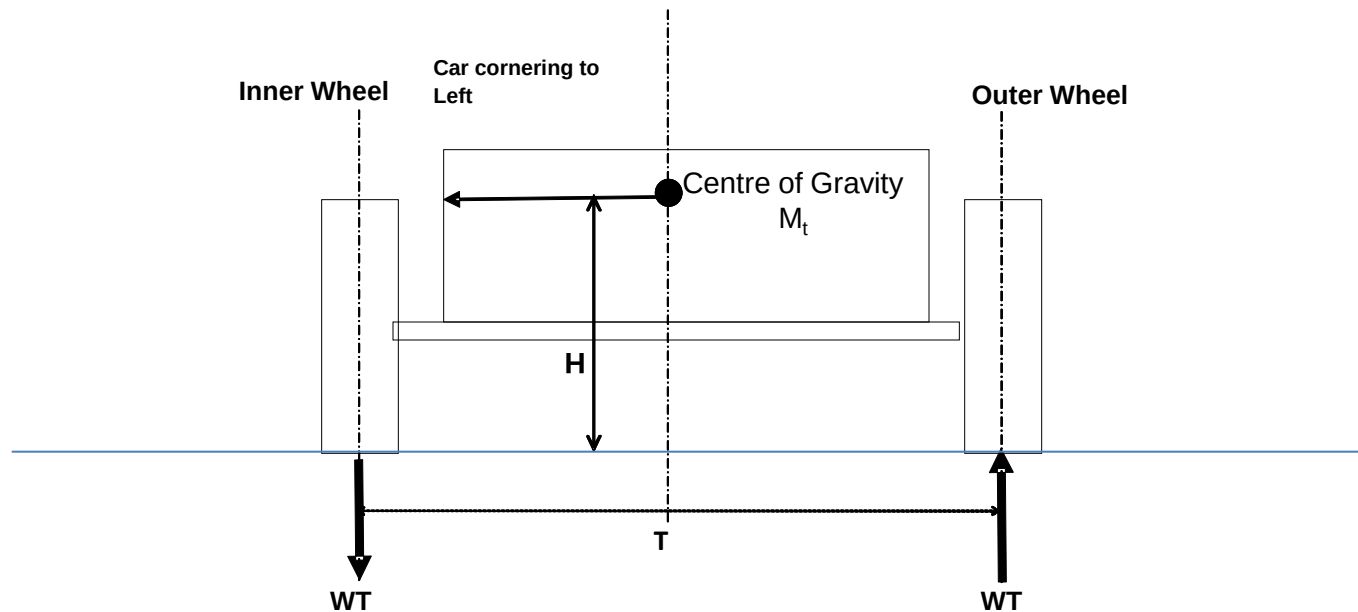


Vehicle Dynamics

DISTRIBUTION OF WEIGHT TRANSFER IN CORNERING

Simple Case of Cornering Car with Rigid Suspension



Simple lateral weight transfer calculation – rigid vehicle

Over turning moment = $M_t a_y H$

Reaction moment = $WT \times T$

For equilibrium Reaction = Overturning moment

$$\text{Giving } WT = \frac{M_t a_y H}{T}$$

Weight transfer calculation for vehicles with suspension

Real cars have suspensions and we need some method of determining the load distribution over the four wheels.

One method that can be used is that adopted by Warren Rowley described in Introduction to Race Car Engineering in which he develops nine simultaneous equations and then solves them using matrix maths in his WAN32 software.

(available on L\Kestrel \Roger Dowden) [Wan32](#)

Three Mass Weight Transfer

Another much simpler method can be used to obtain an effective indication of the effect of changes to the suspension setup on load transfer

This calculates the weight transfer produced by three distinct causes

Weight transfers

1. Weight transfer due to the unsprung mass
2. Weight transfer due to sprung mass which is taken as acting at the roll centre height.
3. Weight transfer due to the sprung mass which is taken as acting about the roll centre

