

PULL & TEAR

FOR ADMINISTRATIVE USE ONLY, CAREFULLY REMOVE PERFORATED STRIP

$$1) \quad A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 0 & 2 \\ 5 & 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & -3 \\ 0 & -1 & -1 \\ 4 & 2 & 1 \end{bmatrix}$$

i) AB

$$= \begin{bmatrix} (1 \times 2) + (-3 \times 0) + (2 \times 4) & (1 \times 1) + (-3 \times -1) + (2 \times 2) & (1 \times -3) + (-3 \times -1) + (2 \times 1) \\ (4 \times 2) + (0 \times 0) + (2 \times 4) & (4 \times 1) + (0 \times -1) + (2 \times 2) & (4 \times -3) + (0 \times -1) + (2 \times 1) \\ (5 \times 2) + (1 \times 0) + (1 \times 4) & (5 \times 1) + (1 \times -1) + (1 \times 2) & (5 \times -3) + (1 \times -1) + (1 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} (2 + 0 + 8) & (1 + 3 + 4) & (-3 + 3 + 2) \\ (8 + 0 + 8) & (4 + 0 + 4) & (-12 + 0 + 2) \\ (10 + 0 + 4) & (5 - 1 + 2) & (-15 - 1 + 1) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 8 & 2 \\ 16 & 8 & -10 \\ 14 & 6 & -15 \end{bmatrix}$$

ii) $A-B$

$$= \begin{bmatrix} 1-2 & -3-1 & 2-(-3) \\ 4-0 & 0-(-1) & 2-(-1) \\ 5-4 & 1-2 & 1-1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -4 & 5 \\ 4 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

iii) $A + B$

$$= \begin{bmatrix} 1+2 & -3+1 & 2+(-3) \\ 4+0 & 0+(-1) & 2+(-1) \\ 5+4 & 1+2 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & -1 \\ 4 & -1 & 1 \\ 9 & 3 & 2 \end{bmatrix} \quad \checkmark$$

iv) B^T

Rows become Columns

$$= \begin{bmatrix} 2 & 0 & 4 \\ 1 & -1 & 2 \\ -3 & -1 & 1 \end{bmatrix} \quad \checkmark$$

Q7

18

18

18

18

18

18

2)

$$\begin{array}{c} T \qquad \qquad P \qquad \qquad T.P \\
 \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+1 \\ 4+2 \\ 6+3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 9 \\ 1 \end{bmatrix}
 \end{array}$$

$$\begin{array}{r} 6 \\ 6 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 6 \\ 6 \\ \hline 0 \end{array}$$

3)

$$\begin{aligned}
 a) \quad x' &= x \cos A - y \sin A + 0z \\
 y' &= x \sin A + y \cos A + 0z \\
 z' &= 0x + 0y + 1z
 \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$b) \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(-A) & -\sin(-A) & 0 \\ \sin(-A) & \cos(-A) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$c) \quad x = 0.707$$

$$y = 1.414$$

$$z = 2$$

$$A = 45$$

and in a simpler form

$$\frac{1}{\sqrt{2}} \quad 1$$

Correct answer.
No simplification requested.

$$x' = 0.707 \times \cos 45 - 1.414 \times \sin 45 = -0.499$$

$$y' = 0.707 \times \sin 45 + 1.414 \times \cos 45 = 1.499$$

$$z = 2$$

$$x' = 0.499$$

$$y' = 1.499$$

$$z = 2$$

$$\begin{array}{r} 16 \\ 16 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 16 \\ 16 \\ \hline 0 \end{array}$$

4)

$$\underline{a} = (2, 1, 3)$$

$$\underline{b} = (3, 2, -3)$$

$$\underline{a} \times \underline{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 3 \\ 3 & 2 & -3 \end{vmatrix}$$

$$= +\mathbf{i}(-3-6) - \mathbf{j}(-6-9) + \mathbf{k}(4-3)$$

$$= +\mathbf{i}(-9) - \mathbf{j}(-15) + \mathbf{k}(1)$$

$$= -9\mathbf{i} + 15\mathbf{j} + 1\mathbf{k}$$

$$\underline{a} \times \underline{b} = (-9, 15, 1)$$

b) The cross product gives us a vector which is perpendicular to both vectors.

The angle between $\underline{a}$ and the cross product is 90° .

$$\theta = \cos^{-1} \left(\frac{(2 \times -9) + (1 \times 15) + (3 \times 1)}{\sqrt{2^2 + 1^2 + 3^2} \times \sqrt{-9^2 + 15^2 + 1^2}} \right)$$

$$\theta = 90^\circ$$

good

$$Q4 = 9$$

$$9$$

$$\frac{2}{9}$$

$$5) \quad \underline{a} = (3, 4, 0) \\ \underline{b} = (8, 6, 0)$$

$$\underline{a} \cdot \underline{b} = (3 \times 8) + (4 \times 6) + (0 \times 0) \\ = 24 + 24 \\ = 48$$

$$\Theta = \cos^{-1} \left(\frac{48}{\sqrt{3^2 + 4^2 + 0^2} \times \sqrt{8^2 + 6^2 + 0^2}} \right)$$

