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Creative mathematics teaching in the eye of the beholder: focusing on teachers' conceptions

Hana Lev-Zamir^{a,b} and Roza Leikin^{a*}

^aUniversity of Haifa, Israel; ^bOranim College of Education, Israel

We consider developing mathematical creativity in each student to be one of the purposes of school mathematics education. We believe that creative teachers are essential to achieving this goal. A creative teacher advances students' motivation, encourages knowledge construction, and promotes creativity. This article presents part of a larger exploratory study that analyses teachers' conceptions of creativity in teaching mathematics and proposes a model for both describing and analysing teachers' conception of creativity in teaching mathematics. From general literature on human creativity we accept fluency, flexibility, originality, and elaboration as main components of the model. Using the grounded theory approach we suggest a system of subcategories. We provide several illustrative examples from teachers' discourse about creativity in teaching and from a mathematics lesson that one of the teachers considers to be creative. The aim of these examples is to demonstrate the manner in which the model was designed.

Keywords: mathematics teaching; creativity; teacher conceptions

Introduction

Creativity is a complex concept, considered by various scholars from different points of view (Haylock 1987; Sriraman 2005; Mann 2006). Modern educational trends are characterised by a shift from the static view of creativity as a personal characteristic that cannot be changed toward the dynamic one, of creativity as a characteristic of personal development (Silver 1997; Milgram and Hong 2009; Subotnik, Pillmeier, and Jarvin 2009). This trend leads to the appreciation of the centrality of creativity in the mathematics classroom (Haylock 1987; Mann 2006; Bolden, Harries, and Newton 2010) and the importance of developing creativity in each and every student, as well as in each and every teacher. There is evidence, however, that teachers' ideas about creativity are limited (Bolden, Harries, and Newton 2010).

The centrality of creativity in child development is rooted in Vygotsky's (1930/1984) argument that creativity ('imagination' in Vygotsky's terms) is one of the basic mechanisms that allow development of new knowledge. A child activates imagination when connecting new and previously known concepts, elaborating the known constructs, or developing abstract ideas. Imagination (or creativity) is a basic component of knowledge construction. Thus, teaching with and for creativity can intensify the learning process.

The need for teacher creativity is also rooted in the ambiguity and uncertainty of the teaching process (Kagan 1992) which expert teachers solve by means of creative

*Corresponding author. Email: rozal@edu.haifa.ac.il

performance (Sawyer 2004). Several recent publications have been devoted to teachers' conceptions of creativity in teaching mathematics. For example, analysing discussions with prospective mathematics teachers, Shriki (2009) argues that their knowledge about creativity is insufficient for a discussion of the subject. Lithner (2008) suggests analysing mathematical activities in the classroom through the lens of creative, as opposed to imitative, thinking. Bolden, Harries, and Newton (2010) analysed written questionnaires and semi-structured interviews with prospective elementary school teachers about their conceptions of creativity, and showed that these conceptions were narrow and associated with particular teachers' actions. However, analysis of creativity in teaching mathematics is clearly overlooked in mathematics education research (Leikin 2009, 2011).

Our study explores conceptions of creativity in teaching mathematics held by in-service teachers. We argue that, in order to develop teachers' capability of teaching with and for creativity, we should better understand their conceptions of creativity. We aimed at devising a model directed at the analysis of what teachers consider to be creative, as expressed in their talk about creativity and in their practice. This model connects theory and practice in the field of creativity, in general, and in the field of teaching mathematics, in particular.

In the following sections we outline the theoretical background on general creativity and on creativity as it relates to teaching mathematics. Then we present several examples that illustrate the way in which subcategories of the model are devised and show the elements of the teaching process that, in the eyes of teachers, indicate creative teaching.

Theoretical background

Creativity as related to teaching mathematics

This study distinguishes between general and specific creativity (Pirto 1999), whereby *general creativity* is associated with using problem-solving patterns in one field for solving problems in another field, whereas *specific creativity* refers to creativity in a particular field by taking into account the logical deductive nature of the field. In the course of the discussion of creativity in mathematics and in teaching mathematics (e.g., Haylock 1987; Ervynck 1991; Silver 1997; Sriraman 2005; Mann 2006), our focus is on a specific type of creativity – creativity in teaching mathematics.

The study is anchored in the belief that teaching is a complex activity that must be creative and directed at students' creativity. Educational literature presents a variety of views on the criteria by which a person can be considered creative, and a diversity of definitions is provided in the research literature for the concept of both general and mathematical creativity (Haylock 1987; Sriraman 2005; Mann 2006). At the same time, by reviewing educational literature between 1966 and 1985, Haylock (1987) demonstrated that the issue of creativity was neglected in mathematics education research. Two decades later, Leikin (2009, 2011) reviewed research publications between 1999 and 2009 and showed that the situation had not changed, and that research on creativity is still secondary to research on mathematical thinking, learning, and teaching. Mann (2006) noted that the lack of an accepted definition of mathematical creativity hinders research efforts.

