



HND/C PHASE TEST SPECIFICATION

Programme:	HND Automotive Engineering / Motorsport Engineering				
Unit:	Mechanical Principles				
Phase Test No:	1	Lecturer:	T Tudor	Internal Verifier:	A Summerfield

Time and Date:	2012	Duration:	90 mins	Contribution to Overall Unit Assessment (%):	25%
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Outcomes being Assessed (from Unit Syllabus):	
Number	Description
Learning Outcome 1:	Investigate complex loading systems
Learning Outcome 2:	{Deflection of beams ('standard cases') { 'Stresses in pressure vessels'

For Attention of Students:

Answer **ALL** questions.

The mark allocation is as indicated – total 40 marks.

A 'formula sheet' is provided for reference.

Assume 'g' = 9.81 m/s^2 .

and $1 \text{ bar} = 10^5 \text{ N/m}^2 (\text{Pa})$

1. Mutually perpendicular stresses of 32MPa (tensile) and 38 MPa (compressive) act at a point in a material. Calculate the magnitude and sense of the strains in the directions of the stresses, given that Poisson's ratio is 0.28 and the Modulus of Elasticity is 208 GPa.

(6 marks)

2. At a certain point in a material there is a strain in the 'x' direction of 0.35×10^{-6} and a strain in the 'y' direction of -0.62×10^{-6} . Calculate the corresponding stresses in each direction and state whether they are compressive or tensile. Take Poisson's ratio to be 0.23 and the Modulus of Elasticity as 80 GPa.

(6 marks)

3. A cube of side length 0.24 m is subjected to an external fluid pressure of 53MPa. If the modulus of elasticity of the cube material is 163.2 GPa and the Poissons ratio is 0.3, calculate the bulk modulus of the material and change in volume.

(8 marks)

4. A 3 m long cantilever beam carries a uniformly distributed load of 350N/m along its entire length. A point load of 1.89 kN also acts at end of the beam. The beam has a flexural stiffness of 18.2 MPa.

Calculate

- i) The corresponding deflection and slope at the end of the beam
- ii) How much the point load may be increased before the deflection exceeds 1.2 mm

(10 marks)

5. A thin walled cylinder acts as an air receiver. It has a diameter of 0.2m, is 0.43m in length and is made from sheet steel with a thickness of 2.2mm. The longitudinal and circumferential joint efficiencies are 70% and 58% respectively, and the maximum allowable stress is 100 MPa. What is the corresponding maximum allowable air pressure?

(10 marks)

Mechanical Principles Formula Sheet 1

Two-dimensional stress and strain equations:

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad \sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad \sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

Three-dimensional strain equations:

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Volumetric Strain:

$$\varepsilon_v = \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\nu)$$

Relationships between elastic constants:

$$E = \frac{\sigma}{\varepsilon} \qquad K = \frac{\sigma}{\varepsilon_v} = \frac{E}{3(1-2\nu)} \qquad G = \frac{E}{2(1+\nu)} \qquad K = \frac{2G(1+\nu)}{3(1-2\nu)}$$

Stress and strains in thin walled pressure vessels:

Cylinders		Spheres
$\sigma_c = \frac{pD}{2t\eta_L}$	$\sigma_L = \frac{pD}{4t\eta_c}$	$\sigma = \frac{pD}{4t\eta}$
$\varepsilon_c = \varepsilon_D = \frac{pD}{4tE}(2 - \nu)$	$\varepsilon_L = \frac{pD}{4tE}(1 - 2\nu)$	
$\varepsilon_v = \frac{pD}{4tE}(5 - 4\nu)$		$\varepsilon_v = \frac{3pD}{4tE}(1 - \nu)$

Stresses in thick walled cylinders: (Lame's Equations)

$$\sigma_R = a - \frac{b}{r^2}$$

$$\sigma_C = a + \frac{b}{r^2}$$

Where a and b are found
by applying boundary
conditions

Q1/ $\sigma_1 = 32 \times 10^6 \text{ N/m}^2$ ✓

$\sigma_2 = -38 \times 10^6 \text{ N/m}^2$ ✓

$\nu = 0.28$

$E = 208 \times 10^9 \text{ N/m}^2$

$\epsilon_1 = \frac{1}{E} (\sigma_1 - \nu \sigma_2) = \frac{1}{208 \times 10^9} (32 \times 10^6 - (0.28 \times -38 \times 10^6))$ ✓

$\epsilon_1 = 2.05 \times 10^{-4}$ ✓ or $205 \mu\epsilon$ (tensile).

$\epsilon_2 = \frac{1}{E} (\sigma_2 - \nu \sigma_1) = \frac{1}{208 \times 10^9} (-38 \times 10^6 - (0.28 \times 32 \times 10^6))$ ✓

$\epsilon_2 = -2.258 \times 10^{-4}$ ✓ or $226 \mu\epsilon$.

