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Applied Mechanics

Assignment 1 – Complex static and dynamic systems

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Nomenclature

Symbol	Description	Unit
m	Mass	Kg
a	Acceleration	m/s^2
r	Radius	m
L	Length	m
θ	Angle of rotation	Radians
T_o	Torque output	Nm
T_i	Torque input	Nm
T_f	Fixing torque	Nm
g	Gravity	m/s^2
I	Moment of inertia	Kgm^2
α	Angular acceleration	$Rads/s^2$
σ_r	Radial stress	Nm^2
σ_c	Circumferential stress	Nm^2
MPa	Stress	MPa
τ	Tau – shearing stress	Nm^2
G θ	Angle of gyration	Radians
J	Second moment of area	m^4

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Q1)

a)

Balance is important because it reduces unnecessary strain acting on a system. Consider a see-saw; the distribution of weight is balanced when each mass of equal weight is placed the same distance apart from the central pivoting point. If one mass was closer than the other, the scale will tip and the heavier mass will fall.

Similarly, for a rotating shaft with masses out of balance, the shaft will tend to follow the greater mass. Thus, shifting the center of gravity and applying additional strain.

For there to be **dynamic balance**, the centrifugal forces acting on a shaft must equal to zero. For example, for a shaft with a single mass there must be an additional mass directly opposite with equal magnitude. This theorem is supported by Newton's third law of motion, for every action there is an equal and opposite reaction (Newton & Chittenden, 1850). This suggests that by adding a mass directly opposite the out of balance mass, the centrifugal forces acting on the shaft will become balanced.

Consider the tyres of a vehicle, when they get balanced at a garage they are placed onto a machine which detects the imbalances. The mechanic then applies counter weights to the inside of the wheel to align the balance.

Otherwise, the out of balance wheel is likely to cause vibration along the kinematic chain because it is rotating about a different axis to the other wheels. The effect this has on the car is that the steering wheel may shake (VanGelder, 2014).

For there to be **static balance** the resultant forces and the sum of the moments must equal zero. Static balance is when the center of gravity and rotation are aligned whilst an object is stationary. The center of gravity for off-center masses is located down the center of the diameter face. Hence, the construction of a vector diagram is about the center of the shaft. A vector diagram can be used to illustrate the angle between masses, the radius to each mass and the mass value.

b)

Table 1 – MRL table to show initial values

Plane	Mass (kg)	Radius (m)	MR (Kgm)	L (m)	MRL (Kgm ²)	θ
A	MA	0.1	0.1MA	0.5	0.005MA	Θ _A
B	0.6	0.12	0.072	1	0.00864	0
C	0.3	0.12	0.036	1.5	0.0648	120
D	MD	0.09	0.09MD	2	0.0162MD	Θ _D

MRL polygon

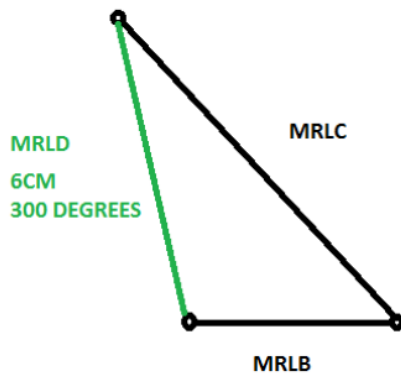


Figure 1 – MRL Polygon to find the value for D

By drawing a Mass-Radius-Length (MRL) polygon with the scale 1cm = 0.01Kgm² it was shown that with a length of 6cm and an angle of 300° that:

$$0.0162MD = 0.06$$

From which MD can be calculated:

$$MD = \frac{0.06}{0.0162}$$

$$MD = 3.7kg$$

Using this value the table can be completed further:

Table 2 – MRL table to show values for plane D

Plane	Mass (kg)	Radius (m)	MR (Kgm)	L (m)	MRL (Kgm ²)	θ
A	MA	0.1	0.1MA	0.5	0.005MA	Θ _A
B	0.6	0.12	0.072	1	0.00864	0
C	0.3	0.12	0.036	1.5	0.0648	120
D	3.7	0.09	0.333	2	0.06	300

In order to calculate the remaining values for the mass on plane A an Mass-Radius (MR) polygon is used using a scale of 1cm = 0.05kgm:

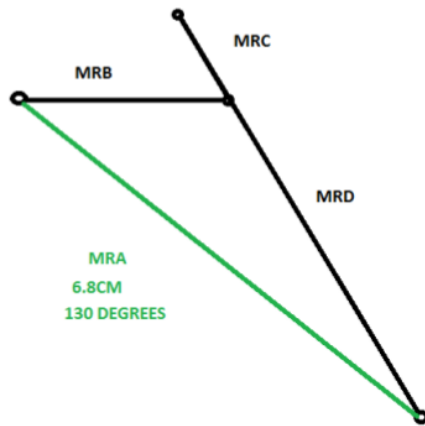


Figure 2 – MR Polygon to find the value for A
The value for MRA was 6.8cm at 130°.

