

Complete, fold and seal
(see note 2 below)

SURNAME: Evans

12 Pages

FORENAME(S): Jacob

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2311160
2	Title of Examination Paper	Mathematics
3	Module Code	ACDB4003
4	Date and Time of Examination	09:10 am 01/05/24
5	Examination Venue	IQ 310
6	Seat Number	004

7	Number of Answer Books handed in	1
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[illegible]

65%
good

(i)

$$i) \quad 3x + 4y - 2z + 2x - 9y + 3z + 5x - 6y + 10z$$

$$7 = 10x \quad \left| \begin{array}{l} 4y - 9y - 6y \\ = -11y \end{array} \right| \begin{array}{l} -2z + 3z + 10z \\ = 11z \end{array}$$

$$x/ = 10x - 11y + 11z$$

$$ii) \quad x(x+5) - (x-4)$$

$$= x^2 + 5x - x + 4$$

$$= x^2 + 4x + 4$$

Careless here

$$iii) \quad 3qr \times 4pq^3 \times 5qr^2$$

$$= 60q^5pr^3 \quad \text{or} \quad 60q^5r^3p$$

$$iv) \quad \frac{12p^2q^3r}{6p^3q^3r^{-3}} = 2p^{-1}r^4 \quad 4q^0 = 1, q \neq 0$$

$$v) \quad \frac{2fg}{16h} \times \frac{8gh^2}{4fh} \times \frac{32fh^3}{64f^3}$$

$$\frac{1}{8} fgh^{-1} \times 2f^{-1}gh \times \frac{1}{2} f^{-2}h^3$$

$$= \frac{1}{8} f^{-2}g^2h^3$$

(2)

$$i) x^2 + 10x = -25$$

$$x^2 + 10x + 25 = 0$$

$$(x+5)(x+5) = 0$$

$$x = -5 \quad (\text{twice})$$

(5)

$$ii) \frac{x-3}{4} = x + \frac{2x-4}{8}$$

no x b.s. by 8

$$8\left(\frac{x-3}{4}\right) = 8x + 2x - 4$$

$$\frac{8x-24}{4}$$

Careless here!

$$2x-6 = x+2x-4$$

$$2x-6 = 3x-4$$

cf

$$0 = x+2$$

$$\therefore \text{therefore } x = -2$$

(2)

(3)

$$a) 4^{2x-1} = 3^{x+1}$$

$$i) \log_4(2x-1) = \log_3(x+1)$$

$$(2x-1)4 \log 2x-1 = (x+1) \log (x+1)$$

$$\log_4(2x-1) = \log_3(x+1)$$

$$\log_4(2x-1)$$

i) $4^{2x-1} = 3^{x+1}$

$$\log_4(2x-1) = \log_3(x+1)$$

$$x = 2x+1$$

$$x = x+1$$

$$4^x$$

$$\log_4 x = \log_3 x$$

$$4^x$$

ii) $\log(x^2+4) - \log(x+8) = \log(x)$

$$x^2+4 - x+8 = x$$

$$\log(x^2+4) = W$$

$$x^2+4 - 2x+8 = 0$$

$$W^{x^2+4}$$

$$x^2 - 2x + 12 = 0$$

$$x =$$

b)

i) $\log_a x^2 +$

$$= 2 \log_a x + \log_a y + \log_a z$$

ii) $\log \left(\frac{x^4 y^2}{z} \right)$

$$= 4 \log x + 2 \log y - \log z$$

(4)

$$a = 2i - 3j + 4k$$

$$b = i + 3j - k$$

$$(2 \times 1)i - 9j - 4k$$

$$2i - 9j - 4k$$

$$2 - 9 - 4 = -11$$

$$ii) \cos^{-1} \theta = \cos^{-1} \left(\frac{a \cdot b}{|a| \times |b|} \right)$$

$$a) \sqrt{2^2 - 3^2 + 4^2}$$

$$b) \sqrt{1^2 + 3^2 - 1^2}$$

$$\sqrt{4 + 9 + 16} = \sqrt{29}$$

$$b = \sqrt{11}$$

$$= \cos^{-1} \left(\frac{-11}{\sqrt{29} \times \sqrt{11}} \right) = 128.02^\circ \text{ (2.d.p.)}$$

$$iii) c = -3i + j + 0k$$

$$3 \cdot 16$$

$$b = i + 3j - k$$

$$c \cdot b = -3 + 3 \text{ (1x0)}$$

$$c \cdot b = -1, -1 \text{ gradient shows perpendicular}$$

iv)

$$= 0 \text{ ! } 4 \cdot 1$$

5) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix}$

$$\text{Det } A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{vmatrix}$$

$$\text{Det } A = 1((-1 \times -2) - (4 \times 1)) - 2((2 \times -2) - (1 \times 3)) + 3((2 \times 4) - (-1 \times 3))$$

$$1(-2) - 2(-7) + 3(11)$$

$$\text{Det } A = 55 \quad 55 \quad 45 \quad \text{unless here!}$$

$$A^{-1} = \left(\frac{1}{\text{det } A} \right) \text{Adj } A$$

(= cofactor)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & +7 & 11 \\ +16 & -5 & -5 \\ 5 & -5 & -5 \end{bmatrix}$$

show how with
cofactor
matrix

M = transpose of cofactor

errors in Calc. Determinant
expressions here!

$$\frac{1}{55} \times \begin{bmatrix} -2 & -16 & 5 \\ -7 & 7 & -5 \\ 11 & -5 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2}{55} & -\frac{16}{55} & \frac{5}{55} \\ -\frac{7}{55} & \frac{7}{55} & -\frac{5}{55} \\ \frac{11}{55} & -\frac{5}{55} & -\frac{5}{55} \end{bmatrix}$$