We elaborate on Haylock's argument that "any definition of mathematical creativity in schoolchildren must refer to both mathematics and creativity" (62) and suggest that any definition of mathematical creativity in mathematics teaching must refer to mathematics, teaching, learning, and creativity. From the general literature on human creativity we adopt fluency, flexibility, originality, and elaboration as main components of creativity in each and every field of human activity (Guilford 1967a, 1967b, 1968; Torrance 1974), including learning and teaching mathematics. *Fluency* relates to the continuity of ideas, flow of associations, and use of basic and universal knowledge. *Flexibility* is associated with changing ideas, approaching a problem in various ways, and producing a variety of solutions. *Originality* is characterised by a unique way of thinking and unique products of a mental or artistic activity. *Elaboration* relates to the ability to describe, illuminate, and generalise ideas. The four components are mutually related, but not all of them need to be present at the same time.

The use of this framework in our study is based on the observation that the four components of creativity are addressed by different researchers as being closely connected to teaching mathematics. Teachers must be *fluent* in lesson management, including the explanations of mathematical ideas they provide for their students (e.g., Leinhardt 1993). Teachers must be *flexible* in reacting to students' unexpected responses (Simon 1997; Leikin and Dinur 2007). Teacher *originality* throughout the school year and the ability to surprise their students increase student motivation (Movshovitz-Hadar 1988). *Elaboration* of students' mathematical ideas is the main mechanism for moving with the students to a new mathematical territory (Lampert 2001). At the same time, teaching mathematics should be aimed at developing students' learning and knowledge to enable *fluent* problem solving, *flexible* mathematical reasoning, the generation of *original* ideas, and *the elaboration* of others' mathematical thoughts (e.g., Polya 1963; Even, Karasenty, and Friedlander 2009). Fluency, flexibility, originality, and elaboration also characterise creative problem solving and problem posing in mathematics (Krutetskii 1976; Silver 1997; Leikin 2009).

In this study we focus on three of the four basic categories – flexibility, originality, and elaboration – because, similar to researchers in the field of mathematics teachers' expertise (e.g., Leinhardt 1993), we consider fluency to be a primary characteristic of the quality of the teachers' knowledge and proficiency. At the same time, although the categories of flexibility, originality, and elaboration are desirable in every mathematics lesson, they are not integral to the teaching process and, thus, may be seen as characteristics that are unique to creative teaching.

The relative nature of creativity in this study

The distinction between *absolute* and *relative* creativity is essential for our study (Leikin 2009; objective and subjective creativity, Lytton 1971). Absolute creativity is associated with great historical works (Vygotsky 1930/1984), with discoveries at a global level. For example, in mathematics, absolute creativity may be represented by the discoveries of Pierre de Fermat or David Hilbert. In pedagogy we can refer, for example, to the works of John Amos Comenius or Johann Heinrich Pestalozzi as absolutely creative works. Relative creativity refers to discoveries by a certain person

in a specific reference group. This type of creativity refers to human imagination that produces something new for that person (Vygotsky 1930/1984).

Creativity in school mathematics obviously differs from the creativity of professional mathematicians (Sriraman 2005). It is considered with respect to the students' previous experiences and to the performance of other students with whom they have a common educational history (Leikin 2009). Liljedahl and Sriraman (2006) argue that at the school level it is feasible for students to offer new (for them) insights/solutions in mathematics, and that these insights are indicators of their creativity.

We believe that teachers may arrive at original and flexible mathematical and pedagogical solutions in the course of their teaching practice. Our design of the model for the analysis of teachers' conceptions of creativity in teaching mathematics adopts a relative perspective of this phenomenon, with specific focus on the teaching and learning of mathematics.

Teacher conceptions

Scheffler (1965) distinguished between teachers' knowledge and beliefs. Beliefs usually express "a disposition to act in a certain way under certain circumstances" (76). In contrast to beliefs, which are propositional in nature, knowledge is operational. People do not require proofs for their beliefs, but they can prove the facts that they know (Scheffler 1965). Following Thompson (1992), who described teachers' conceptions of the nature of mathematics as their knowledge and beliefs concerning the discipline of mathematics, the present study focuses on teachers' conceptions of creativity in teaching mathematics. In this paper, teachers' mathematical conceptions refer to their vision of creative mathematical content. Teachers' pedagogical conceptions refer to their awareness of didactic and psychological aspects of creativity in teaching and learning mathematics.

