



The likelihood of subcontractor payment

Downstream progression via the owner and contractor

The likelihood of subcontractor payment

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Abstract

Purpose – Subcontractor payments typically come through the contractor, though there can be exceptions to this, and their timing and quantum can be affected by the upstream payment practices of the owner to the contractor, as well as the payment practices of the contractor. The purpose of this paper is to study the linked effect of late and incomplete payments of both the owner and contractor on what the subcontractor receives.

Design/methodology/approach – The paper's analysis develops on an existing Markov chain formulation of owner payments. The probability of getting payment from an owner or contractor is represented as a function of time since claim submission. Such functions are established through goodness of fit tests using actual project data. The downstream progression of payment from owner to contractor to subcontractor is treated as a collection of series and parallel systems, for which the likelihood of payment is assessed.

Findings – A model that enables subcontractors to calculate the likelihood of getting their claims paid, based on owner and contractor historical payment practices, is developed. Subcontractors are able to calculate the conditional and unconditional probabilities of their claims being paid at any time after claim submission. The model may be used with historical payment records, or with identified typical owner and contractor payment types.

Practical implications – The paper presents a practical method by which a subcontractor is able to calculate age-dependent probabilities of outstanding claim amounts being paid. Such information feeds into the subcontractor's tendering practices before entering a new project, and in the subcontractor's contract administration practices in terms of pursuing claims.

Originality/value – The modelling of the owner-contractor-subcontractor payment linkage is original. No similar modelling exists in the literature.

Keywords Construction industry, Subcontracting, Payments, Likelihood, Markov chains

Paper type Research paper

Introduction

Construction projects typically involve multiple participants linked contractually – the project owner, the head/general/main contractor, subcontractors and suppliers. In traditional delivery, there is a head contract between the owner and the contractor, and subcontracts to suppliers and subcontractors (Carmichael, 2000). Further subletting of work to sub-subcontractors (and below) may also be carried out, for example, in cases where simultaneous construction work is carried out, or the work is highly complicated or specialised (Sozen and Kucuk, 1999). Subcontracting, covering specialist trades, materials and labour and labour-only work, is common in the construction industry. The practice of subcontracting allows a number of benefits for contractors including



a reduction in a contractor's overheads and the ability to use firms that have the facilities and skilled labour to complete that work efficiently and economically (Sozen and Kucuk, 1999).

In general, the contractor is responsible for the satisfactory completion of the work by a subcontractor and for the payment of subcontractors' claims. Typically conditions of contract for a subcontractor involve the subcontractor doing the work, submitting or lodging a claim, and being paid at a nominated later time or times. In some countries and on some projects it is unlawful or not permitted for contractors to withhold payments to subcontractors because of delayed and incomplete payments from the owner. For example, the United States Department of Transportation has regulations requiring that general contractors pay their subcontractors after the completion of subcontracts, even if they have not received their own payment from the owner (Touran *et al.*, 2004). The security of payments legislation operating in the states of Australia also nullifies the effect of the "pay when paid" and "paid if paid" clauses in a contract, in order to protect subcontractors from the practice of owners and contractors arbitrarily delaying or denying payment (Uher and Brand, 2008b).

Contractors are expected to manage their financial affairs knowing that they may need to pay a subcontractor prior to receiving payment for the same work from the owner. However, experience shows that some contractors do delay payments to subcontractors in order to improve their cash flow, especially when payments from the owner are late and incomplete. Unscrupulous contractors may take advantage of smaller subcontractors by delaying or deducting payment without good reason (Uher and Brand, 2008b). Payments to the subcontractor are, therefore, not only subjected to a contractor's payment behaviour, but are also affected by an owner's payment behaviour through its flow-on effect of influencing the contractor's behaviour when payments from the owner are late or incomplete. Subcontractors, usually because of limited financial resources, may not always be able to recover delayed payments through formal dispute resolution mechanisms and therefore could be said to be exposed to the largest (in proportional terms) financial risks associated with payment uncertainties.

The linked effect of owners' late and incomplete payments to contractors on subcontractors' cash flow and financial performance is of particular interest. This paper presents a method for subcontractors to estimate the likelihood of getting their claims paid based on historical payment behaviour of owners and contractors. It provides a practical tool for subcontractors to convert past payment behaviour of owners and contractors into a decision-support tool prior to embarking on a new project, and in its contract administration practices in terms of how it conducts its affairs regarding claims. Typical owner and contractor payment types are identified in order to provide a fast and simple analysis. The paper's analysis extends findings from the Markov chain formulation of owner payment histories in Carmichael and Balatbat (2010).