Example Data

- **Worked Example**

- **Data**

•	Tf	Front track	1.270 m
•	Tr	Rear track	1.335 m
•	L	Wheel base	2.248 m
•	H	CG height above roll axis	0.350 m
•	Msf	Front sprung mass	312 kg
•	Msr	Rear sprung mass	328 kg
•	Musf	Front total unsprung mass	60 kg
•	Musr	Rear total unsprung mass	70 kg
•	r	Rolling radius of tyre	0.277m
•	hf	Front RC height	10 mm
•	hr	Rear RC Height	50 mm
•	Kwr	Rear wheel rate	21875 N/m
•	Kwf	Front wheel rate	20483 N/m
•	Karr	Rear anti roll bar wheel rate	4500 N/m
•	Karf	Front anti roll bar wheel rate	5000 N/m
•	a_y	Lateral acceleration	6m/s ²
•	Front suspension - unequal length double wishbones		
•	Rear suspension - unequal length double wishbones		

Nomenclature Changes

Note that for these lateral weight transfer calculations H will be taken as the height of the CG above Roll axis and H' the height of the CG above the ground

1.

Weight transfer due to the unsprung mass which is taken as acting at the height of the wheel centre and is resisted by the tyres alone

$$WT_{usf} = \frac{M_{usf} \times a_y \times r}{T_f} \quad \text{Equation 1f}$$

$$WT_{usr} = \frac{M_{usr} \times a_y \times r}{T_r} \quad \text{Equation 1r}$$

2.

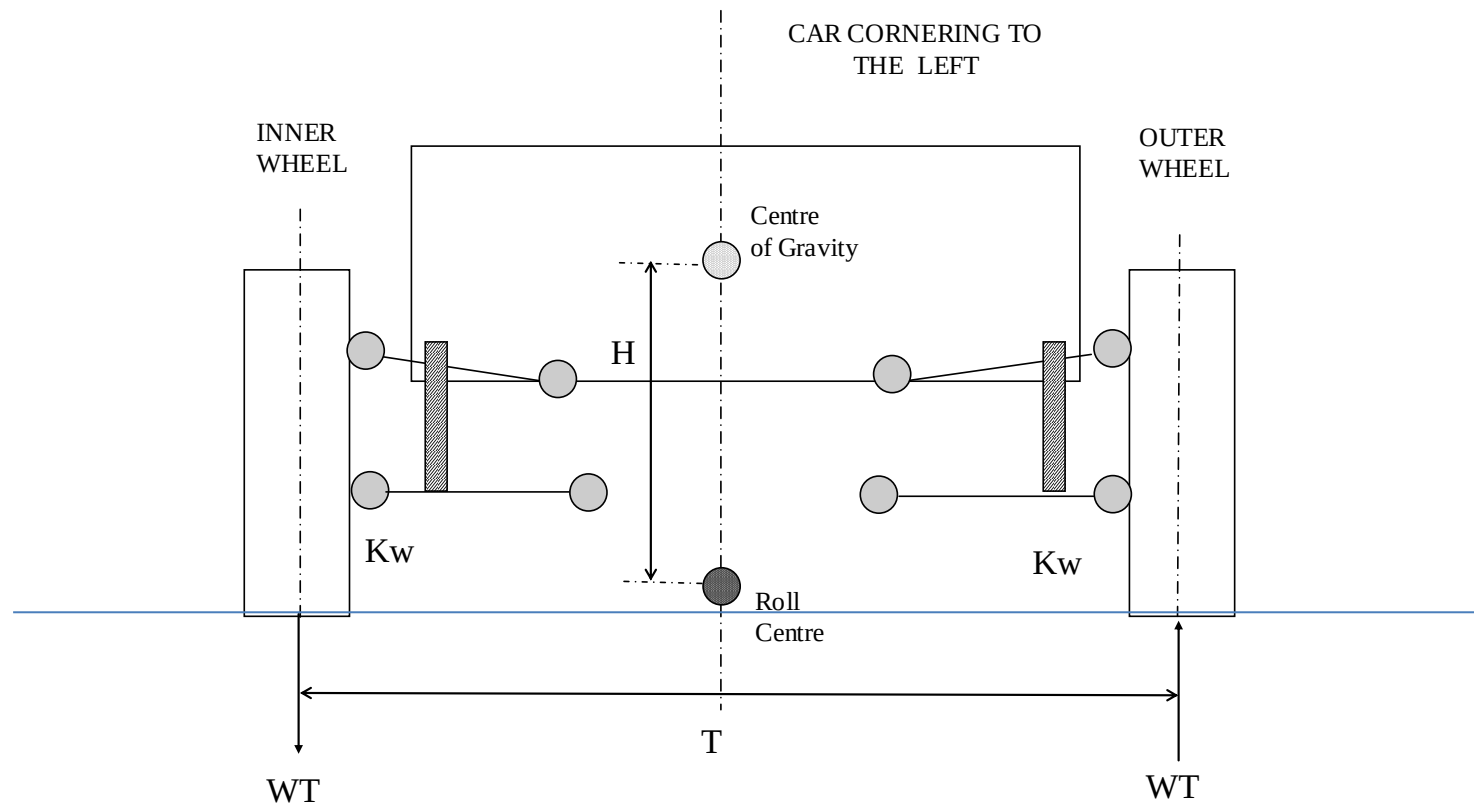
Weight transfer due to sprung mass which is taken as acting at the roll centre height and which is resisted by the tyres alone.

