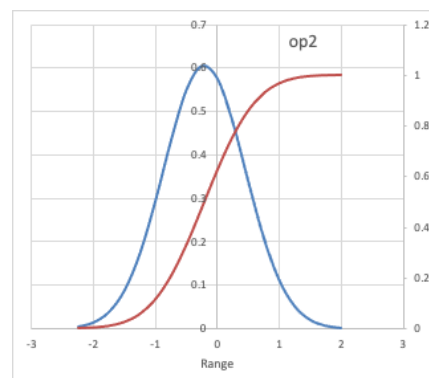
As shown in the graph 2.3, the normal distribution curve demonstrates that probability of error between -0.81 to +0.17 is 95% that means the probability of estimate will be true size $M \pm 2SD$ is 95%. Even the bell curve moves to the negative values which states that the mean of the error is high in negative values. If the mean of the error is 0 than the bell curve peak will be on the X=0 axis.

Finally, the op1 is more accurate about his estimation rather than other operators. Operator 1 measurements are almost over estimation so I can say that it is due to some instrumental delay or error.

Operator 2



Graph 2.4: Operator 2 linear regression

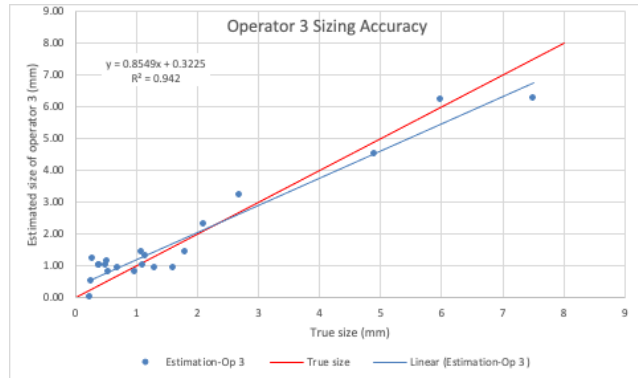


Graph 2.5: Operator 2 normal distribution

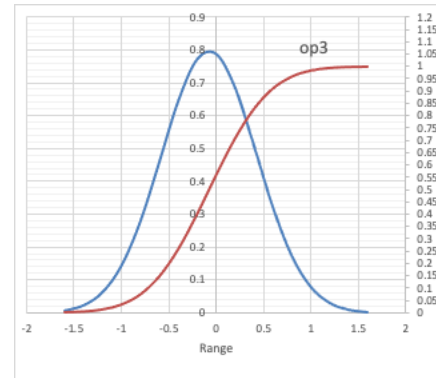
As shown in the graph 2.4 and 2.5, the operator 2 measure the flaw and his estimation is miscellaneously distributed over the trendline. His estimation is not accurate as operator 2 because his some of the measurements are over estimation while some of them are under estimation. Which

means op2 is not accurate about the measurement of flaw size. Even the correlation index value is smaller than operator 1. Normal distribution curve is also wide as compared to op1. If we compare the 95% probability than the range is -1.5 to 1.08 mm which is quite larger than op1.

Operator 3



Graph 2.5: Operator 3 linear regression

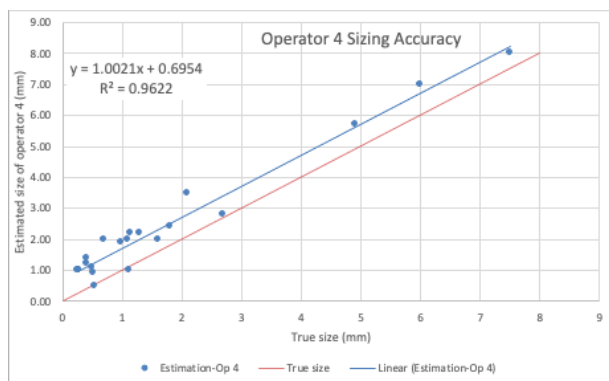


Graph 2.6: Operator normal distribution

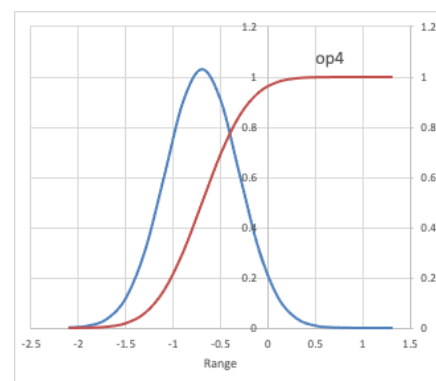
As shown in the graph 2.5 and 2.6, the operator 3 is not more accurate because the value of correlation index is smaller than the op1. The good thing for op3 is that the peak of the histogram curve lied on the X=0 axis that means the mean of the error is smaller than op 1 and 2.