The study

The *purpose of the study* presented in this paper is to design a model for the characterisation of teachers' conceptions of creativity in teaching mathematics.

Eleven elementary and middle school mathematics teachers (Grades 4 to 8; student ages 11 to 14), leaders in their schools, participated voluntarily in this study. The data were collected using a qualitative research paradigm, through observations of lessons and individual, semi-structured interviews with the teachers. The interviews preceded and followed the lesson observations. The first author of this paper carried out all data collection.

During the interviews, the teachers were asked to think aloud about creative mathematics teaching, and provide examples of expressions of creativity in mathematics teaching. They were asked to invite the researcher to (4–5) lessons that they themselves considered to be creative. Before each lesson, the teachers outlined the creative elements that might be of particular interest and worthy of observation. After each lesson, teachers provided a reflective account of what had happened during the lesson. The researcher attempted to be as neutral as possible during data collection, and did not provide any evaluative comments on what had been said during the interviews or during the lessons. She asked the teachers to

clarify their ideas when they were not entirely clear, and to verify the reasons for the decisions taken before and during the lessons. During the study, the teachers did not participate in any teacher education framework associated with creativity in mathematics teaching. Altogether 48 lessons were observed and 107 interviews were conducted. The data were video- and audio-recorded, and all the recordings were transcribed.

Data analysis

As noted earlier, we designed the model by devising subcategories for the three components of creativity – flexibility, originality, and elaboration. Development of the subcategories was based on the grounded theory approach (Strauss and Corbin 1990), using content analysis of teachers' discourse by repeated observations of the video records of lessons and interviews, and repeated reading of the transcripts.

First, teachers' statements about creativity in teaching mathematics, as well as their actions and interactions with students during the lessons, were classified as fitting into one of the three categories – flexibility, originality, and elaboration. The category of flexibility included talk and actions related to transformations of various kinds associated with teaching mathematics and with different solutions to a particular mathematical or pedagogical problem. The category of originality included statements and actions related to teachers' and students' ideas that were unusual or unconventional with respect to the regular curriculum being taught and the commonly accepted socio-mathematical norms (Cobb 2000) in the classroom. The category of elaboration comprised statements and actions related to moving to a higher mathematical and pedagogical level than before. Secondly, all the data classified as belonging to a particular category – flexibility, originality, or elaboration – were categorised in order to create subcategories that characterise teachers' conceptions of flexibility, originality, and elaboration in teaching mathematics. The data analysis was inductive, first identifying the initial subcategories based on some of the data collected for a particular teacher, then refining and expanding the initial subcategories based on more data collected with the same teacher, and finally, to refining the devised sub-categories by the analysis of data collected with all the participating teachers. Furthermore, all the subcategories devised for the various categories of creativity were analysed for commonalities and differences across the categories.

It should be noted that some of the activities described by teachers as examples of creativity in teaching mathematics would not have been recognised as such by us without their explanation. However, while analysing the data we distanced ourselves from making judgments about the tasks the teachers had chosen, by alluding to the 'relativity' of creativity.

In this paper we present several examples that illustrate the way in which we designed the model. The examples are borrowed mainly from the data collected with a teacher named Gili (pseudonym), as her views were fairly common and coincided with those of many teachers who participated in the study. When some of the subcategories of flexibility, originality, and elaboration were missing in Gili's talk and actions, we used examples from other teachers.

Devising the model

Examples and data analysis

Gili is an elementary school teacher with 30 years of experience in teaching mathematics to 3rd, 4th, 5th, and 6th graders. She is considered by her colleagues as a creative and leading teacher in her school. According to her report, she enjoys teaching because “she can apply her imagination and improvise all the time.” When asked how she sees creativity in mathematics teaching, she answered:

1-Gili: I participate in courses for teachers... I listen and come up with something new... [1-1] I look for an idea I can implement in my class and enjoy it [1-2] and develop it in directions that are suitable to me [1-3]... A teacher must be ready for surprises [1-4]. For example, I can define the purpose of a lesson but a student asks a question that I must address immediately [1-5]... I can switch and change planning because of students' remarks [1-6] and I share it with everyone [in the class]...

[A creative teacher is] a person who *knows mathematics and loves it and wants to teach* what she loves and knows [1-7]... I always try to *develop* something that *is beyond what is written in books* [1-8]... It is a very important way to enhance students' knowledge in *attractive ways* [1-9]... I don't miss any opportunity to *do mathematics in a different way* [1-10]... [Teachers'] *creativity leads students to thinking* [1-11]. One can view each *student's discovery and each generalisation he made that was not known before as his creation* [1-12].