From which MA is calculated by:

$$0.1MA = 0.34$$

$$MA = \frac{0.34}{0.1}$$

$$MA = 3.4kg$$

Therefore, the table can be completed:

Table 3 – MRL table to show values for plane A

Plane	Mass (kg)	Radius (m)	MR (Kgm)	L (m)	MRL (Kgm ²)	θ
A	3.4	0.1	0.34	0.5	0.017	130
B	0.6	0.12	0.072	1	0.00864	0
C	0.3	0.12	0.036	1.5	0.0648	120
D	3.7	0.09	0.333	2	0.06	300

Q2)

Task one

a)

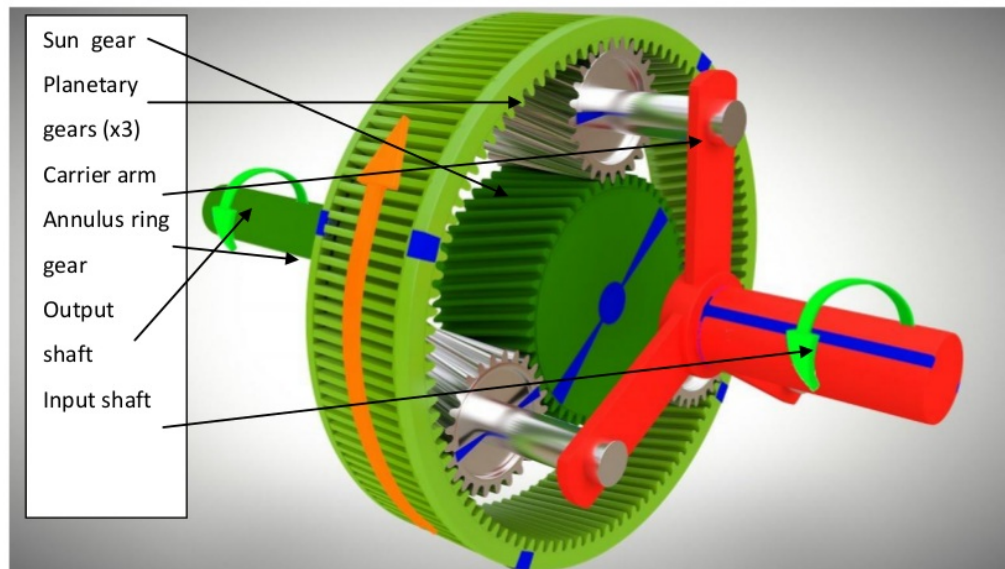


Figure 3 – Illustration to show the components that make up an epicyclic gear

b)

Figure 3 shows the arm carrier is the input and the annulus or sun is the output. Each of them operates in opposite directions. This allows the machine to work in direct drive or reverse. The gear ratio can be increased by allowing more movement in the train, or it can be reduced by fixing members. Reducing the train will increase torque and reduce speed.

Increasing the train will reduce torque and increase speed.

There are five basic principles that cover the outcomes of the epicyclic gear system (Tube, 2015):

- 1) When there is no reactionary and all components are free to rotate, the gearing is in neutral.
- 2) When the planet carrier is the output the annulus ring gear is locked meaning gear reduction.
- 3) When the planet carrier is the input the gearing is in overdrive.
- 4) When the planet carrier is the reactionary the gearing is in reverse.
- 5) When two members are locked the gearing is in direct drive.

c)

When the carrier arm is fixed, the sun and annulus ring gear rotate.

- Fixed frame assembly
- Input and output rotate in opposite directions.
- Gear ratio 1:10

When the sun gear is fixed, the annulus and carrier arm rotate.

- Input and output rotate in same direction.
- Gear ratio 1:2

When the annulus is fixed, the sun and carrier arm rotate.

- Input and output rotate in same direction.
- Gear ratio 1:12

The gear ratios are approximate and are dependant on drive direction (Desvaux, et al., 2017).

d) When there is no reactionary all gears are free to rotate. This is referred to as the law of neutral (Brejcha, 1973). 

e) Common applications for an epicyclic gearbox can be found in automatic transmission vehicles such as cars, trucks and tanks. In cars, for example, the gear stick shifts bands and clutches which add tension and apply the desired gearing.