If I talk about 95% error probability, then again it ranges lie in -1.05 to +0.95 which is also larger than op1.

Operator 4



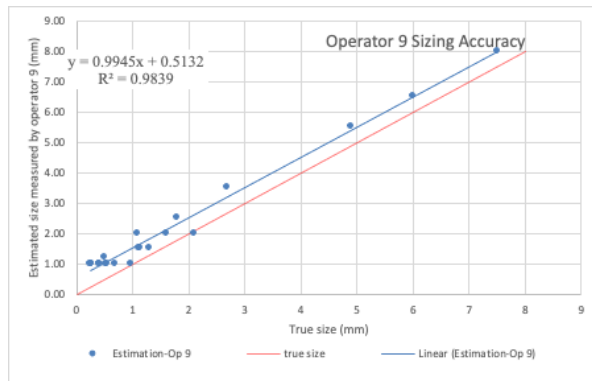
Graph 2.7: Operator 4 linear regression



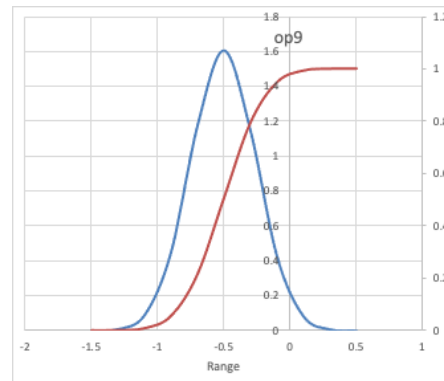
Graph 2.8: Operator 4 normal distribution

Graphs 2.7 and 2.8 show the results of the operator 4. From the linear regression graph the slope of the trendline is 1.0021 which is almost linear to the true size value line for the up to 9mm size of the flaw. And the Y-intersect of the trendline is 0.6954 which means the error between these two lines is 0.6954mm that can be explain by the instrumental error like setting the absolute position when starting the inspection. Next if I consider about correlation index which is 0.9622 which is also very near to the 1. The probability of 95% error lie between -1.45mm to 0.06mm. That means the error can be found between -1.45mm to +0.06 mm has 95% probability.

Operator 9



Graph 2.9: Operator 9 linear regression



Graph 2.10: Operator 9 Normal distribution

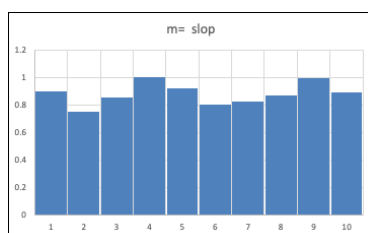
In graph 2.9 and 2.10, the error scattering points are more fitted with the trendline as like as op1. Furthermore, the value of R^2 is also very near to 1. Which means the operator has measured the flaw size quite accurate and trustworthy. The slope of the trendline is also nearly 1 and the Y-intersect value is 0.5132. From the Y-intersect I can assume that there are some equipment limitations, and or the operator has not set the absolute position of gauge.

If I talk about the 95% error estimation probability, the values lie between -0.99 to -0.02 mm. which means the range of the error is small and hence the prediction will be more within these range.

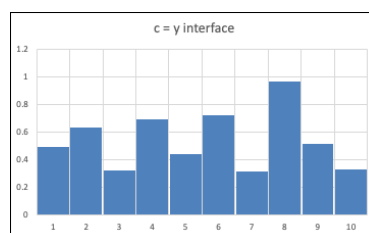
So, as per my perception the operator 9 has estimated very accurate and trustworthy about flaw size.

Other operators like 5,6,7,8, and 10 have no significant considerable outcomes so these graphs are not discussed. The graphs for these operators are mentioned in the appendix.