$$WT_{srcf} = \frac{M_{sf} \times a_y \times h_f}{T_f} \quad \text{Equation 2f}$$

$$WT_{srcr} = \frac{M_{sr} \times a_y \times h_r}{T_r} \quad \text{Equation 2r}$$

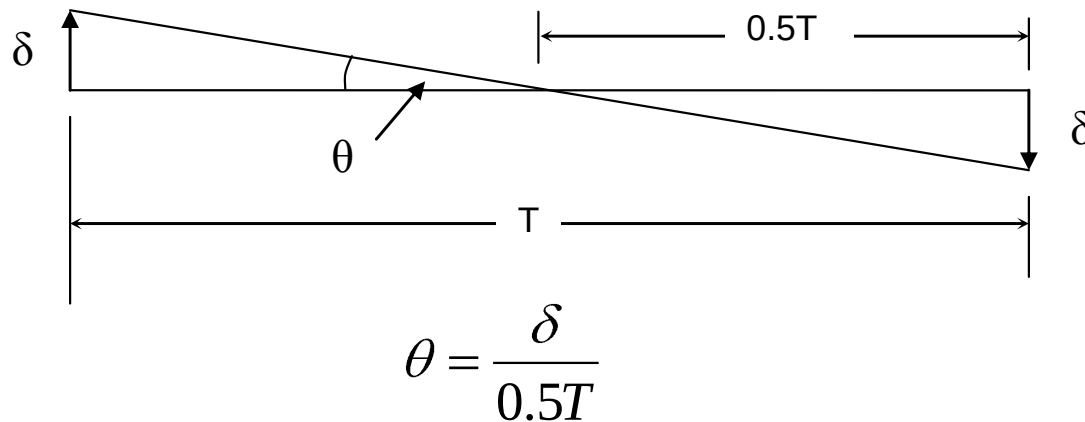
3.

Weight transfer due to the sprung mass which is taken as acting about the roll centre and which is resisted by the springs.



Vehicle roll

The diagram below symbolises a lateral axis of the body. The car has rolled through θ radians causing a deflection of inner and out wheels of δ m. As a wheel rate of K_w is defined (which is the effective spring rate at the wheel) the wheel displacement can be taken as the effective spring displacement.



$$\delta = \frac{T\theta}{2} \quad \text{Eqn 3}$$

Roll Moment

Anti- roll moment produced by the springs = M_{ϕ}

$$M_{\phi} = \delta \times K_w \times T \quad \text{Equ 4}$$

$$M_{\phi} = \frac{T\theta}{2} \times K_w \times T$$

$$M_{\phi} = \frac{T^2\theta \times K_w}{2} \quad \text{Equ 5}$$

Roll Stiffness

Roll stiffness of the front suspension = $K\phi$ Nm/Rad

$$K\phi = \frac{M\phi}{\theta}$$

Giving

$$K\phi = \frac{(Tf^2 \times Kw_f)}{2}$$

Roll stiffness at each end

$$K_{\phi f} = \frac{(T_f^2 \times K_{wf})}{2} \quad \text{Eqn 6f}$$

and similarly for the rear

$$K_{\phi r} = \frac{T_r^2 \times K_{wr}}{2} \quad \text{Eqn 6r}$$

.

