

~~Q3a)~~ Q1) a)

$$i) y = \frac{x^2}{2x+1}$$

$$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$\frac{dy}{dx} = \frac{2x^2 + 2x}{(2x+1)^2}$$

$$ii) y = \sqrt{x^2+1}$$

$$\frac{dy}{dx} = (x^2+1)^{-\frac{1}{2}}$$

$$= \frac{1}{2} (x^2+1)^{-\frac{1}{2}} \times 2x$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2+1}}$$

$$iii) y = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x$$

$$= 4.5x^{\frac{1}{2}} + \frac{3}{1} - 6$$

$$= 4.5x^{\frac{1}{2}} - 3$$

$$iv) y = \sec^3 4x$$

$$\frac{dy}{dx} = 3 \sec^2(\sec x \tan x) \times 4$$

$$= 12 \sec^2(\sec x \tan x)$$

$$v) y = (x + \sin 2x)$$

$$vi) y = \frac{1-10x}{(2x-1)^2}$$

$$y = -9(2x-1)^{-2}$$

$$\frac{dy}{dx} = 18(2x-1)^{-3} \times 2$$

$$= 36(2x-1)^{-3}$$

$$\frac{dy}{dx} = \frac{36}{(2x-1)^3}$$

$$b) y = \frac{1}{x} + \cos 2x, \text{ find } \frac{d^5 y}{dx^5}$$

$$\frac{dy}{dx} = -2 \times 1 \sin 2x - \frac{1}{x^2}$$

$$\frac{dy}{dx} = -2 \sin(2x) - \frac{1}{x^2}$$

$$\frac{d^2 y}{dx^2} = -4 \cos 2x + \frac{2}{x^3}$$

$$\frac{d^3 y}{dx^3} = \cancel{8 \sin 2x} 8 \sin 2x - \frac{6}{x^4}$$

$$\frac{d^4 y}{dx^4} = 16 \cos 2x + \frac{24}{x^5}$$

$$\frac{d^5 y}{dx^5} = -32 \sin 2x - \frac{120}{x^6}$$

$$c) f(x) = \operatorname{cosec} x - \cot x$$

$$f'(x) = \frac{1}{\operatorname{cosec} x - \cot x} \times (-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x)$$

$$= \frac{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x}$$

Q2a)

$$i) \int \frac{2x}{4+3x^2}$$

$$u = 4 + 3x^2 + 4$$

$$\frac{du}{dx} = 6x \quad \therefore dx = \frac{1}{6x}$$

$$\Rightarrow \frac{1}{3} \int \frac{1}{u} du$$

$$= \ln(u)$$

$$\frac{\ln(u)}{3} = \frac{\ln(3x^2+4)}{3}$$

$$= \frac{\ln(3x^2+4)}{3} + c$$

ii)

iii)

3) a)

$$i) \left(\frac{1+i}{1-i} \right)^3$$

$$\frac{1+i}{1-i} = \cancel{(i)^3} (i)^3$$

4.

$$= \cancel{i^3} i^3$$

$$= -i$$

ii)

$$\frac{3}{(1+i)(2+i)}$$

$$= \frac{3}{1+3i}$$

$$= \frac{3(1-3i)}{10}$$

$$= \frac{3}{10} - i \frac{9}{10}$$

iii)

$$\frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$$

$$= -i - 1 + 3i + 1 - 2i$$

$$= -i + 3i - 2i$$

$$= 0$$

$$iv) 2i(i-1) + \sqrt{3} - i + (1+i)(1+i)$$

b)

c) Quotient $1+3i$ and $1+i$

$$\left(\frac{1+3i}{1+i} \right) \left(\frac{1-i}{1-i} \right)$$

$$= \frac{-3i^2 + 3i - i + 1}{-i^2 + 1}$$

$$= \cancel{1} + \cancel{2i} - 3i + 2i$$

$$= -i$$