Q2/ $\epsilon_x = 0.35 \times 10^{-6}$ ✓

$\epsilon_y = -0.62 \times 10^{-6}$ ✓

$\nu = 0.23$

$E = 80 \times 10^9 \text{ N/m}^2$

$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{80 \times 10^9}{1-(0.23)^2} [0.35 \times 10^{-6} + (0.23 \times -0.62 \times 10^{-6})]$ ✓

$\sigma_x = 17.52 \text{ kPa}$ ✓

$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{80 \times 10^9}{1-(0.23)^2} [-0.62 \times 10^{-6} + (0.23 \times 0.35 \times 10^{-6})]$ ✓

$\sigma_y = -45.57 \text{ kPa}$ ✓

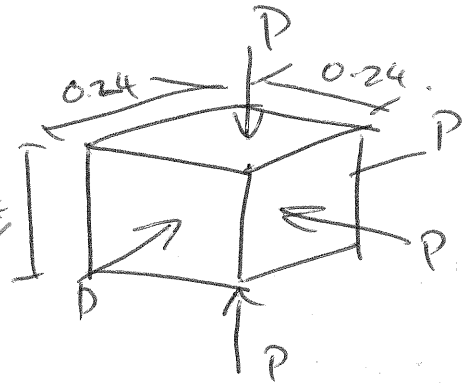
Q3/ $\ell = 0.24 \text{ m}$

$P = 53 \times 10^6 \text{ Pa}$

$K = 136 \times 10^9 \text{ Pa}$

$K = \frac{E}{3(1-2\nu)} = \frac{163.2 \times 10^9}{3(1-(2 \times 0.3))} = 136 \times 10^9 \text{ Pa} \checkmark$

$V = 0.24^3 = 0.013824 \text{ m}^3 \checkmark$



$K = \frac{\sigma}{\epsilon_v} = \frac{-P}{\epsilon_v} \checkmark$

$\epsilon_v = \frac{-P}{K} = \frac{-53 \times 10^6}{136 \times 10^9} = -3.897 \times 10^{-4} \checkmark$

$\epsilon_v = \frac{\Delta V}{V}$

$\Delta V = \epsilon_v V = -3.897 \times 10^{-4} \times 0.013824$

$\Delta V = -5.387 \times 10^{-6} \text{ m}^3$

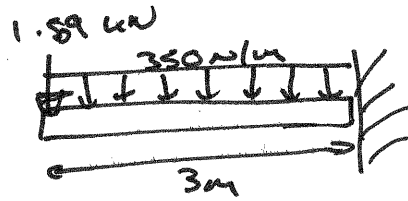
$\Delta V = -5387 \text{ mm}^3 \checkmark$

Q4/ $L = 3\text{m}$

$w = -350 \text{ N/m}$

$F = -1890 \text{ N}$

$EI = 18.2 \times 10^6 \text{ N/m}^2$



Slope: $\frac{dy}{dx} = \frac{F L^2}{2EI} + \frac{w L^3}{6EI}$ ✓

$\frac{dy}{dx} = -\frac{1890 \times 3^2}{2 \times 18.2 \times 10^6} + \frac{-350 \times 3^3}{6 \times 18.2 \times 10^6}$ ✓

$\frac{dy}{dx} = \cancel{-4.673 \times 10^{-4}} - 8.654 \times 10^{-5} = -5.538 \times 10^{-4}$ ✓

Deflection: $y = \frac{F L^3}{3EI} + \frac{w L^4}{8EI}$ ✓

$y = \frac{-1890 \times 3^3}{3 \times 18.2 \times 10^6} + \frac{-350 \times 3^4}{8 \times 18.2 \times 10^6}$ ✓

$y = -9.346 \times 10^{-4} - 1.947 \times 10^{-4} = -1.129 \times 10^{-3} \text{ m}$
 $= -1.129 \text{ mm}$ ✓

i) $y_{pc} = y_{max} - y_{ude}$

$y_{pc} = 1.2 \times 10^{-3} - 1.947 \times 10^{-4} = 1.0053 \times 10^{-3} \text{ m}$ ✓

$y_{pc} = \frac{F L^3}{3EI} \Rightarrow F = \frac{y_{pc} \times 3 \times EI}{L^3}$ ✓

$F = \frac{1.0053 \times 10^{-3} \times 3 \times 18.2 \times 10^6}{3^3} = 2032.9 \text{ N}$

$\therefore \Delta F = 2032.9 - 1890 = \underline{142.94 \text{ N}}$ ✓

Q5/ ✓

$$D = 0.2 \text{ m}$$

$$l = 0.43 \text{ m}$$

$$t = 2.2 \times 10^{-3} \text{ m}$$

$$n_L = 0.7$$

$$n_c = 0.58 \checkmark$$

$$\sigma_{\max} = 100 \times 10^6 \text{ N/m}^2$$

$$\sigma_c = \frac{pD}{2tn_L} \checkmark$$

$$p = \frac{2tn_L\sigma_c}{D} \checkmark$$

$$p_{\max_c} = \frac{2 \times 2.2 \times 10^{-3} \times 0.7 \times 100 \times 10^6}{0.2} \checkmark$$

$$p_{\max_c} = 1.54 \times 10^6 \text{ N/m}^2$$

$$p_{\max_c} = 15.4 \text{ Bar} \checkmark$$

$$\sigma_n = \frac{pD}{4tn_c} \checkmark$$

$$p = \frac{4tn_c\sigma_n}{D} \checkmark$$

$$p_{\max_n} = \frac{4 \times 2.2 \times 10^{-3} \times 0.58 \times 100 \times 10^6}{0.2} \checkmark$$

$$p_{\max_n} = 2.552 \times 10^6 \text{ N/m}^2$$

$$p_{\max_n} = 25.5 \text{ Bar.}$$

$$\therefore p_{\max} = 15.4 \text{ Bar} \checkmark$$