$$\Theta = 16.26^\circ$$

$$\cos = \frac{8}{8}$$

$$6) \quad \underline{a} = (3, 2, 6)$$

$$|\underline{a}| = \sqrt{3^2 + 2^2 + 6^2} \\ = \sqrt{9 + 4 + 36} \\ = \sqrt{49} \\ = 7$$

$$\hat{a} = \frac{\underline{a}}{|\underline{a}|}$$

$$\hat{a} = \left(\frac{3}{7}, \frac{2}{7}, \frac{6}{7} \right)$$

$$\cos = \frac{7}{7}$$

$$7) A = \begin{bmatrix} 1 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & 3 & -1 \end{bmatrix}$$

$$\begin{aligned} |A| &= +1(-2-6) - 1(-1-2) + 6(3-2) \\ &= +1(-8) - 1(-3) + 6(1) \\ &= -8 + 3 + 6 \\ &= 1 \end{aligned}$$

$$\Delta 7 = 7$$

$$\begin{pmatrix} 7 \\ 7 \end{pmatrix}$$

$$\begin{aligned} 8) \quad 1x + 1y + 6z &= 1 \\ 1x + 2y + 2z &= 2 \\ 1x + 3y - 1z &= 4 \end{aligned}$$

$$\begin{matrix} A & X & K \\ \begin{bmatrix} 1 & 1 & 6 \\ 1 & 2 & 2 \\ 1 & 3 & -1 \end{bmatrix} & \begin{bmatrix} x \\ y \\ z \end{bmatrix} & = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \end{matrix} \quad \begin{matrix} A^{-1} \\ \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix} \end{matrix}$$

~~$$AX = K$$~~

~~$$A^{-1}A$$~~

$$AX = K$$

$$A^{-1}AX = A^{-1}K$$

$$IX = A^{-1}K$$

$$X = A^{-1}K$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 & 19 & -10 \\ 3 & -7 & 4 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} (-8+38-40) \\ (3-14+16) \\ (1-4+4) \end{bmatrix} = \begin{bmatrix} -10 \\ 5 \\ 1 \end{bmatrix}$$

$$x = -10 \quad y = 5 \quad z = 1$$

TEST

$$\begin{aligned} 1(-10) + 1(5) + 6(1) &= 1 \\ -10 + 5 + 6 &= 1 \end{aligned}$$

$$1 = 1$$

$$LHS = RHS$$

$\therefore$ correct

$$\Delta 8 = \frac{9}{9}$$

$$\begin{pmatrix} 9 \\ 9 \end{pmatrix}$$

TYNNWCH A
RHWYGWCH

AR GYFER DEFYDD GWEINYDDOL YN UNIG, TYNNWCH Y STRIBYN TYLLOG I FFWRDD YN OFALUS



PRIFYSGOL CYMRU
Y Drindod Dewi Sant
UNIVERSITY OF WALES
Trinity Saint David

Llyfr Atebion Arholiadau

24 Tudalen

Cyfarwyddiadau i Ymgeiswyr

1. Rhwch yr holl wybodaeth y gofynnir amdani yn y blychau isod.
2. Rhwch eich enw yn y lle a ddarperir i'r dde a seliwch i guddio eich enw.
3. Rhaid i'r atebion fod mewn llawysgrifen **ddarlennadwy**. Gall yr Arholwyr naill ai anwybyddu sgript annarllennadwy, neu drefnu iddi gael ei theipio a chael yr ymgeisydd i dalu.
4. Ysgrifennwch eich atebion mewn inc.
5. Dechreuwch bob ateb ar dudalen newydd. Gwnewch unrhyw fras nodiadau yn y llyfr atebion a thynnw llinell drwyddynt.
6. Ni ddylid rhwygo unrhyw ran o'r llyfr hwn; rhaid ei gyflwyno yn gyfan.
7. Os defnyddir mwy nag un llyfr, rhwymwch hwy gyda'i gilydd gan ddefnyddio'r tagiau sydd ar gael oddi wrth y gwylwtr/gwylwyr.
8. Rhaid dychwelyd pob llyfr. Ni chaniateir mynd â llyfrau atebion, wedi eu defnyddio neu heb eu defnyddio, allan o'r lleoliad arholi o dan unrhyw amgylchiadau.

1	Rhif Myfyrwr	
2	Teitl y Papur Arholiad	
3	Cod y Modwl	
4	Dyddiad ac Amser yr Arholiad	
5	Lleoliad yr Arholiad	
6	Rhif Sedd	
7	Nifer y Llyfrau Atebion a gyflwynwyd	

Rhesitrwch rifau'r cwestiynau a ymgeisiwyd gennych	
At Ddibenion y Swyddfa	
Cyfanswm	

(See the other side for English version)

A wnewch chi lenwi'r rhan hon a'i SELIO cyn i'r arholiad ddechrau.

Cyfenw _____ (LLYTHRENNAU BREISION) Enw(au) cyntaf _____

PLYGWCH
lenwch y rhan hon, llynnwch y strbyn, plygwch fel a ddangosir a SELIWCH