Information about the probability that a subcontractor can recover all, or part of its money for the work it has done is considered crucial for the management of a subcontractor's financial affairs. If, pre-project, a subcontractor sees little chance of being reimbursed fully or on time for its work, it may decide not to undertake the work. Alternatively, by estimating the proportion of its claims that is likely to be recovered, the subcontractor may select to adjust its tender price accordingly.

In analysing owner or contractor past payment behaviour, only delays and incomplete payments due to payer-initiated causes are considered, and not other causes.

On this basis, Tran *et al.* (2011) have identified and classified typical payment types, and these can be used in the absence of any other data to give a fast and simple analysis.

The method advanced in this paper is applicable for subcontractors, sub-subcontractors, specialist trades, suppliers and others which are subject to multiple payment links. It is also applicable for subcontractors working with multiple contractors simultaneously. Subcontractors are able to estimate the likelihood of recovering outstanding entitlements at any stage during the project, given a current outstanding amount. This information is particularly useful for cash planning purposes.

The paper first reviews related studies on subcontractor payments and established legislation for protecting subcontractors' rights in getting payments. The existing literature on owner-contractor payment modelling is extended to incorporate the probability of subcontractors being paid. Case study data are used to demonstrate the calculation of the probability of being paid, taking into account alternatives in owner and contractor payment practices. Typical historical payment behaviour of owners and contractors, in the form of payment likelihood functions, are given so that they can be readily used by subcontractors.

The modelling of the owner-contractor-subcontractor payment linkage is original. No similar modelling exists in the literature. The paper will be of interest to owners, contractors, subcontractors and others involved in payment links on projects.

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Background

The issue of construction contractors and subcontractors suffering payment delays and payment incompleteness is common to many countries. Late payments, reduced payments and even non-payment happen to contractors and subcontractors, for example in Turkey (Sozen and Kucuk, 1999), India (De, 2001), Saudi Arabia (Assaf *et al.*, 2001), China (Wang *et al.*, 2006; Wu *et al.*, 2008; Wu *et al.*, 2011) and Australia (Carmichael and Balatbat, 2010). Owners delaying or withholding part of payments in order to improve their own financial position, referred to as "opportunistic" behaviour by Wu *et al.* (2011, p. 16), is naturally disliked by contractors (Wu *et al.*, 2011; Ingirige and Sexton, 2006; Greenwood and Yates, 2006). It has become an entrenched culture in parts of the construction industry.

In a study by Hinze and Tracey (1994), two thirds of the subcontractors stated that the percentage retained by contractors from subcontractors was not the same as that withheld by owners from contractors, or that they were not aware of the retained amount withheld by the owner from the contractor. Chen and Chen (2005) show that contractor payment conditions to subcontractors are far more complicated than those to suppliers. Wu *et al.* (2008) report on escalating payment arrears in China and point to, as major causes, deficiencies in the credit system, unfair market conditions, shortfalls in the legal system, looseness in implementing regulations, and local governments initiating projects without sufficient funding arrangements in place. Wu *et al.* (2011) claim that payment arrears seems to arise from a deliberate choice of opportunism, rather than any ignorance of the potential of "partnering" or long-term collaboration.

There appears to be little difference in the incidence and degree of late payment between contractors and subcontractors, rather differences appear to be more related to company size. In Australia, firms with less than five employees and turnover less than \$500,000 receive late payments more often than firms with more employees and higher turnover (Brand and Uher, 2010).

Contractors may have little knowledge of how good a paymaster a potential owner will be when entering into an agreement (Brand and Uher, 2010). Only approximately one fifth of contractors and subcontractors in Mainland China are able to access another party's payment records before entering into an agreement (Wu *et al.*, 2011).

Despite the fact that the quantum of claims made by subcontractors against contractors is similar to those made by contractors against owners (Uher and Brand, 2005), subcontractors are often subjected to unclear contract payment terms. Standard forms of conditions of contract, which usually state when payments are due, are not always used for subcontracts. Also, the interest rate which applies for the unpaid portion of a claim is less frequently stated in subcontracts than in head contracts (Uher and Brand, 2005).

In Australia, there exist mechanisms to protect contractors' rights (when dealing with owners) and subcontractors' rights (when dealing with contractors) in the form of payment bonds and legislated adjudication for progress payments. Security of payment legislation ensures that contractors and subcontractors fully receive their lawful entitlement for the work they have done and materials supplied; the legislation applies to late and incomplete payments from the payer and insolvency of the payer. Recent amendments to the legislation can additionally require an owner to withhold money from a contractor, pending determination of a subcontractor adjudication application against the contractor (Bannon and Gillard, 2011).