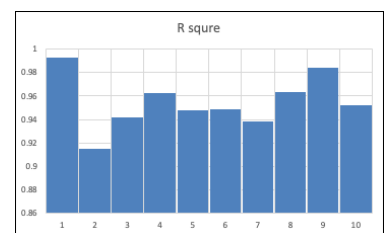
3. Conclusion



Graph 2.11: slope comparison

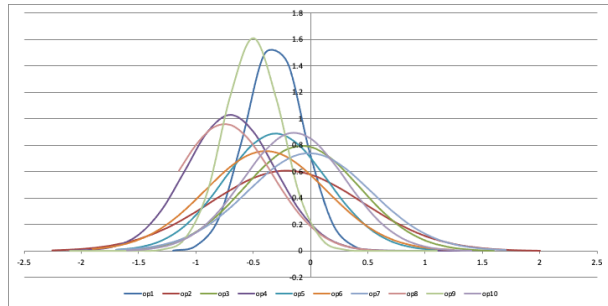


Graph 2.12: intersect comparison

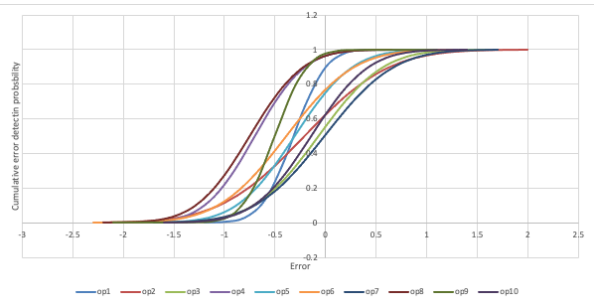


Graph 2.13: R^2 comparison

From the graph 2.11, measurement of operators 4 and 9 measure the flaw with same instrumental or equipment or maneuverer error. Because the linear estimation line (trendline) and true size line are almost parallel for this range of flaw (0 to 9 mm). From intersect graph 2.12, these values indicate the how much error will be there from starting of the inspection. Because if true size is zero then still the trendline shows some values which is the initial error. From R^2 graph, 2.13, it represents that the operator 1 and operator 9 data have maximum correlation between true size and estimation error. Op1 and op9 measured data are very accurately fitted in their prediction as compared to other operators.



Graph 2.14: Histogram curve comparison



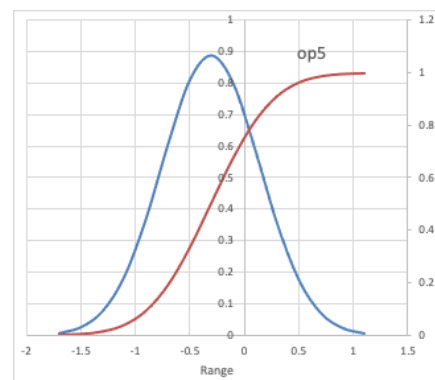
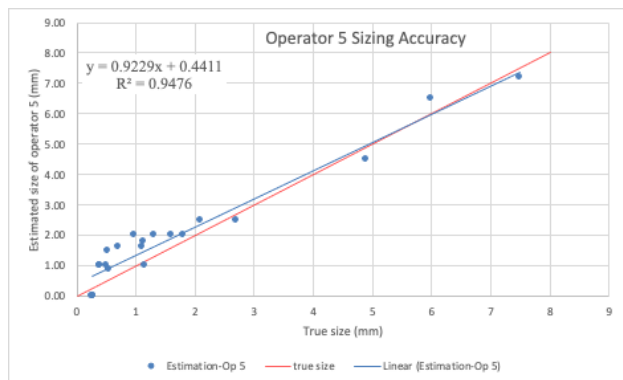
Graph 2.13: Cumulative probability curve comparison

From graph 2.14 and 2.15, operators 1 and 9 data have very narrow histogram curve which means the probability to found same range is quite more than other operators which data have wide histogram curve.

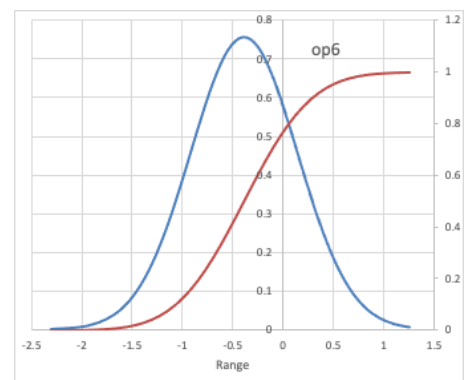
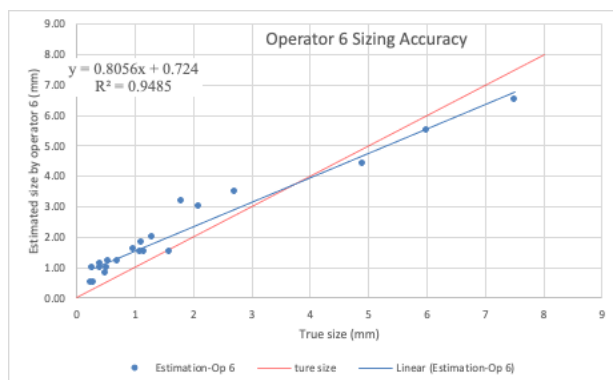
Finally, from my knowledge and analysis, my opinion about operator accuracy is that the operator 9 measured the flaw size very accurate and trustworthy than other operators.