Wind turbines also use epicyclic gearboxes for its high gear ratios allowing fast movement of the turbine compared to the output. Without the high gear ratio, the wind turbine is likely to suffer the effect of "yawing" as a result of the gyration of rotation (Rivkin, 2013). Whereby not enough or too much rotation is allowed.

f) Epicyclic gearboxes are good for high gear ratios whereby a fast input speed is achieving a slower output speed.

When the input and output rotate in opposite directions, the planets will follow the gear with the greater speed. Increasing the sun gear speed will slow down the output speed. When the input and output speed is equal, the planet carrier stops rotating.

Increasing the sun gear speed further beyond that of the ring gear will reverse the rotation of the planet carrier.

A differential is used to transmit power from one input to the other keeping a balance for steering. For example, each track of a tank has different inputs. This allows the tank to turn. As one track speeds up and the other slows down the tank will rotate to the side of the slowed-down track. When the track inputs are the same the tank will move forward.

The benefit of using an epicyclic gear train rather than a simple or compound gear train is that at any given moment the epicyclic gear train has more teeth in contact with the input and output gears. By increasing the contact between gears there is less chance of slippage, thus reducing wear as a result of evenly distributed stresses. Epicyclic gearboxes tend to be more efficient, allowing a greater torque output. The disadvantage of using an epicyclic gear train is that they are more complex and are more expensive to install and repair.

Task two

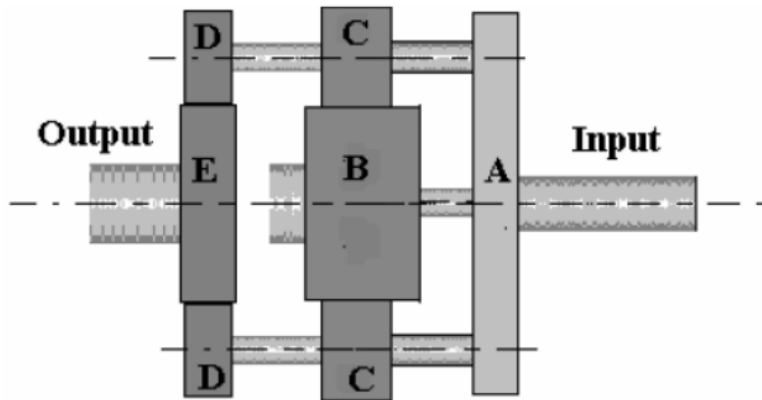


Figure 4 – Compound epicyclic gearbox

Table 4 – Q2 task two initial values

TA	?	TD	50 teeth
TB	50 teeth	TE	200 teeth
TC	200 teeth		

Using the formula:

$$T_A = T_B + 2T_C$$

$$T_A = 50 + 2(200)$$

$$T_A = 450$$

Table 5 – Calculating gear ratio

		A	B	C/D	E
1	Rotate gears +1 revolution	+1	+1	+1	+1
2	Rotate B, Fix the arm	0	$+1 \cdot \left(\frac{450}{50}\right)$	$-1 \cdot \left(\frac{200 \times 50}{200 \times 50}\right)$	$-1 \cdot \left(\frac{200}{50}\right)$
3	Add lines 1 & 2	1	9	-1	-4

Since B is fixed and C and D are rigidly connected, the gear ratio is 1:4.

Q3)

Gear reduction ratio = 30:1

Drum diameter = 0.8m

Motor torque = 104Nm

Motor efficiency = 85%

Moment of inertia of motor and gears = 0.04Kgm²

Moment of inertia of drum and cables on drum shaft = 25Kgm²

If the torque of the motor is 104 Nm and the gear reduction is 30:1:

$$\text{Gearbox torque} = 30 \times 104$$

$$\text{Gearbox torque} = 3120$$

$$\text{Output torque @ 85\% efficiency} = 3120 \times 0.85$$

$$\text{Output torque @ 85\% efficiency} = 2652$$

Since,

$$T_f = T_o - T_i$$

$$T_f = 2652 - 104$$

$$T_f = 2548 \text{ Nm (Fixing torque)}$$

$$\text{Total inertia} = 0.04 + 25 = 25.04 \text{ Kgm}^2$$

If

$$T = I \cdot \alpha$$

$$\alpha = \frac{T}{I}$$

Therefore,

$$\alpha = \frac{2652}{25.04}$$

$$\alpha = 105.9 \text{ rads/s}^2$$

Since,

$$T = I \cdot \alpha + mgr + T_f$$

$$2652 = (22.04 \times 105.9) + (500 \times 9.81 \times 0.4) + 500a \times 0.4 + 2548$$

$$2652 = (2334.036) + (19.62) + 200a + 2548$$

$$2652 = (2334.036) + (19.62) + 200a + 2548$$

$$2652 = 6844.036 + 200a$$

$$\frac{2652 - 6844.036}{200} = a$$

Therefore, the vertical acceleration of the load due to the applied torque is:

$$a = -20.96 \text{ m/s}^2$$

Q4)

a)