Since the introduction of this security of payment legislation, some statistics have been gathered (Uher and Brand, 2005, 2008a, b; Brand and Uher, 2010). These statistics indicate a reducing industry frequency of late payments over the years, presumably because of the high success of claimants, especially of subcontractor claimants, in adjudication cases (Uher and Brand, 2008b). Nevertheless, it appears that contractors and subcontractors are not taking full advantage of the legislation in protecting their own rights (Brand and Uher, 2010). Many firms surveyed remain undecided about whether the legislation has created a fair and balanced payment standard. Smaller firms, in terms of employees and turnover, are less satisfied that the legislation has brought a fair and balanced payment standard than larger firms (Brand and Uher, 2010). Industry is still undecided on the time and cost efficiency aspects of the adjudication process (Uher and Brand, 2008a).

Probability of payment as a function of time

Of direct relation to this paper is the Markov chain formulation of owner payment practices developed by Carmichael and Balatbat (2010). In that formulation, an owner's payment practices are characterised by the probability of outstanding amounts being paid at any time following claim submission by the contractor. (Below, payers are generalised beyond owners to also include contractors, subcontractors, etc. and payees are generalised beyond contractors to include, respectively, subcontractors, sub-subcontractors, etc. that is entities in the contractual links.) This probability decreases as a claim's age increases, implying that the likelihood of being paid reduces with time. The better the payer's payment practices the higher the probability of a claim being paid, and hence the higher the desirability of working with that payer.

The Carmichael-Balatbat Markov chain formulation of late and incomplete payments uses a summary of outstanding money against time after claim submission. In this formulation, states are defined as the period of time by which an amount is overdue. There are n transient states corresponding to n time periods i , $i = 0, 1, 2, \dots, n - 1$.

The choice of the value of n , that is how far into the future the analysis is done, and the choice of time period unit, are up to the discretion of the payee, and does not affect the method. An absorbing state, “To be resolved”, is introduced to represent amounts which are conceded as possibly not forthcoming or need pursuing through some dispute resolution process. Another absorbing state, “Paid”, is used to represent amounts which are received from the owner. An absorbing state is one in which an amount entering cannot exit to another state. From the summary of outstanding amounts against time after claim submission, transition probabilities between states can be calculated. Accordingly a transition matrix, containing transition probabilities between transient states and from transient states to absorbing states, can be populated.

Standard Markov chain calculations give information about the probability of absorption in each of the absorbing states, for any amount starting in a transient state. Standard notation uses “NR” to denote the matrix of these probabilities (Carmichael and Balatbat, 2010). The entries in NR are the age-dependent probabilities of a claim being paid or needing to be resolved (perhaps through some dispute resolution forum). The first column of NR gives the probability of a claim of age t being paid; rows correspond to increasing values of t . Figure 1 shows plots of the first column of NR matrices based on data collected from 10 recently completed construction projects.

The first column entries in the NR matrix, represent the payment characteristic of a payer (here an owner, contractor, etc.). The time interval (here, week) and that nominated as the time by which claims are judged as in need of resolution (here, eight weeks) in Figure 1 are for demonstration purposes only. The form of the analysis is unaffected by these choices.

Let the probability of an outstanding amount of age t following claim submission being paid be $\rho(t)$, and the amount not being paid be $\varphi(t)$. Then:

$$\rho(t) = 1 - \varphi(t) \quad (1)$$

The probability that an outstanding amount aged t ending up being paid is assumed here to follow a Rayleigh distribution given by:

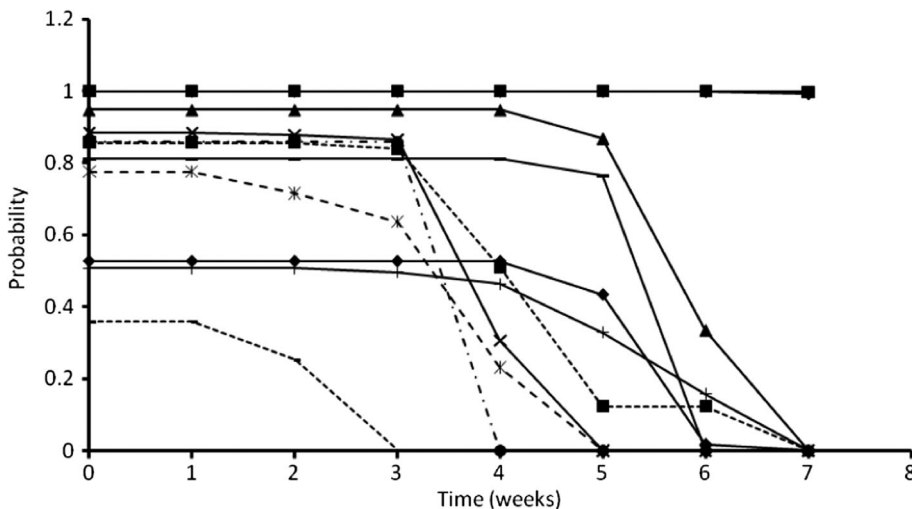


Figure 1.
Payment profiles
(probability of amounts in
each of the age categories
ending up being paid
versus age categories) –
actual project data;
ten different projects
(11 data sets)

$$\rho(t) = \beta \exp \left[-\frac{kt^2}{2} \right] \quad (2)$$

Distributions such as Weibull and exponential, and the usual Rayleigh distribution, start at 1 when $t = 0$, and do not fit the plot for the first column of NR, and hence are not suitable. The parameter β is introduced to adjust the starting point of the curve to better fit with the plot of the first column of NR. In terms of application and simplicity in modelling, it is considered desirable that the probability distribution function has no more than two shape parameters; the distribution in equation (2) has two parameters, namely β and k . Typical payment behaviour is characterised by the values of these two parameters. The inclusion of the t^2 term in the exponential component allows a curve, giving flexibility to the modelling.

The probability of non-payment is given as:

$$\varphi(t) = 1 - \rho(t) = 1 - \beta \exp \left[-\frac{kt^2}{2} \right] \quad (3)$$

The rate of the non-payment probability, that is, the gradient of the $\varphi(t)$ plot is derived by taking the derivative of $\varphi(t)$ with respect to t :

$$\lambda(t) = \frac{d\varphi(t)}{dt} = \beta kt \exp \left[-\frac{kt^2}{2} \right] \quad (4)$$

Having data for $\rho(t)$ or $\varphi(t)$, $\lambda(t)$ can be calculated.

The plot of $\lambda(t)$ shows the time t at which the slope of the non-payment curve is steepest, that is, the change in the non-payment probability is at a maximum. This information gives a subcontractor the time to wait before perhaps starting some payment recovery mechanism.

Example data sets

Consider some actual project data on late and incomplete payments. Having no more than a historical payment profile summary in hand, the probability of non-payment and accordingly the rate of non-payment probability can be plotted.

Example data set (a)

Project: the contractor was engaged to carry out formation and bridge construction work for a new 25 km rail freight line. The project was terminated by the owner after one year. The project had a low claim acceptance level by the owner, varying from approximately 20 to 90 per cent of claims submitted; well into the project, the amount not paid was approximately 65 per cent of the total claimed.

Payments from the owner to the contractor followed the payment profile of Table I for claimed amounts up to eight weeks late.

A best fit curve using MATLAB (Mathworks, Inc. 2009) gives $\beta = 0.3956$ (95 per cent confidence interval 0.3041-0.4870) and $k = 0.3699$ (95 per cent confidence interval 0.1410-0.5988). Figure 2 shows actual and best-fit plots for the owner's payment profile (probability of non-payment). Figure 3 shows the corresponding theoretical rate of non-payment probability.

Example data set (b)

Project: the project involved the conversion of a roundabout to traffic signals with the duplication and reconstruction of four approaches over a two-year period for local government. Extensive drainage work and the signalisation and upgrade of other minor intersections, were also involved. On average, progress payments were received six days earlier than the contractual requirement of 28 days. However, the approval of variation and extension of time claims took much longer; the large amount of time that elapsed between when the variation work was done and payment approval occurring

Total claimed amount (\$K)	Outstanding amount (\$K) at							
	1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks	7 weeks	8 weeks
201,800	201,800	173,470	129,740	129,740	129,740	129,740	129,740	129,740

Table I.
Owner payment profile;
example data set (a)