4. Appendix

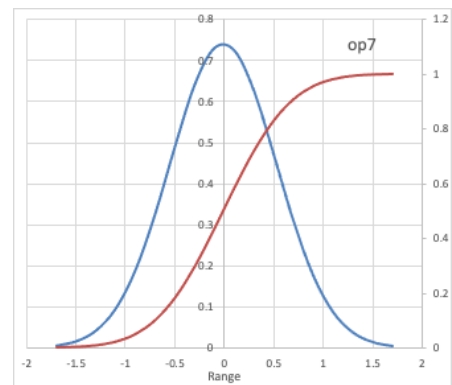
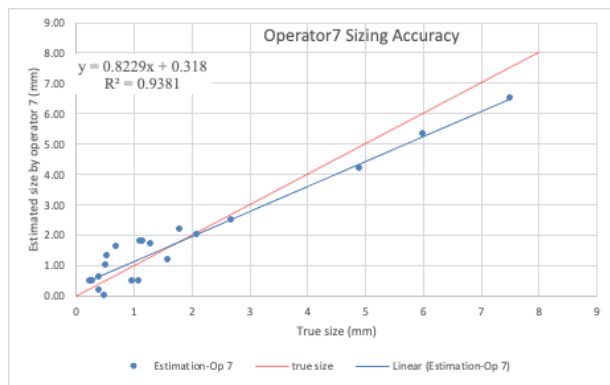
Operator 5



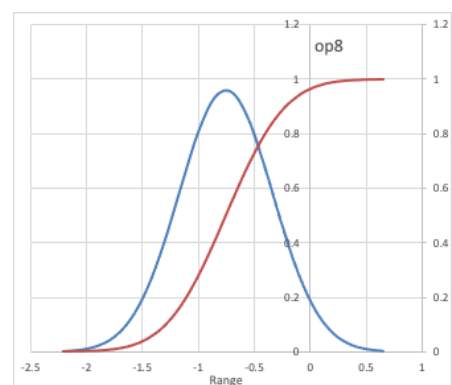
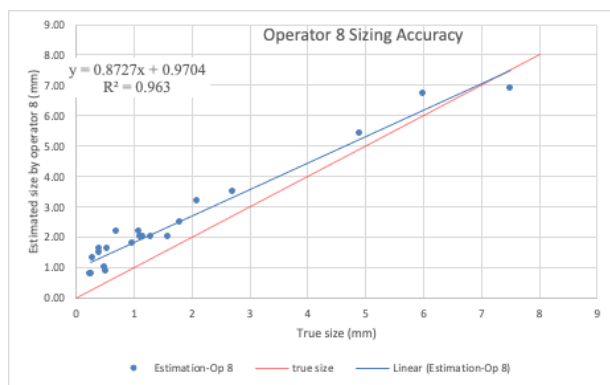
Operator 6



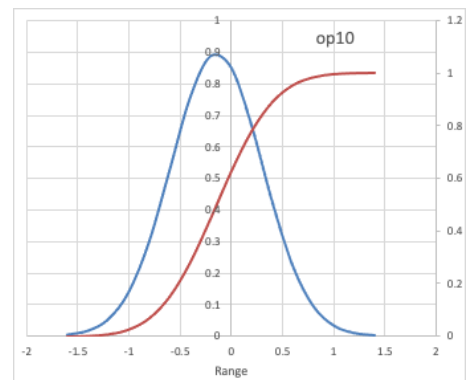
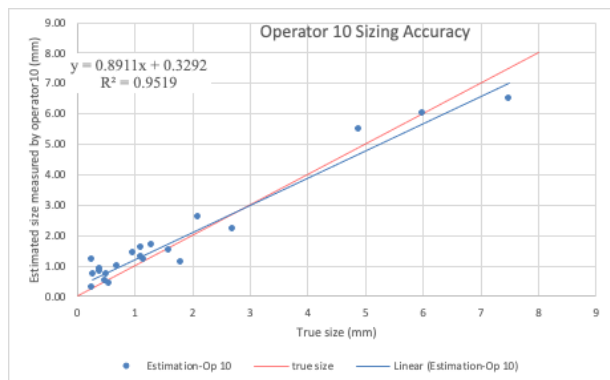
Operator 7



Operator 8



Operator 10



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