The excerpt contains clear indicators of Gili's conception of creativity in teaching mathematics as associated with the generation of new ideas that go beyond the school textbooks [e.g., 1-1, 1-8]. We consider this as an indicator of her views on *originality* in teaching mathematics. New ideas can be generated to suit teachers' own enjoyment and preferences [1-2, 1-3] or to be interesting to the students and stimulate their mathematical reasoning [1-9, 1-11, and later 2-1]. Gili's view on creativity, as it relates to changing the trajectory of the planned lesson in response to various 'surprises', including students' unexpected questions or remarks [1-4, 1-5, 1-6], is a clear indicator of her connecting creativity in teaching mathematics with *pedagogical flexibility* (Leikin and Dinur 2007; Simon 1997). In Gili's eyes, only a knowledgeable teacher who loves the profession can be creative [1-7]. The fact that Gili considers doing mathematics in different ways [1-10] to be essential is an indication of her *mathematical flexibility*. Finally, in this excerpt Gili points out that creativity in teaching mathematics can be seen in students' discoveries (for us, indicators of originality) and generalisations (for us, indicators of elaboration) [1-12].

This excerpt (1-Gili) reveals that Gili's conception of creativity in teaching mathematics includes *teacher actions* directed at students' interesting and meaningful learning and at *opening opportunities for students'* own creativity. We found that these two directions of talk about creativity are typical for most teachers who participated in this study. We therefore introduce the distinction between teacher-directed conceptions of creativity in mathematics teaching and student-directed conceptions of creativity.

Teacher-directed conceptions of creativity

Teacher conceptions that include the connections between creativity in teaching mathematics and teaching-related acts that make teachers themselves creative, we

call teacher-directed conceptions of creativity (or teacher-directed creativity). As noted above (in the analysis of excerpt 1-Gili), teacher-directed creativity is aimed at generating student enjoyment [1-9, 2-1], encouraging students' mathematical reasoning [1-11], suiting the content to students and the curriculum [3-3, below], and developing students' own creativity [1-12]. Teacher-directed creativity has been described by the teachers in different ways, most frequently by reference to changing the mathematics in the tasks and changing the pedagogy (setting) associated with mathematical tasks.

Changing the mathematics in the task

2-Gili: I give them such a pattern (see Figure 1). Here (on the left) there is $\frac{1}{6}$ and $\frac{3}{6}$, and they need to think how many times $\frac{1}{6}$ makes $\frac{3}{6}$, and put this number in the circle... This seems to me *more interesting than the ordinary exercises* [2-1]... It is a kind of idea that came into my mind... I must have seen it somewhere with *addition*... [2-2]

Task 1

Find numbers in the vertexes of the square so that their product will be equal to the numbers on the sides of the square.

Gili started from an addition-related task [2-2] and transformed it into a multiplication-related one (Task 1). She used this (new) task to exercise multiplication of fractions in a more interesting way than the "ordinary exercise" [2-1]. We treat this task transformation as an indicator of Gili's flexibility in the field of mathematics directed at teaching.

This type of talk was also typical of other teachers who provided examples of changing the mathematics in tasks.

Another teacher, Erica, showed a different aspect of changing the mathematics in a task to match the topic being taught in class: numbers. Erica described how she used the same task with different numbers in different school grades:

3-Erica: Many times I take an activity that I did in one class, it can be an activity from grade 2 that involves addition and subtraction [of natural numbers] up to 100. *I change the activity* [3-1], *I match the numbers* [3-2] *to grades 5 or 6* [3-3] *using fractions or decimals*.

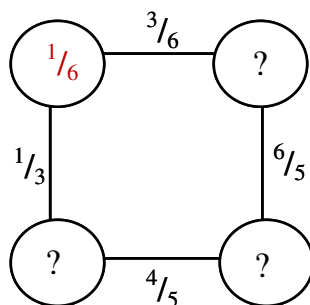


Figure 1. Task 1.

We must point out that, in our opinion, the mathematical transformations of the tasks were mostly minor, related mainly to changes in operations [2-2] or to changing the numbers [3-2] used in the tasks.

Changing the pedagogy associated with the task

In the following example, Gili describes how she changed the instructional setting in an activity in which students were asked to find the ratio between the diameter and the circumference of a circle.

4-Gili: With one class I taught [the relationship between the circumference and the diameter]... They cut [paper circles], measured, pasted... [4-1] *I was not satisfied with what they learned* [4-2]. *I was looking for something else* [4-3]...