.

Total Roll Stiffness for the Car

Total roll stiffness for the car = $K_{\phi t}$

$$K_{\phi t} = \frac{(T_f^2 \times K_{wf})}{2} + \frac{(T_r^2 \times K_{wr})}{2}$$

So

$$K_{\phi t} = \frac{(T_f^2 \times K_{wf}) + (T_r^2 \times K_{wr})}{2} \quad \text{Nm/Rad} \quad \text{Eqn 7}$$

Roll Angle

When cornering, an overturning moment (Mot) is produced by d'Alemberts force acting on the centre of mass about the roll axis with a moment arm of H m.

$$Mot = Ms \times a_y \times H \quad \text{Nm} \quad \text{Eqn 8}$$

This is resisted by the springs and the roll angle produced in cornering is therefore:

$$\theta = \frac{Mot}{K_{\phi t}} \quad \text{Radians}$$

Roll Angle

The roll angle produced by a lateral acceleration of $a_y = \theta$ Radians

$$\theta = \frac{2 \times M_s \times a_y \times H}{(f^2 \times K_{wf}) + (Tr^2 \times K_{wr})} \quad \text{Radians} \quad \text{Eqn 9}$$

Solutions to example

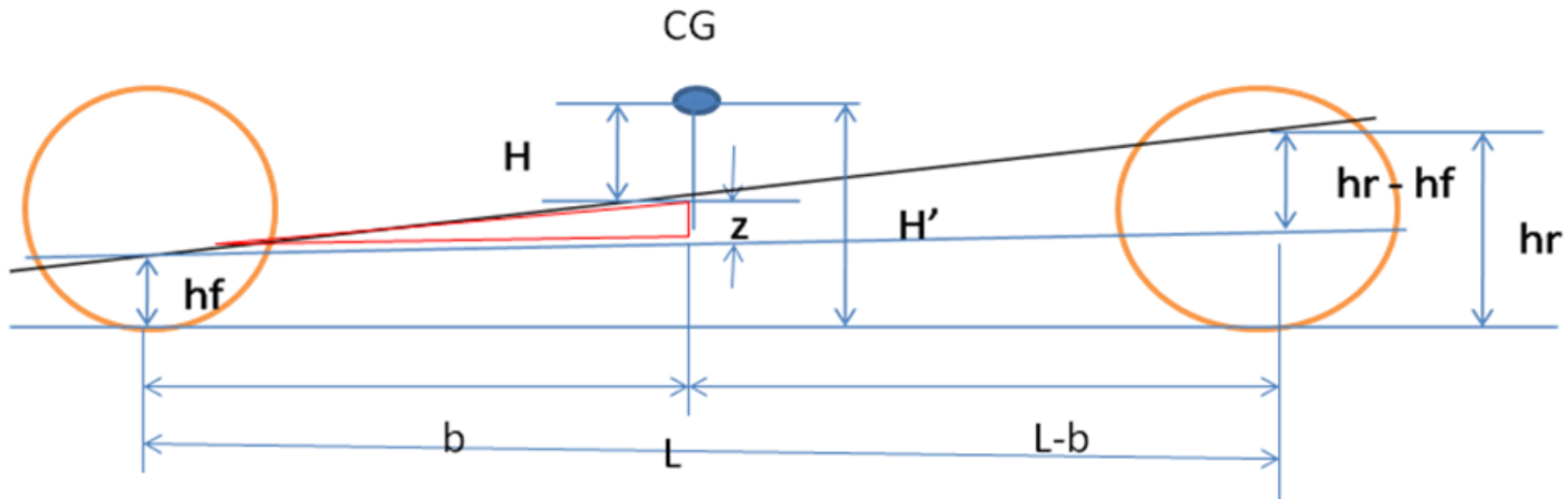
- $W_{tuf} = 78.52$
- $W_{tusr} = 87.15$
- $W_{Tsrcf} = 14.74$
- $W_{Tsrcr} = 72.71$
- $K_{\phi tf} = 16518.5 \text{ Nm/rad}$
- $K_{\phi tr} = 19\,493.1 \text{ Nm/rad}$
- $K_{\phi t \text{ tot}} = 36011.6 \text{ Nm/rad}$
- $M_{ot} = 1344 \text{ Nm}$
- $\theta = 0.0373 \text{ rad (ie } 2.14 \text{ deg)}$