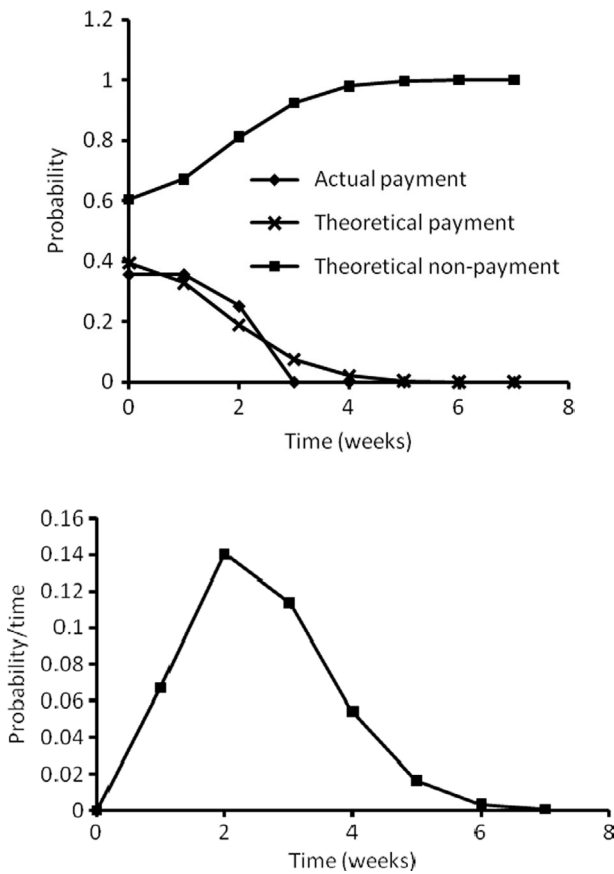


Figure 2.
Probability of payment
and probability of
non-payment;
example
data set (a)

Figure 3.
Theoretical rate of
non-payment probability;
example data set (a)

led to work verification issues and a further reduction in approved values. The payment history followed Table II.

A best fit curve gives $\beta = 0.8588$ (95 per cent confidence interval 0.6695-1.0480) and $k = 0.1424$ (95 per cent confidence interval 0.0623-0.2225). Similarly to the case example (a), the largest change in the probability of non-payment, $\varphi(t)$, is at $t = 2$ weeks.

Statistical analysis – goodness of fit test

A Kolmogorov Smirnov goodness of fit test (K-S test) (Stephens, 1974; Meyer and Rasche, 1989) is performed to assess the validity of the distribution assumption of equation (2). For this purpose, the payment profiles of 11 project data sets based on actual projects are summarised. The entries of the first column of the NR matrices, as shown in Figure 1, are calculated following the Markov formulation given in Carmichael and Balatbat (2010). The parameters β and k of each of these 11 data sets are estimated along with their 95 per cent confidence intervals, using the curve-fitting tool in MATLAB.

The K-S test is performed in R program (R Development Core Team, 2009) to calculate the difference between the theoretical and actual $\rho(t)$. The hypotheses for the K-S one sample test are formally defined as follows:

- H_0 The data points come from the distribution function as in equation (2).
- H_a The data points do not come from the distribution function as in equation (2).

The test statistic quantifies the distance D between the empirical distribution function of the sample (entries of the first column of NR) and the theoretical distribution function (equation (2)). The R program gives the value of D and accordingly each case's p -value, which is the probability of obtaining a value of D at least as extreme as the calculated D , assuming that the null hypothesis is true. (The use of the symbol P here is not to be confused with the use of this symbol to describe transition matrices in Markov chains.)

As can be seen in Table III, all of the 11 project data sets have a p -value greater than 0.05, implying that there is little evidence against the null hypothesis (Hogg *et al.*, 2005). Therefore, it is concluded that the Rayleigh distribution function as in equation (2) is a reasonable representation of the probability of payment.

Values of β and k for typical owner payment behaviour

A study of the classification of owner payment behaviour by Tran *et al.* (2011) established that there are six main types of owners when characterised in terms of their payment histories. Each owner type may be characterised by a representative payment profile – the first column of the NR matrix. Following this classification, the corresponding estimated β and k parameters for each typical payment behaviour are shown in Table IV.

Table II.
Payment profile
summary; example
data set (b)

Total claimed amount (\$K)	Outstanding amount (\$K) at							
	1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks	7 weeks	8 weeks
19,000	19,000	15,010	11,700	5,540	4,260	4,260	4,260	4,260

Project data set	β	k	D	p-value
1	0.9998	0.0001921	0.375	0.6272
2	0.9608	0.0967000	0.250	0.9640
3	1.0000	0.0433300	0.375	0.6272
4	1.0000	0.1278000	0.375	0.6272
5	0.8587	0.1423000	0.375	0.6272
6	0.9836	0.1546000	0.500	0.2700
7	0.5642	0.0578200	0.250	0.9640
8	0.3958	0.3709000	0.625	0.08787
9	0.9244	0.0581700	0.375	0.6272
10	0.6004	0.0604600	0.375	0.6272
11	0.9999	0.00004701	0.625	0.08787