I took the students to the gym and asked several students to stand in a straight line [holding hands] and some of other students were asked to stand in a circle so that the *line of the students served as a diameter*... *The number of students in the circle was approximately three times* [the number of students in the diameter] [4-4]. We repeated this activity with a different number of children standing in the diameter... and the same *discovery* was made... [4-5]

[To receive a more exact ratio] they later continued to investigate the connection between the diameter and the circumference *by measuring hoops and ropes* [4-6]... At the end [of the activity], the students realised there is a [constant] ratio between the circumference and the diameter [4-7].

Like other teachers, Gili has a rich repertoire of examples for changes in the learning setting directed at meaningful learning by her students. In the above excerpt [4-Gili], she started by cutting out paper circles and measuring them [4-1], then she took her students to the gym where they stood in a circle [4-4], and later she asked the students to measure hoops [4-6]. The changes in the instructional setting described in this excerpt (4-Gili) result in a discovery activity for the students [4-5, 4-7]. At the same time, we learn that the main motive for these transformations is Gili's dissatisfaction with the effectiveness of the initial setting [4-2, 4-3].

These changes clearly demonstrate Gili's pedagogical flexibility, which she exercises in order to raise the quality of her students' mathematical understanding. We are uncertain, however, about who owns the originality of the mathematical tasks and activities she employed, because we do not have sufficient information about the sources of these ideas.

The idea of mathematical activities in which students participate as 'actors' has recurred in several interviews, and we consider it as an indicator of creativity in teaching mathematics.

5-Eva: [My] lesson on parallel lines is an example of a creative lesson. It was not something guided by the book... I asked a group of students to form parallel lines in the school playground. They organised two equal groups and stood *[in parallel lines]* [5-1]: students from one group facing students from the other group. Then I asked: "If two students from one group join the other group, can you create parallel lines?"... They *discovered* [5-2] that parallel lines do not have to be of equal length.

In the above excerpt Eva, (5th grade teacher) asked her students to investigate the properties of parallel lines “constructed from students” [5-1] in the school playground. She argues that this activity has led her students to understand that straight lines (including parallel lines) need not be of equal length.

- 6- Lihi: In one of the lessons I gave students a hand-shaking problem: “Eight friends met at the party and shook hands with each other once. How many times did friends shake hands at this party altogether?” I asked 8 students to *demonstrate the solution* [6-1]. At the beginning it was a mess. They did not have a plan [for hand-shaking]. Then, as I suggested, they stood in a line and *discovered* [6-2] that each student shakes one hand fewer than the previous student, and that there is a rule [6-3].

Lihi (5th grade teacher) gives an example of solving a problem where ‘mathematical play’ demonstrates the solution to the problem [6-1]. Again, the example (6-Lihi) illustrates that teachers connect changes in the classroom setting with creativity in teaching mathematics. For us, these changes indicate teachers’ pedagogical flexibility, which can lead to potentially meaningful mathematical experiences for their students. Moreover, when addressing creativity in teaching mathematics and describing ‘creative activities’, teachers usually indicated that these activities led to mathematical discoveries by the students [4-5, 5-2, 6-2, 6-3]. Clearly, then, these discoveries are connected with students’ creative acts in mathematics lessons.

In addition to the setting transformations presented in the above examples (4-Gili, 5-Eva, and 6-Lihi), when describing creativity in teaching mathematics, other teachers described activities that enabled students to work with various types of manipulatives (similar to 4-1) and models of mathematical objects (similar to 4-6).

Student-directed conceptions of creativity

We refer to teachers’ conceptions about the connections between creativity in teaching mathematics and mathematical activities that promote student creativity, including their mathematical flexibility, originality (imagination), and elaboration, as student-directed conceptions of creativity (or student-directed creativity). Examples of teacher statements that reflect student-directed creativity can be seen above [1-10, 1-12, 4-5, 4-7, 5-2, 6-2, 8-1, 8-2, 8-3]. In this section, we present an example from Gili’s lesson as being typical of the student-directed examples provided by teachers. The example demonstrates Gili’s view of how teachers’ actions can enhance the potential for the development of students’ mathematical creativity. As in other examples collected in this study, student-directed creativity clearly requires teacher flexibility, and is based on suggestions originally provided to the students by the teacher.

Students generate their own mathematical exercises

As noted earlier, Gili was asked to invite the first author of this paper (Hana) to observe lessons that Gili believed were creative. In a 4th grade mathematics lesson

that took place on 20.05.2008, Gili asked students to use the digits “of the day” to create numeric expressions of different values. During the interview Gili reported:

7-Gili: I begin with an activity: “Today’s Date.” *Not in the sense of* [implementing this activity] *first time* ... [7-1] *but in the sense of* [leading students to] *search for exercises* [7-2] ... and *I have no idea where it is going to take the class* [7-3].

