



Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold the flap and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either legibly copy your script or have it typed and the expense of typing is charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

[illegible]

79%
✓ ✓ ✓ good

✓ at Feb 25

Q1

a)

$$i) \frac{d}{dx}(x+3) = 1 \quad \checkmark$$

(2)

$$ii) \frac{d}{dx}(12x^3 + 10x^{\frac{1}{2}}) = 36x^2 + 5x^{-\frac{1}{2}} \quad \checkmark$$

(3)

$$iii) \frac{d}{dx}(x^{-5} + x^3) = -5x^{-6} + 3x^2 \quad \checkmark$$

(3)

$$iv) \frac{d}{dx}(\sin(8x))$$

$$8x \rightarrow \sin x \quad \text{or} \quad 8x = u \Rightarrow \sin(u) \quad \checkmark$$

$$= \cos(8x) \cdot 8$$

$$= 8\cos(8x) \quad \checkmark$$

(2)

$$v) \frac{d}{dx}(x \cos x) = \cos x - x \sin x \quad \checkmark$$

(5)

x	$\cos x$
1	$-\sin x$

$$vi) \frac{d}{dx}(2x-3)^{10} \quad u^{10}, u = 2x-3$$

$$= 10(2x-3)^9 \cdot 2$$

$$= 20(2x-3)^9 \quad \checkmark$$

(3)

$$vii) \frac{dy}{dx}(3x + \tan x) = 3 + \sec^2 x \quad \checkmark$$

(1)

note either $y = f(x)$ or $\frac{d}{dx} f(x) =$

$\Rightarrow \frac{dy}{dx} = f'(x)$

Q1

$$b) \frac{d}{dx} (x^{-1} + \cos(2x)) = -x^{-2} - 2\sin(2x)$$

$$\frac{d^2 y}{dx^2} = 2x^{-3} - 4\cos(2x)$$

$$\frac{d^3 y}{dx^3} = -6x^{-4} + 8\sin(2x)$$

4

$$c) \frac{dy}{dx} \left(\frac{x-1}{4+x^2} \right) = \frac{1(4+x^2) - 2x(x-1)}{(4+x^2)^2}$$

$$= \frac{4+x^2 - 2x^2 + 2x}{(4+x^2)^2}$$

$$= \frac{-x^2 + 2x + 4}{(4+x^2)^2}$$

$$\left. \frac{dy}{dx} \right|_{x=0} = \frac{-(0)^2 + 2(0) + 4}{(4+(0)^2)^2}$$

$$= \frac{4}{16}$$

$$= \frac{1}{4}$$

6

29

Q2

$$i) \int 10x + x^3 dx = 5x^2 + \frac{1}{4}x^4 + C$$

(3)

$$ii) \int x - 3e^x dx = \frac{1}{2}x^2 - 3e^x + C$$

(3)

$$iii) \int x^{-1} + 4 dx = \ln x + 4x + C$$

(3)

$$iv) \int 2x(3x+4) dx = \int 6x^2 + 8x dx$$

$$= 2x^3 + 4x^2 + C$$

(3)

$$v) \int_0^2 3x^2 + 2x dx = \left[x^3 + x^2 \right]_0^2$$

$$\Rightarrow 2(2)^3 + (2)^2 - (0)^3 + (0)^2$$

$$= 12$$

(3)

Q2

b)

$$\int_0^1 \frac{x}{\sqrt{x^2+1}} dx$$

Change limits

$$u = (1)^2 + 1 = 2$$

$$u = (0)^2 + 1 = 1$$

good

$$\text{Let } u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$\Rightarrow \frac{du}{2} = x dx$$