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Table III.
K-S goodness of fit test for the Rayleigh distribution assumption

Owner type	β	Upper bound	Lower bound	k	Upper bound	Lower bound
1	0.5550	0.5851	0.5248	0.00784	0.01275	0.002933
2	0.3069	0.3976	0.2163	0.07464	0.13330	0.015980
3	0.7558	0.9197	0.5919	0.05669	0.09189	0.021500
4	0.3363	0.3587	0.3138	0.37640	0.44360	0.309100
5	0.7802	0.8482	0.7122	0.09610	0.11760	0.074560
6	0.6963	0.8226	0.5700	0.17540	0.25710	0.093670

Table IV.
Shape parameters of the six representative owner types of Tran *et al.* (2011); “upper” and “lower” bounds refer to 95 per cent confidence intervals

Conditional probability of subcontractor payment

Consider now the linked effect from owner to contractor to subcontractor. Consider the probability that the contractor pays its subcontractor, given that the owner has made its payment to the contractor for the same work item. Define A as the event that the owner pays an outstanding amount of age t to the contractor; define B as the event that the contractor pays an outstanding amount of age t to the subcontractor.

The conditional probability that the contractor pays its subcontractor, given that the owner has already paid the contractor, is given as:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad (5)$$

When payments from the owner are late and incomplete, there are four possible scenarios regarding the contractor’s behaviour, namely.

Scenario 1 (best scenario)

The contractor always pays its subcontractor punctually and in full regardless of payment from the owner. In this case the conditional probability P(B|A) equals P(B) and equals 1.

Scenario 2

The contractor delays and/or withholds part of the owner payment, responding to every subcontractor claim with exactly the same treatment, in terms of delay and partial payment, that it gets from the owner (a “pay when paid” situation applies). For example, if the owner pays a claim one week late then the contractor would also pay

its subcontractor one week late. When the owner pays the contractor, the contractor also pays its subcontractor. In this case, payment from the contractor to subcontractor reflects exactly the owner payment characteristics in both timing and quantum:

$$P(A \cap B) = P(A) \quad (6)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1 \quad (7)$$

The conditional probability that the subcontractor gets an outstanding amount paid in this case is 1. That is, if the owner pays the contractor, then the subcontractor also gets paid.

Scenario 3 (worst scenario)

The contractor tries to delay and withhold payments to the subcontractor even if the owner has made its payment on time and in full, in order to improve its own cash flow to the detriment of the subcontractor. Payment to the subcontractor is completely independent of the payment from the owner to the contractor. In this case, A and B are independent since they reflect the owner's and the contractor's separate payment behaviour:

$$P(A \cap B) = P(A)P(B) \quad (8)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B) \quad (9)$$

Thus, the conditional probability $P(B|A)$ reflects the contractor payment behaviour.

Scenario 4

A contractor's response to each subcontractor claim may not be consistent. The contractor may not always act as in any of the scenarios above, but rather may exhibit behaviour which is a combination of the above scenarios. For some claims, the contractor pays its subcontractor when the owner pays; for other claims, the contractor does not pay even if the owner has paid. This scenario is in between Scenarios 2 and 3. Then the conditional probability $P(B|A)$ has an upper bound of Scenario 2 and a lower bound of Scenario 3. Figure 4 shows the possible range for the value of $P(B|A)$.

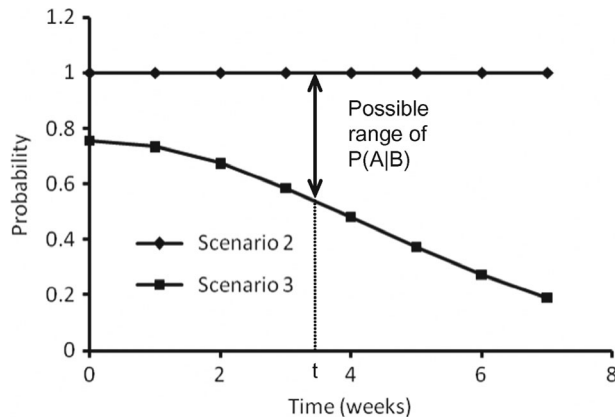


Figure 4.
Conditional probability
of payment to the
subcontractor (Scenario 4)

Using a probability of payment that follows equation (2), the conditional probability that the subcontractor receives its claimed amount, given that the owner has paid the contractor, can be written as:

$$\beta_c \exp \left[-\frac{k_c t^2}{2} \right] \leq P(A|B) \leq 1 \quad (10)$$

where β_c and k_c are parameters characterising the contractor payment behaviour. The subscript c denotes contractor.