We can see fractions like $\frac{5}{20} = \frac{2}{8}$... and there are simple exercises like $2 \times 5 = 2 + 8$... and there are more, but it is not so important what I arrive at. *I am sure they will have wonderful ideas of their own* [7-4] ...

Gili claims that creative teaching does not necessarily have to include the teachers’ new (original) ideas [7-1], but can be designed to stimulate students to generate their own original ideas [7-4]. This requires the teacher to be flexible, that is, ready for unpredictable situations and prepared to adjust the course of the lesson to the students’ ideas [7-3]. Gili was excited and curious to see “where it would take the students”, confident that they would have many suggestions.

During the lesson, the students raised different suggestions, searched for examples, and listened to the suggestions of their classmates. Gili asked clarification questions and challenged students by hinting at other possibilities.

Episode 1 (8-Gili)

The date is written on the board: 20.05.2008

Student 1: $8 \times 5 = 20 + 20$

Student 2: $\frac{5}{20}$ is $\frac{1}{4}$ and $\frac{2}{8}$ is also $\frac{1}{4}$. [8-1]

Gili: $\frac{5}{20} = \frac{2}{8}$ [writes on the board]. Are they equal? [8-2]

Student 2: [Yes they are] because both equal $\frac{1}{4}$.

Students 3,4,5,6: $\frac{8}{2} \times 5 = 20$, $5 \times 20 = 20 + 80$, $5 \times 8 = 20 \times 2$ [8-3], $5 \times 8 - 20 = 20$

Student 7: Gili [students in Israel address teachers with their first names], what did you find?

Gili: When I saw the date in the morning, I immediately ... wrote things down ... something that you didn’t say. *Maybe they [my examples] are too simple for you* [8-4] ... [writes on the board] $2 \times 5 = 2 + 8$, and also $20:5 = 8:2$.

Gili presented students with a task that required them to create their own exercises. To generate two or more different exercises with the same result, students needed to be flexible mentally [8-1, 8-2, 8-3]. Moreover, after one student presented an example and other students were required to generate different ones, they had to be flexible to do so (Students 3, 4, 5, 6). The ‘Today’s Date’ task was therefore directed at developing students’ mathematical flexibility. Note also that management of the lesson required Gili to be sensitive to her students and flexible in her lesson management. We see that during Episode 1, she asked only minor clarification questions [8-2]. She also encouraged students’ independent reasoning by saying that her examples were simpler than those provided by the students themselves [8-4].

Episode 2 (9-Gili)

Student 5: This is $[20:5 = 8:2]$ the same thing that is on the board [points to $\frac{5}{20} = \frac{2}{8}$].

Gili: It is similar but not exactly the same. *This belongs to a topic that is called 'ratio' and you will learn about it in the 6th grade.* [9-1]. . . . Can you find a fraction that equals $\frac{2}{5}$? Those who do, tell me what helped you find it.

Student 2: No. It is like $\frac{20}{5} = \frac{8}{2}$

In the after-the-lesson interview, Gili reflected on the responses of Student 5 and Student 2, and explained her pedagogical considerations related to the management of the discussion in Episode 2.

10-Gili: There was much *creativity and thinking* [10-1]. I had no idea where it would go [10-2]. . . It was fascinating in my opinion . . . This activity enables *connecting many subjects* [10-3] – arithmetic operations, fractions, identities . . . *they think, they create* [10-4]. They listened and related to various suggestions and even *touched, without knowing it, on the topic of ratio* [10-5]. . . *I didn't follow up on it because this is related to the 6th grade and I didn't want to confuse them* [10-6].

In Episode 2, Student 5 observed equivalence between the statements $20:5 = 8:2$ and $\frac{5}{20} = \frac{2}{8}$. In Gili's opinion this discovery was premature for her 4th grade students [10-6], and chose to stop the discussion about this conjecture, because according to the curriculum it belonged to content learned in the 6th grade [9-1, 10-5]. From the excerpt (10-Gili) we learn once again that Gili connects creativity during the lesson with student thinking and student creativity [10-1, 10-4]. In her view, to be creative students must connect mathematical knowledge that allows their creative ability to reveal itself [10-3]. To enable students to be creative during lessons, teachers should allow them to proceed in any direction they want [i.e., plan to be flexible, 10-2]. As a result, a rich collection of expressions was generated by the students, so that overall Gili learned with her students. Figure 2 summarises the student-generated mathematical expressions related to the 'Today's Date' task.