Finding H the height of the CG above the roll axis

In practice the distance of the CG above the roll axis cannot be measured directly and so needs to be calculated using the following data:

- H' Height of CG above ground
- M_f Total of two front weighing scales readings in kg
- M_r Total of two rear weighing scales readings in kg
- h_f Height of front roll centre
- h_r Height of rear roll centre
- L Wheel base

To find H the CG height above roll axis
when H' , M_f , M_r , h_f and h_r are known



Finding H Height of CG above roll axis

Example : Assume

$$M_f = 144 + 145 = 300 \text{ kg}$$

$$M_r = 165 + 177 = 342 \text{ kg}$$

$$L = 2351 \text{ mm}$$

$$H' = 340 \text{ mm}$$

Find CG position fore and aft

$$642 * b = 2351 * 342$$

$$\text{ie } b = 1255$$

So CG is 1255mm from the front wheel ie the length of the red triangle

Finding H the height of the CG above the roll axis

By similar triangles therefore:

$$\frac{143}{2351} = \frac{Z}{1255}$$

Giving $Z = 76 \text{ mm}$

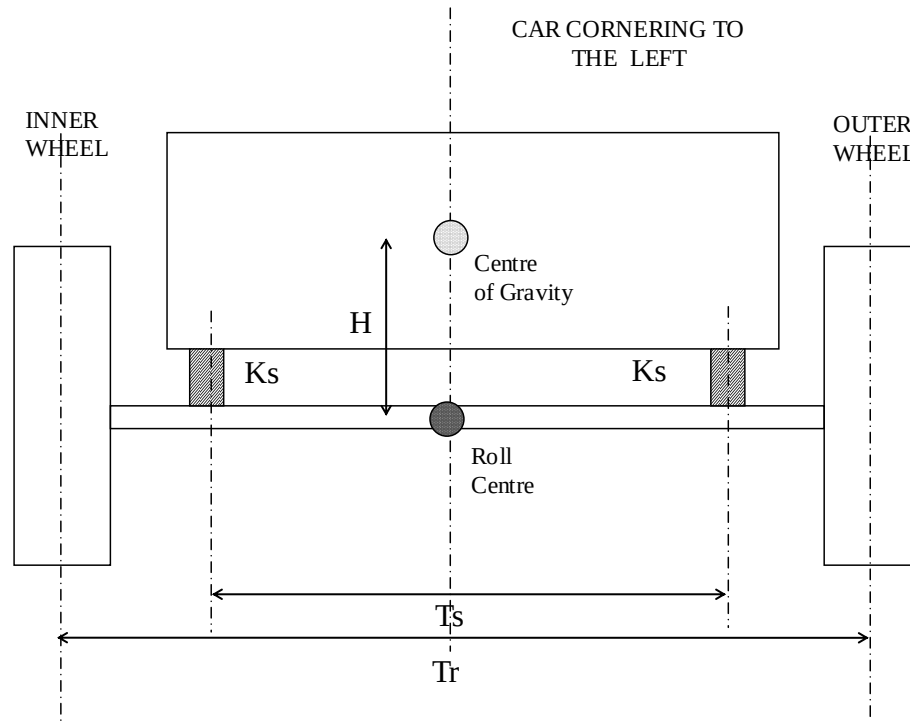
Where Z is the height of the triangle at the position of the CG

The height of the CG above the ground H' is 340mm

This give a value of H of $340 - (76+95) = 169\text{mm}$

Modification when using a live axle

Where the vehicle has a rear axle the spring track is used instead of the wheel track and spring rate is used instead of the combined wheel and tyre rate. The body now compresses the springs across the spring track T_s rather than the wheel track.