Unconditional probability of payment to subcontractor – series system

Consider the probability of payment to a subcontractor as an unconditional probability. The downstream payment from the owner to the contractor and from the contractor to the subcontractor can be modelled as a system connected in series as shown in Figure 5.

Let $\rho_1(t)$ be the probability of payment reflecting the owner payment behaviour (entries of the first column of the NR matrix obtained from historical payment records of the owner). Let $\rho_2(t)$ be the probability of payment reflecting the contractor payment behaviour (entries of the first column of the NR matrix obtained from historical payment records of the contractor). $\rho_1(t)$ and $\rho_2(t)$ are assumed to be probabilistically independent; this can be attained where the historical payment information for owner-contractor and contractor-subcontractor is taken from unrelated projects.

Let $\rho_s(t)$ be the probability that the subcontractor gets its claimed amount paid at time t following claim submission. The subscript s denotes subcontractor. Because of probabilistic independence:

$$\rho_s(t) = \rho_1(t)\rho_2(t) \quad (11)$$

where $\rho_i(t)$, $i = 1, 2$, expressed in the form of equation (2), are:

$$\rho_1(t) = \beta_1 e^{-(k_1 t^2/2)} \quad (12)$$

$$\rho_2(t) = \beta_2 e^{-(k_2 t^2/2)} \quad (13)$$

It follows that the probability that the subcontractor can recover all of its entitlement decreases with time at a greater rate than that in the contractor's case. Thus, the probability of payment to the subcontractor is:

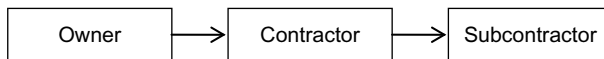
$$\rho_s(t) = \rho_1(t)\rho_2(t) = \beta_1 e^{-(k_1 t^2/2)} \beta_2 e^{-(k_2 t^2/2)} = \beta_1 \beta_2 e^{-((k_1 + k_2)t^2/2)} \quad (14)$$

Accordingly the probability of non-payment to the subcontractor is:

$$\varphi_s(t) = 1 - \rho_s(t) = 1 - \beta_1 \beta_2 e^{-((k_1 + k_2)t^2/2)} \quad (15)$$

Hence the rate of non-payment probability is:

$$\lambda_s(t) = \frac{d(1 - \rho_s(t))}{dt} = \beta_1 \beta_2 (k_1 + k_2) e^{-((k_1 + k_2)t^2/2)} \quad (16)$$



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Figure 5.
Payment as a series system

The equivalent shape parameters for the owner-contractor-subcontractor series system are:

$$\beta_s = \beta_1 \beta_2 \quad (17)$$

$$k_s = k_1 + k_2 \quad (18)$$

If the payment behaviour of both the owner and the contractor are known to the subcontractor, then the subcontractor can, from equations (14) and (15), estimate the probability of getting an outstanding amount aged t paid or not paid. The time at which the change in the probability of non-payment is highest can be found using equation (16).

Example

Consider a situation where a subcontractor is working with an owner with a historical payment profile as in the case example data set (a) and a contractor with a historical payment profile as in the case example data set (b) above. The estimated values for the β and k parameters are:

$$\text{Owner: } \beta_1 = 0.3956 \quad k_1 = 0.3699$$

$$\text{Contractor: } \beta_2 = 0.8588 \quad k_2 = 0.1424$$

Then:

$$\text{Subcontractor: } \beta_s = \beta_1 \beta_2 = 0.3397 \quad k_s = k_1 + k_2 = 0.5123$$

The probability that the subcontractor gets its claim paid, as a function of time, is given by:

$$\rho_s(t) = \beta_s e^{-(k_s t^2 / 2)} = 0.3397 \times e^{-(0.5123 t^2 / 2)}$$

The requirement of having historical payment data for the owner and the contractor can be eased if the subcontractor, from its own experience or others' experiences, knows the typical payment types of the owner and the contractor. The result of the analysis remains faithful when the payment behaviour of an owner of a certain type (in the sense of Table IV types – Tran *et al.*, 2011) is combined with payment behaviour of a contractor of a different type (in the sense of Table IV types).

Consider an example of a subcontractor working with an owner of Type 3 and a contractor of Type 4 (in the sense of Table IV). The equivalent shape parameters of the three-component series system, calculated using equations (16) and (17) are:

$$\beta_s = \beta_1 \beta_2 = 0.7558 \times 0.3363 = 0.2542$$

$$k_s = k_1 + k_2 = 0.05669 + 0.3764 = 0.43309$$

The probability of payment curve for the subcontractor is shown in Figure 6. The probability of payment curves representing the owner and the contractor payment behaviour are also included for comparison purposes.