Note that solving and discussing the 'Today's Date' task, which had been allowed only ten minutes in Gili's planning, lasted half an hour, testifying to her flexibility.

Some notes on elaboration

Although the categories of flexibility and originality were clearly represented in teachers' talk, reflecting both teacher-directed and student-directed conceptions of creativity, we were disappointed to realise that the category of elaboration was underrepresented both in the teacher interviews and in the lessons Hana was invited to observe. We did not find any indication of elaboration-related acts, either in

20.05.2008	$8 \times 5 = 20 + 20$	$5 \times 20 = 20 + 80$	$5 \times 8 = 20 \times 2$	$5 \times 8 - 20 = 20$
	$\frac{20}{5} = \frac{8}{2}$	$\frac{2}{5} = \frac{8}{20}$	$\frac{5}{20} = \frac{2}{8} = \frac{1}{4}$	$\frac{8}{2} \times 5 = 20$
		$20:5 = 8:2$ is as $\frac{5}{20} = \frac{2}{8}$		

Figure 2. Student-generated expressions for the 'Today's Date' activity.

teachers' talk or in the lessons, in relation to teacher-directed conceptions of creativity. However, we found, several excerpts that can serve as evidence that some teachers do connect creativity with mathematical activities that lead students to elaboration of mathematical ideas, including posing mathematical problems and generalisation. The next example (11-Debby) illustrates teachers' talk of this kind:

11-Debby: A teacher must think how to balance each activity and lead students to other places [11-1]. One can begin with something simple, then go up [in difficulty] gradually... [11-2] start from an activity of investigation and then develop... conclusions and generalisations... [11-3] by investigating, folding paper, making changes in the original problem, they will arrive at conclusions [11-4]... it is better than formulations that I would give them.

In this excerpt, Debby connects creativity in teaching mathematics with going to "other [meaning new] places" [11-1], raising the level of mathematics [11-2], generalising [11-3], and making changes in the original problems to arrive at conclusions [11-4]. All these are acts of elaboration and represent a particular teacher's ideas about what teachers should encourage students to do in order to make teaching mathematics creative.

Summary and discussion

This paper attempts to characterise teachers' conceptions of creativity in teaching mathematics. Individual interviews and lessons involving elementary-school mathematics teachers were analysed. To illustrate the main findings of this study, we chose several examples, mostly from the material generated by one particular teacher (Gili), and added other examples to help us draw a more complete picture of teachers' conceptions of creativity in teaching mathematics. The paper focuses on flexibility, originality, and elaboration as main indicators of creativity. We analysed the teachers' discourse, their actions, and interactions with the students in light of these three categories, as devolved from the theoretical point of view. Based on this analysis, we devised specific characteristics of creativity in teaching mathematics that characterise teachers' conceptions. In this section, we summarise our findings, propose a model of creativity in mathematics teaching, and discuss this model.

We found that teachers' conceptions of creativity in teaching mathematics consist of two main (specific) types: (1) teacher-directed, perceiving creativity in teaching to be acts by teachers that make them creative; and (2) student-directed, connecting creativity in teaching mathematics with opportunities provided for the development of students' creativity (Figure 3). This distinction is similar to that of Bolden, Harries, and Newton (2010), which distinguishes between creative teaching and creative learning. In our view, the term 'creative learning' requires research focusing on the processes in which students are involved during the teaching of mathematics. Insofar as our focus is on teachers' conceptions and we do not analyse the cognitive processes associated with learning mathematics in a 'creative classroom', we prefer to use the terms teacher-directed/student-directed conceptions of creativity.

Teacher-directed conceptions of creativity can be of a mathematical or pedagogical kind. Teachers' mathematical flexibility is related to the transformation of mathematical tasks, the solving of problems by the teacher in multiple ways, or to

Teachers' conceptions of creativity in teaching mathematics		
	Teacher-directed conceptions	Student-directed conceptions
Flexibility	<i>Mathematical flexibility</i> - Teacher transforms mathematical tasks: changing operations [2-2], changing the numbers [3-2] - Doing mathematics in different ways [1-10] - Using mathematical models [4-1, 4-6] <i>Pedagogical flexibility</i> - Transforming instructional settings [4-Gili, 5-Eva, 6-Lihi] - Adjusting the planned learning trajectory to students' needs and responses [1-4, 1-5, 1-6] - Suiting the content to students at different stages of learning [3-3, 9-1, 10-3, 10-5]	Students generate various solutions to a particular problem [1-10], different from those presented/generated previously [Episode 1]
Originality	<i>Mathematical originality</i> - Generating mathematical tasks beyond the textbook [1-1, 1-8] <i>Pedagogical originality</i> - Generating new* ideas suited to the teachers' own preferences [1-2, 1-3] - Generating new* ideas in order - to make the lesson interesting and enjoyable for students [1-9, 2-1] - to stimulate students' mathematical reasoning [1-11]	- Students generate new** ideas, new exercises, and discover new** facts [1-12, 4-5, 4-7, 5-2, 6-2, Figure 2] - Students suggest rare/insightful solutions to a problem [no example found in this study]
Elaboration		Students generalise mathematical ideas, and raise the level of mathematical discussion [11-1, 11-2, 11-3; 11-4]
* new for teachers ** new for students <i>relatively new</i>		