A thick cylinder has a wall thickness greater than 5% of its internal diameter. In contrast, a thin cylinder has a wall thickness less than 5% of its internal diameter. For example, a cylinder with an internal diameter of 50mm must have a wall thickness greater than 2.5mm to be a thick cylinder or less than 2.5mm for a thin cylinder. For both cylinder types we can assume that the hoop and longitudinal stresses are constant throughout the system.

A thick cylinder is better suited for systems where the internal stresses acting on the vessel are high. This includes the constant or cyclic pressures and the applied temperatures. Typical examples include the pipework supplying houses, industrial storage tanks and the hydraulic drums near hydraulic presses.

Thin walled cylinders are better suited for supplying low volume fluids where the stresses acting on the wall is low e.g. extraction air ducts.

b)

i)

A steel pipe of internal diameter 30mm and external diameter 120mm is subject to an internal pressure of 12MPa and an external pressure of 5MPa.

$$\sigma_r = a + \frac{b}{r^2}$$

$$\sigma_c = a - \frac{b}{r^2}$$

Converting pressure units from Pascals to Newton meters squared.

$$\text{If } 1 \text{ Pascal} = 1 \text{ Newton},$$

$$\text{Internal pressure} = -12 \text{ MNm}^2$$

$$\text{External pressure} = -5 \text{ MNm}^2$$

When,

$$1) \quad r = 0.015, \quad \sigma_r = -12 \text{ MNm}^2$$

$$2) \quad r = 0.06, \quad \sigma_r = -5 \text{ MNm}^2$$

Using the formula for σ_r :

$$1) \quad -12 \times 10^6 = a + \frac{b}{(0.015)^2}$$

$$2) \quad -5 \times 10^6 = a + \frac{b}{(0.06)^2}$$

From 2)

$$-5 \times 10^6 = a + \frac{b}{(0.06)^2}$$

$$-5 \times 10^6 = a + 277.7b$$

$$a = -5,000,277.7$$

Sub into 1)

$$12 \times 10^6 = -5,000,277.7 + 4444.4b$$

$$5,000,277.7 \quad \boxed{-1} \quad 2 \times 10^6 = 4444.4b$$

$$\frac{-6,999,722.3}{4444.4} = b$$

$$b = -1574.95$$

Therefore,

If $\sigma_c = a - \frac{b}{r^2}$

When, $r = 0.015$

$$\begin{aligned}\sigma_c &= -5,000,277.7 - \left(\frac{-1574.95}{0.015^2} \right) \\ \sigma_c &= -5,000,277.7 - (6,999,777.778) \\ \sigma_c &= 1,999,500.078 \\ \sigma_c &= 1.9MNm^2\end{aligned}$$

When, $r = 0.06$

$$\begin{aligned}\sigma_c &= -5,000,277.7 - \left(\frac{-1574.95}{0.06^2} \right) \\ \sigma_c &= -5,000,277.7 - (-437,486.1) \\ \sigma_c &= 5,437,763.8 \\ \sigma_c &= 5.4MNm^2\end{aligned}$$

ii)

