

1803210

Stress analysis
(ACHA5015)

09/05/22

$$\epsilon_x = 22 \times 10^{-6}$$

$$\epsilon_y = -13 \times 10^{-6}$$

$$\nu = 0.28$$

$$E = 170 \times 10^9$$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$\text{ii) } \sigma_x = \frac{170 \times 10^9}{1-0.28^2} (22 \times 10^{-6} - (0.28 \times -13 \times 10^{-6}))$$

$$\sigma_x = 4.73 \text{ MPa (tensile)}$$

$$\sigma_y = \frac{170 \times 10^9}{1-0.28^2} (-13 \times 10^{-6} - (0.28 \times 22 \times 10^{-6}))$$

$$\sigma_y = -3.53 \text{ MPa (compressive)}$$

$$\text{iii) } \sigma_x = 83 \times 10^6$$

$$\sigma_y = 42 \times 10^6$$

$$\nu = 0.28$$

$$E = 170 \times 10^9$$

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y)$$

$$\epsilon_x = \frac{1}{170 \times 10^9} (83 \times 10^6 - (0.28 \times 42 \times 10^6))$$

$$\epsilon_x = -6.92 \times 10^{-5}$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x)$$

$$\epsilon_y = \frac{1}{170 \times 10^9} (42 \times 10^6 - (0.28 \times 83 \times 10^6))$$

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$$\varepsilon_y = 1.1 \times 10^{-4}$$

$$2i) \sigma_x = 105 \times 10^6$$

$$\sigma_y = 203 \times 10^6$$

$$\tau_{xy} = 125 \times 10^6$$

$$\sigma_{1,2} = \frac{(\sigma_x + \sigma_y)}{2} \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + \tau_{xy}^2}$$

$$\sigma_1 = \frac{(105 \times 10^6 + 203 \times 10^6)}{2} + \sqrt{\frac{(105 \times 10^6 - 203 \times 10^6)^2}{4} + 4 \times (125 \times 10^6)^2}$$

$$\sigma_1 = 154 \times 10^6 + \frac{\sqrt{9.604 \times 10^{15} + 6.25 \times 10^{16}}}{2}$$

$$\sigma_1 = 154 \times 10^6 + 268.52 \times 10^6$$

$$\sigma_1 = 422.52 \times 10^6$$

$$= 422.52 \text{ MPa}$$

$$\sigma_2 = \frac{(105 \times 10^6 + 203 \times 10^6)}{2} - \sqrt{\frac{(105 \times 10^6 - 203 \times 10^6)^2}{4} + 4 \times (125 \times 10^6)^2}$$

$$\sigma_2 = 154 \times 10^6 - 268.52 \times 10^6$$

$$\sigma_2 = -114.52 \times 10^6$$

$$= -114.52 \text{ MPa}$$

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$$2ii) \tau_{max} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{max} = \frac{422.52 + 114.52}{2}$$

$$\tau_{max} = 268.52 \text{ pa}$$

$$\tau_{min} = -\frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}}{2} = -\left[\frac{\sigma_1 - \sigma_2}{2}\right]$$

$$\tau_{min} = \frac{422.52 - 114.52}{2}$$

$$\tau_{min} = 154 \text{ pa}$$

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3i) Bending moment

$$EI \frac{d^2y}{dx^2} = +1969.4[x] - \frac{450[x-0.5]^2}{2} + \frac{450[x-1.5]^2}{2} - \frac{800[x-1.5]^2}{2} - \frac{2000[x-1.5]^2}{2} + \frac{800[x-2.5]^2}{2}$$

3ii) Slope

$$EI \frac{dy}{dx} = \frac{1969.4[x]^2}{2} - \frac{450[x-0.5]^3}{6} + \frac{450[x-1.5]^3}{6} - \frac{800[x-1.5]^3}{6} - \frac{2000[x-1.5]^3}{6} + \frac{800[x-2.5]^3}{6} + Ax + B$$

3iii) Deflection

$$EIy = \frac{1969.4[x]^3}{6} - \frac{450[x-0.5]^4}{24} + \frac{450[x-1.5]^4}{24} - \frac{800[x-1.5]^4}{24} - \frac{2000[x-1.5]^4}{6} + \frac{800[x-2.5]^4}{24} + Ax^2 + Bx + C$$

3iv) $y=0$ $x=0 \rightarrow B=0$ $y=0$ $x=3.1$

$$EIy = \frac{1969.4[3.1]^3}{6} - \frac{450[3.1-0.5]^4}{24} + \frac{450[3.1-1.5]^4}{24} - \frac{800[3.1-1.5]^4}{24} -$$

$$\frac{2000[3.1-1.5]^3}{6} + \frac{800[3.1-2.5]^4}{24} + A(3.1) + 0$$

$$EIy = 1344.44 - 856.83 + 122.88 - 218.45 - 1365.3 + 4.32 + 3.1A$$

$$= -968.94 + 3.1A$$

$$-3.1A = -968.94$$

$$A = -968.94$$

$$-3.1$$

$$A = 312.56$$

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$$3v) EI = 1.53 \text{ MPa} = 1.53 \times 10^6$$

$$x = 1.55 \text{ m}$$

$$1.53 \times 10^6 y = \frac{1969.4(1.55)^3}{6} - \frac{450(1.55-0.5)^4}{24} + \frac{450(1.55-1.5)^4}{24} -$$

$$\frac{800(1.55-1.5)^4}{24} - \frac{2000(1.55-1.5)^3}{6} + A(1.55) + B$$

$$1.53 \times 10^6 y = ~~1969.4(1.55)^3~~ + 1199.51 - 17.73 - 714.063$$

$$y = 3.057 \times 10^{-4}$$

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$$4i) \sigma_c = \frac{PD}{2t\eta_L}$$

$$t = \frac{PD}{2\sigma_c \eta_L}$$

$$t = \frac{(14.5 \times 10^6)(0.28)}{2(150 \times 10^6)(0.53)}$$

$$t = 2.55 \times 10^{-3} \\ = 2.55 \text{ mm}$$

$$d = 280 \text{ mm} = 0.28 \text{ m}$$

$$L = 600 \text{ mm} = 0.60 \text{ m}$$

$$\eta_L = 53\% = 0.53$$

$$\eta_c = 62\% = 0.62$$

$$P = 14.5 \text{ bar} = 14.5 \times 10^6 \text{ N/m}^2$$

$$\sigma_{\max} = 150 \text{ MPa} = 150 \times 10^6 \text{ N/m}^2$$

$$4iii) t_{\min} = 1.09 \text{ mm}$$

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$$4ii) d = 280 \text{ mm} = 0.28 \text{ m}$$

$$L = 600 \text{ mm} = 0.60 \text{ m}$$

$$n_1 = 53\% = 0.53$$

$$n_2 = 62\% = 0.62$$

$$p = 14.5 \text{ bar} = 14.5 \times 10^5 \text{ N/m}^2$$

$$\sigma_{\max} = 150 \text{ MPa} = 150 \times 10^6 \text{ N/m}^2$$

$$\sigma_L = \frac{PD}{4t\eta_c}$$

$$t = \frac{PD}{4\sigma_L\eta_c}$$

$$t = \frac{(14.5 \times 10^5)(0.28)}{4(150 \times 10^6)(0.62)}$$

~~$$t = 1.09 \times 10^{-3} \text{ m}$$~~

$$t = 1.09 \times 10^{-3} \\ = 1.09 \text{ mm}$$