Figure 3. The model of teachers' conception of creativity in teaching mathematics.

the use of different mathematical models when teaching mathematics. Teachers' mathematical originality is related to the generation of original mathematical tasks. Teachers' pedagogical flexibility is related to transformations of didactic settings and the adaptation of tasks to the students' cognitive level, whereas teachers' pedagogical originality is expressed in the generation of new pedagogical ideas. Student-directed conceptions of creativity comprise students' mathematical flexibility (e.g., solving problems in multiple ways); students' mathematical originality is expressed in the production of original ideas. We also found some signs of connections between student-directed conceptions of creativity and students' elaborative acts, such as generalisation of mathematical ideas and raising the level of mathematical discussion. Based on these observations we argue that creativity in teaching mathematics leads teachers and students to move "to a different mathematical territory" (Lampert, 2001). It is clear that student-directed creativity requires the teacher to be flexible and attentive, original and sensitive. In other words, student-directed creativity in teaching mathematics requires teachers to be creative. The complexity of the relationship between teacher-directed and student-directed

creativity is reflected in Jeffrey and Craft (2004), who suggest that the interplay between creative teaching and teaching for creativity renders it difficult to treat these facets separately.

Note also that whereas Holland (in Mann 2006) addresses sensitivity as a component of creativity, which is expressed in “constructive criticism of standard methods” (Mann 2006, 238), we argue that in order to develop mathematical creativity in students a teacher should be sensitive to students as described by Jaworski’s (1994) construct *Sensitivity to Students*. This sensitivity requires teachers to take into account students’ needs and the level of their knowledge and understanding at any given moment of the lesson, and relates to the teacher’s understanding of students’ responses (Jaworski 1994).

We believe that creativity is of a dynamic nature, and that it is possible to develop teachers’ knowledge and skills with the aim of enhancing both student-directed and teacher-directed creativity. Our study contributes to this suggestion in two ways: (a) we portray teachers’ conceptions of creativity in order to ascertain the starting point for developing their conceptions of creativity in teaching mathematics; and (b) the terminology we developed here is borrowed from the theoretical perspective on creativity, as well as from teachers’ talk, and should therefore be useful in our communications with mathematics teachers in elementary schools.

The study started with theoretical definitions and distinctions related to creativity. Our empirical investigation connects theory with teacher practice. It should be noted that without the teachers’ explanations, at times we would not have recognised what they themselves considered to be creative, or we might have tended to consider it simplistic. For example, we expected teachers to describe creativity in teaching mathematics in terms of their own interesting insights about teaching and learning mathematics, insights associated with elaboration based on their students’ responses in a classroom context. However, contrary to our expectations, we were barely able to categorise any of the teacher’s descriptions of creative teaching as elaboration. Evidently, teachers refer to ‘specific creativity’ (Piirto 1999) situated in their practice, and sometimes bound by the school principal’s requirements or by the curricula. Our awareness of the relativity of creativity in a person or an act (Leikin 2009) helped generate respect for the teachers’ views and conceptions. Teacher-directed creativity is relative (Sriraman 2005; Leikin 2009) to the routines of the community of practice (Lave and Wenger 1991) to which the teachers belong, to the prescribed learning activities that educational authorities require from them, and to their own practices, which may include some unusual (creative) moments.

We are continuing our research, basing our work on the categories devised in this paper. The categories described here serve as large themes for a more comprehensive study, in which we continue to analyse lessons perceived by teachers as creative, and compare the conceptions of teachers from different educational backgrounds, as expressed in their talk about creativity in teaching mathematics, in their actions, and in interactions with students. Using these categories, we are also carrying out a comparative study to shed light on the differences between conceptions of creativity held by different teachers. The differences revealed help us explain variations in the quality of the lessons that these teachers consider to be creative. Thus, we argue that the model introduced in this paper has both descriptive and explanatory power (in the terms of Schoenfeld, 2002). We are continuing the search for new large themes and relevant subcategories, in order to refine our understanding.

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