Roll angle for a car with a live axle

The analysis is exactly the same but Spring track, T_s replaces T_r and K_{wr} is equal to K_{sr} (unless some suspension leverage exists due to the positioning of the Spring) and the deflection δ of the wheel is now the direct deflection of the spring on the axle.

The equation for θ then becomes :

$$\theta = \frac{2 \times M_s \times H \times a}{\left(f^2 \times K_{wf} \right) + \left(s^2 \times K_{wr} \right)} \quad \text{Rad} \quad \text{Eqn 10}$$

Weight transfer due to sprung mass acting about the roll axis

Knowing the value of θ , the weight transfer due to the sprung mass which is taken as acting about the roll centre axis can be determined.

Although still a simplification it can be assumed that the load transfer distribution front to rear for the sprung mass acting about the roll axis will be in proportion to the roll stiffness front to rear.

At each end of the car the increased spring load produced by the upward deflection of the outer wheel is equivalent to the weight transfer of the sprung mass acting about the roll axis, WTs . The chassis is assumed to be infinitely stiff and so roll angle θ is assumed to be the same at each end of the car.

$$WTs = \delta \times K_w$$

Weight transfer WTs due to sprung mass at each end of the car

$$\delta = \frac{\theta \times T}{2}$$

So

$$WT_{sf} = \frac{\theta \times T_f \times K_{wf}}{2} \quad \text{Eqn 11f}$$

and

$$WT_{sr} = \frac{\theta \times T_r \times K_{wr}}{2} \quad \text{Eqn 11r}$$

Weight transfer with live rear axle

With a live rear axle the load increase at the spring is not the weight transfer experienced at the wheel. As the body rolls relative to the axle the outer spring is compressed and the spring load increase is F_{sr} then:

$$F_{sr} = \frac{\theta \times T_s \times K_{sr}}{2}$$

The increased wheel load produced by this increased spring load is WT_{srl}

To find the weight transfer reacted at the wheel,

Equate the moment produced by the increased wheel load WT_{srl} with moment produced by increased spring load F_{sr}

$$WT_{srl} \times Tr = F_{sr} \times Ts$$

Substituting for F_{sr} gives

$$WT_{srl} = \frac{Ts^2 \times \theta \times K_{sr}}{2 \times Tr} \quad \text{Eqn 13r}$$

Overall weight transfers

Total front weight transfer = $1f + 2f + 11f$

Total rear weight transfer = $1r + 2r + 11r$

and for a car with a live rear axle

Total rear weight transfer = $1r + 2r + 13r$

Determination of individual wheel loads:

The following suffixes are used to identify the wheel loads .

o = outer i = inner z = vertical y = lateral

So, for example, F_{zof} = Vertical load on the outer front tyre

The front and rear, outer wheel loads can now be determined by taking the static wheel loads and adding the load transfers to them.

Similarly the inner wheel loads can be determined by subtracting the load transfers from their static loads.

Example - Front Outer Wheel Load

$$F_{zof} = (M_{sof} + M_{usof}) \times 9.81 + \text{Total Front weight Transfer}$$