Stress profile

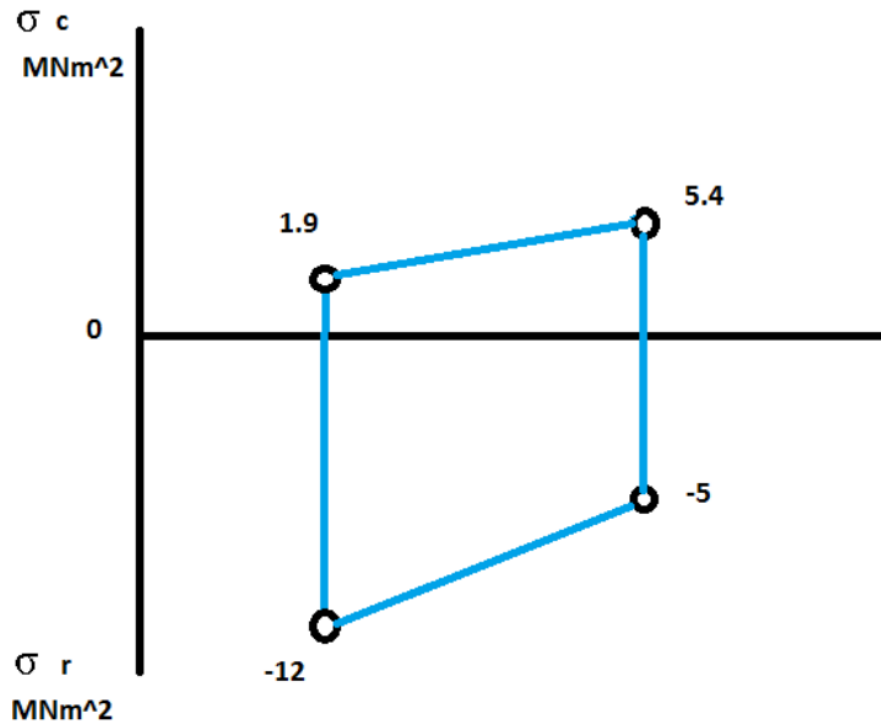


Figure 5 – Stress profile of radial and circumferential stresses acting on steel pipe

Table 6 – Stress profile values

Radius (m)	σ_r (MNm^2)	σ_c (MNm^2)
0.06	-5	5.4
0.015	-12	1.99

Q5)

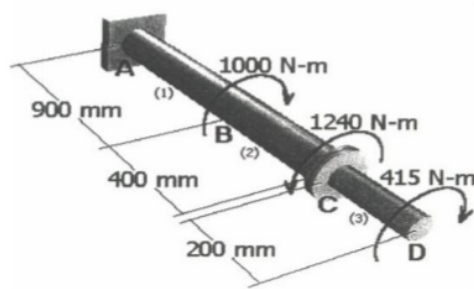


Figure 6 – Torsion diagram

i)

$$T_1 = -415 \text{ Nm}$$

$$T_2 = -415 + 1240 = 825 \text{ Nm}$$

$$T_3 = -415 + 1240 - 1000 = -175 \text{ Nm}$$

From the formula:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

Using:

$$\frac{T}{J} = \frac{\tau}{r}$$

Section 1

Since,

$$\tau_1 = \frac{T_1 \times r_1}{J_1}$$

Where,

$$J_1 = \frac{\pi D^4}{32}$$

$$J_1 = \frac{\pi \times 0.03^4}{32} = 7.95 \times 10^{-8} \text{ m}^4$$

$$\tau_1 = \frac{-415 \times 0.015}{7.95 \times 10^{-8}}$$

$$\tau_1 = -78.3 \text{ MNm}^2$$

Section 2

Where,

$$J_2 = \frac{\pi \times 0.045^4}{32} = 4 \times 10^{-7} \text{ m}^4$$

$$\tau_2 = \frac{825 \times 0.0225}{4 \times 10^{-7}} = 46.4 \text{ MNm}^2$$

Section 3

Where,

$$J_3 = \frac{\pi \times 0.045^4}{32} = 4 \times 10^{-7} m^4$$

$$\tau_3 = \frac{-175 \times 0.0225}{4 \times 10^{-7}} = -9.8 MNm^2$$

Therefore, the maximum shear stress is $-78.3 MNm^2$ in section one of the shaft. The minus is included to indicate the direction of torque in accordance to Ampere's circuital law (Rebeiro & Knight, 2018).

ii)

From

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\theta = \frac{T \cdot L}{J \cdot G}$$

Section 1

$$\theta_1 = \frac{-415 \times 0.2}{7.95 \times 10^{-7} \times 75 \times 10^9} = -0.00139 \text{ Radians}$$

Section 2

$$\theta_2 = \frac{825 \times 0.4}{4 \times 10^{-7} \times 75 \times 10^9} = 0.011 \text{ Radians}$$

Section 3

$$\theta_3 = \frac{-175 \times 0.9}{4 \times 10^{-7} \times 75 \times 10^9} = -0.00525 \text{ Radians}$$

iii)

$$\text{Total angle of twist at free end} = -0.00139 + 0.011 - 0.00525$$

$$\text{Total angle of twist from A to D} = 0.00436 \text{ Radians}$$

In degrees,

$$\frac{0.00436}{2\pi} \times 360 = 0.25^\circ$$

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