⑥

$$Ax = b$$

$$x = A^{-1}B$$

$$\begin{aligned} x + 2y + 3z &= 5 \\ 2x - y + z &= 0 \\ 3x + 4y - 2z &= 2 \end{aligned}$$

Matrix becomes

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 4 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

$$1 \begin{vmatrix} (-1 \times -2) - (1 \times 4) \\ +2 & -4 \end{vmatrix} - 2 \begin{vmatrix} (-2 \times 2) - (1 \times 3) \\ -4 & -3 \end{vmatrix} + 3 \begin{vmatrix} 2 \times -2 \\ (2 \times 4) \end{vmatrix}$$

Cofactors

$$-2 \quad 7 \quad 8 \quad 3 \quad 8$$

$$8 \quad -3$$

$$\begin{aligned} -2 + 14 + 24 \\ -2 + 14 \quad 33 \end{aligned}$$

$$\begin{aligned} -2 \quad -7 \quad 5 \\ -16 \quad -11 \quad -2 \\ 5 \quad -5 \quad -5 \end{aligned}$$

$$\det A = 33$$

$$\det A = 45$$

$$\frac{1}{33} \begin{bmatrix} -2 & -7 & 5 \\ -16 & -11 & -2 \\ 5 & -5 & -5 \end{bmatrix}$$

$$\frac{10}{33} \quad \frac{-10}{33}$$

$$\frac{1}{45} \times \begin{bmatrix} -2 & -16 & 5 \\ -7 & -11 & -5 \\ 5 & -5 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} (-2 \times 5) \\ (-16 \times 0) \\ (5 \times 2) \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 10 \end{bmatrix}$$

$$-15 + 3$$

④ $y = 5x - 3$

2

i) $\frac{dy}{dx} = 5$

ii) $y = 4x + 10x^{\frac{1}{2}} + \frac{5}{6}$

$\frac{dy}{dx} = 4 + 5x^{-\frac{1}{2}}$ or $\frac{dy}{dx} = 4 + \frac{5}{x^{\frac{1}{2}}}$

2

iii) $y = \frac{2}{x^5} + 9x^3$

$y = 2x^{-5} + 9x^3$

2

$\frac{dy}{dx} = -10x^{-6} + 27x^2$

iv) $y = \cos(20x)$ $u = \cos$

~~$\frac{dy}{dx} = -\sin(20x)$~~

$\frac{dy}{dx} = -20 \sin(20x)$

2

v) $y = x^2 \cdot \sin(x)$

$\frac{dy}{dx} = 2x \cos(x)$ ~~no need product rule here~~

①

v.i) $y = (8x+3)^5$

$u = 8x+3$ ~~$dy = u^5$~~

~~$\frac{dy}{dx} = (8x+3) \cdot 5$~~

$\frac{du}{dx} = 8$

$\frac{dy}{du} = 5u^4$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 40(x+3)^4$

①

v.ii) $y = 2e^x + \frac{1}{x}$

$\frac{dy}{dx} = 2e^x + \frac{1}{x^2}$

②

b) $y = \frac{x+3}{1+x^2}$

$u = x+3$
 $v = 1+x^2$

$\frac{du}{dx} = 1$

$\frac{dv}{dx} = 2x$

$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{dy}{dx}$

$\therefore = \frac{(1+x^2) \cdot 1 - 2x(x+3)}{(1+x^2)^2}$ ~~Cancel here!~~

$= \frac{1+x^2 - 2x^2 - 6x}{(1+x^2)^2} = \frac{-x^2 - 6x + 1}{(1+x^2)^2}$

When $x = 2$

$$\frac{-(2)^2 - 3(2) + 1}{(1 + 2)^2}$$

Method of
error in
simplifying.

$$\frac{4 - 6 + 1}{25}$$

$$= \frac{dy}{dx}$$

when $x = 2$

$$= \frac{-1}{25}$$

8

i) $\int 4x + x^3 \cdot dx$

$$= \frac{4x^2}{2} + \frac{x^4}{4} + C$$

$$= 2x^2 + \frac{x^4}{4} + C$$

ii) $\int 2x + 3e^x \cdot dx$

$$= \frac{2x^2}{2} + 3e^x + C$$

iii) $\int \left(\frac{1}{x} + 7\right) \cdot dx$

$$\int x^{-1} + 7 \cdot dx$$

$$= \ln x + 7x + C$$

iv) $\int (x^2 - 1)(3x + 4) \cdot dx$

$$\int 3x^3 + 4x^2 - 3x - 4 \cdot dx$$

$$= \frac{3x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2} - 4x + C$$

$$v) \int_3^6 3x^2 + 2x$$

$$\begin{aligned} & (3(6^2) + 2(6)) - (3(3^2) + 2(3)) \\ &= 108 + 12 - (27 + 6) \\ & \quad 120 \quad \quad \quad 33 \\ & 120 - 33 = 87 \end{aligned}$$

$$\text{or } \frac{3x^3}{3} + \frac{2x^2}{2} = (x^3 + x^2) \Big|_3^6$$

$$\begin{aligned} & ((6)^3 + (6)^2) - ((3)^3 + (3)^2) \\ & \quad 216 + 36 \quad \quad \quad - \quad \quad \quad 27 + 9 \\ & \quad 252 \quad \quad \quad \quad \quad \quad \quad \quad \quad 36 \\ & = 216 \end{aligned}$$

This is a good effort Jacob
Well Done!