$$\int_1^2 \frac{x}{u^{1/2}} du$$

$$\int_1^2 x u^{-1/2} du$$

~~scribbles~~

$$I = \int_{u=1}^2 \frac{\frac{du}{2}}{\sqrt{u}}$$

2

$$= \frac{1}{2} \int_1^2 u^{-1/2} du$$

$$= \frac{1}{2} \left[\frac{u^{1/2}}{1/2} \right]_1^2$$

$$= \frac{1}{2} [2\sqrt{u}]_1^2$$

$$= \frac{1}{2} [2\sqrt{2} - 2\sqrt{1}]$$

$$= \frac{1}{2} \cdot 2(\sqrt{2} - 1)$$

$$= \sqrt{2} - 1$$

Q2

$$uV - \int v \frac{du}{dx}$$

$$c) \int x \sin(2x) dx$$

$$u = x \quad \frac{du}{dx} = \sin 2x$$

$$\frac{du}{dx} = 1 \quad v = -\frac{1}{2} \cos 2x$$

Done by thinking what differentiation would get this result

$$= -\frac{1}{2} x \cos 2x - \int -\frac{1}{2} \cos 2x dx$$

$$= -\frac{1}{2} x \cos 2x - \left(-\frac{1}{4} \sin(2x) \right) + C$$

$$= -\frac{1}{2} x \cos 2x + \frac{1}{4} \sin(2x) + C$$

5

$$ii) \int_1^e x \ln x dx$$

$$\ln x \quad x$$

$$\frac{1}{x} \quad \frac{1}{2} x^2$$

$$\left[\frac{1}{2} x^2 \ln x \right]_1^e - \int_1^e \frac{x^2}{2x} dx$$

$$= \int_1^e \frac{1}{2} x dx$$

$$\left[\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 \right]_1^e$$

$$\left(\frac{1}{2} (e)^2 \ln e - \frac{1}{4} (e)^2 \right) - \left(\frac{1}{2} (1)^2 \ln 1 - \frac{1}{4} (1)^2 \right) = \left(\frac{e^2}{2} - \frac{e^2}{4} \right) - \frac{1}{4}$$

$$= 1.59726$$

$$2.097 = 1.59726 \text{ to 2 d.p.}$$

4

Q3

$$i) (3+2i)(1-i)$$

$$3+2i-3i-2i^2$$

$$3-i+2$$

$$= 5-i$$

(2)

$$ii) \frac{3}{2+i}$$

$$\frac{3}{2+i} \cdot \frac{2-i}{2-i} = \frac{6-3i}{4-i^2}$$

$$= \frac{6-3i}{5}$$

$$= \frac{6}{5} - \frac{3i}{5}$$

(2)

$$iii) \frac{4}{i^5} = -4$$

$$-1 \times -1 \times -1 \times -1 \times -1$$

$$\frac{4}{i^2 i^3} = \frac{4}{(-1)(-1)} \cdot \frac{1}{i}$$

$$= 4 \cdot \frac{1}{i}$$

$$= 4 \cdot \frac{1}{i} \cdot \frac{i}{i}$$

$$= -4i$$

(0)

Q4)

$$i) 3^{2x} = 2^{x+1}$$

$$2x \ln 3 = (x+1) \ln 2$$

$$2x \ln 3 = x \ln 2 + \ln 2$$

$$2x \ln 3 - x \ln 2 = \ln 2$$

$$x(2 \ln 3 - \ln 2) = \ln 2$$

$$x = \frac{\ln 2}{2 \ln 3 - \ln 2}$$

$$x = 0.46 \text{ to 2 d.p.}$$

(4)

$$ii) 2 \ln x = \ln(5x-6)$$

$$\ln x^2 = \ln(5x-6)$$

$$e^{\ln x^2} = e^{\ln(5x-6)}$$

$$x^2 = 5x - 6$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2)$$

Both sols valid

$$\therefore x=3 \vee x=2$$

(2)

2

2

✓ 10

3

3

c) i) $P = Z\left(\frac{3}{5} - \frac{4j}{5}\right) + \frac{4}{5} + \frac{3j}{5} = 2 - j$
ii) $Q = \frac{3}{5} - \frac{4j}{5} + \frac{4}{5} - \frac{3j}{5} = -\frac{1}{5} - \frac{7j}{5}$

This is a good effort
Hannah, well done!