Final individual corner loads

Corner ←

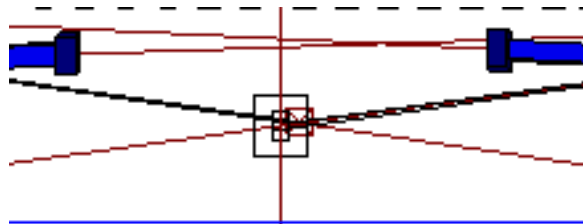
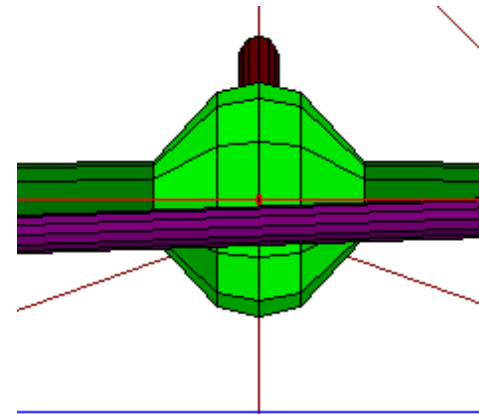
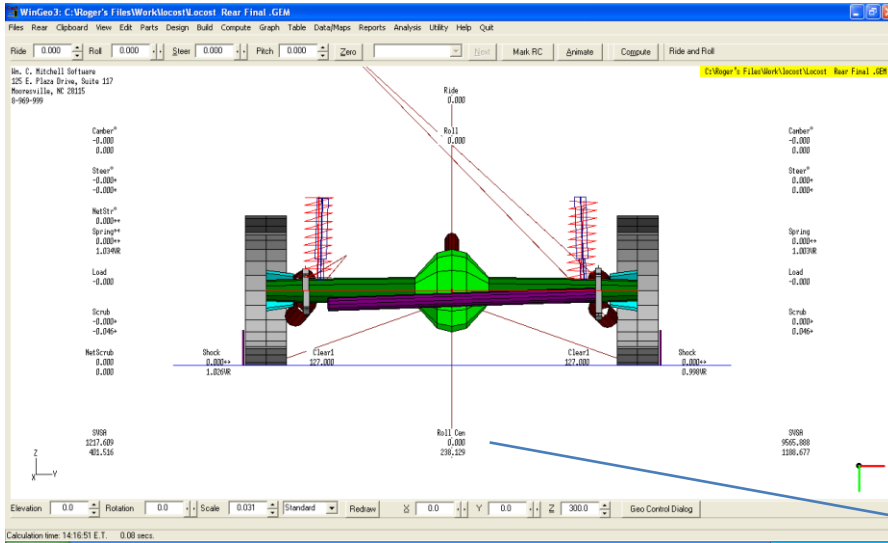
Front ↑

Fzif	Fzof
Fzir	Fzor

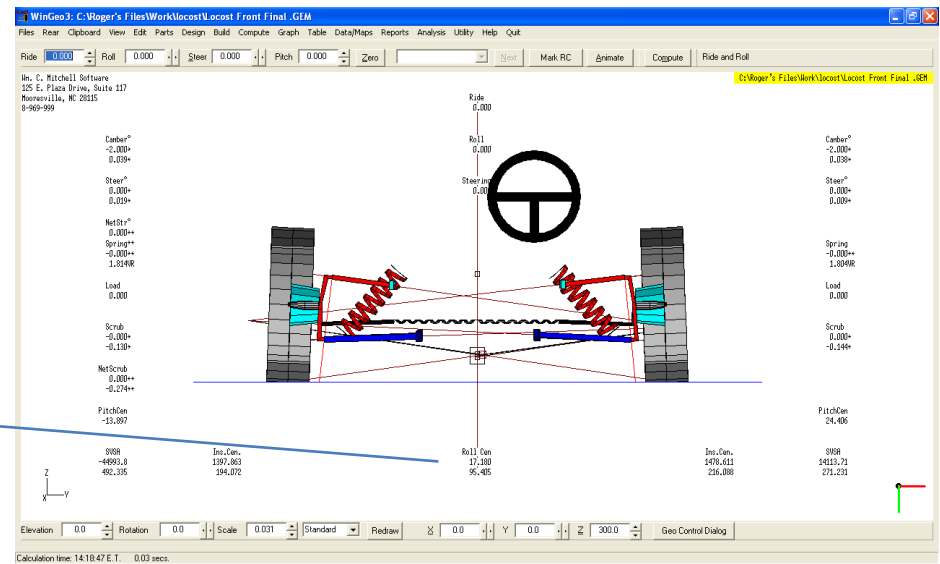
Worked Example

Take the data from your Locost test to determine the load distribution that would be achieved at 0.8g Lateral acceleration

Use h_f and h_r from WinGeo3 data and CG height of sprung mass above ground of 0.299m



Roll Cen
17.180
95.405



Refinement to roll distribution method

A more accurate development of this method used to determine the full roll angle and load transfers, taking into account tyre deflection can be found in *Car Suspension and Handling* by Bastow, D, Howard, G and Whitehead.

The use of wheel displacement potentiometers will only allow the measurement of the roll angle of the chassis on the wheels, however, so can only be used for verification of the simpler form of the calculations which has been used.