As anticipated, the probability of a subcontractor getting paid for an outstanding amount aged t weeks is smaller than that when the subcontractor works for the contractor only without the upstream link to the owner. For example, consider an outstanding amount aged three weeks: if the subcontractor receives payment directly from the owner, then this amount has a 58 per cent probability of being paid;

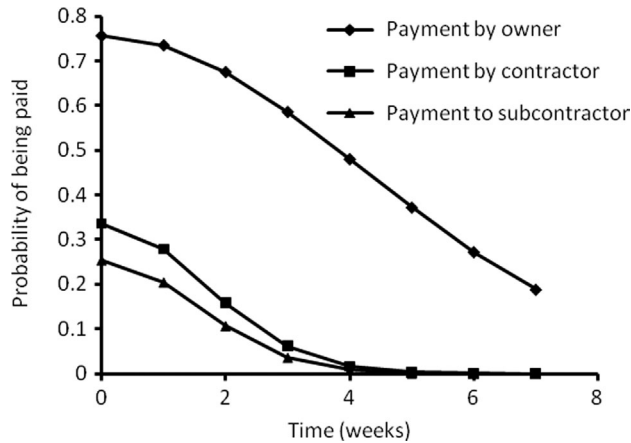


Figure 6.
Unconditional probability of the subcontractor being paid

if the subcontractor works through the contractor, then this amount has a 6.2 per cent probability of being paid; if the project involves all three parties (owner, contractor, subcontractor), then there is only a 3.6 per cent chance that the subcontractor will have this amount paid.

The probability that the subcontractor gets its claim paid has an upper bound given by the contractor's payment profile. The better the owner payment behaviour, the smaller the gap between $\rho_s(t)$ and $\rho_2(t)$. The value of $\rho_s(t)$ cannot be larger than the corresponding value given by $\rho_2(t)$.

Sensitivity

The above formulation of $\rho_s(t)$ can be shown to be insensitive to small changes in the distribution (equation (2)) parameters. Consider the example shown in Figure 6 – owner of Type 3 (characterised by parameters β_1 and k_1) and contractor of Type 4 (characterised by parameters β_2 and k_2). By multiplication, a proportional change in β_1 or β_2 will cause the same proportional change in $\rho_s(t)$. Figures 7 and 8 show the sensitivity in the probability of subcontractor payment to changes in the parameters β_1 and k_1 , which are related to the owner payment behaviour. Similar plots are obtained by allowing β_2 and k_2 , which are related to the contractor payment behaviour, to change.

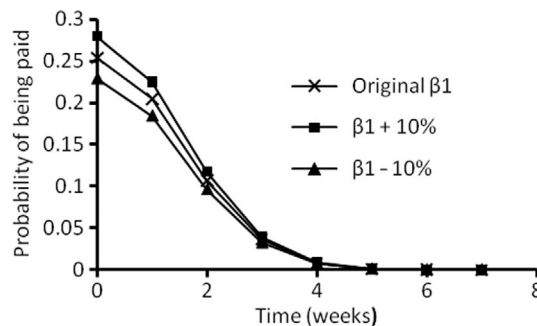
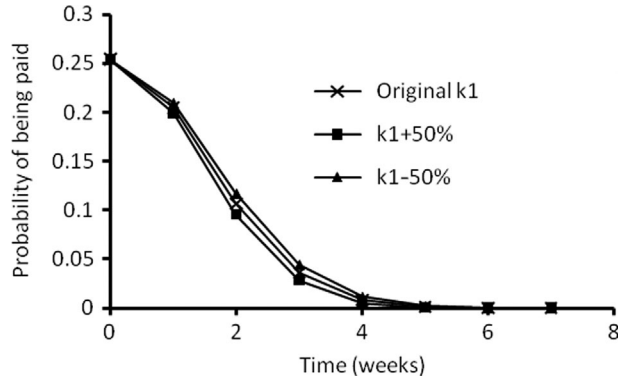


Figure 7.
Example influence of changes in β_1 on $\rho_s(t)$

Figure 8.
Example influence of
changes in k_1 on $\rho_s(t)$



The insensitivity of $\rho_s(t)$ to small changes in β and k indicates that relatively good estimation of payment probabilities can be achieved with β and k estimates which need not be precise.

Extension to a four-component series system – payment to sub-subcontractors and suppliers

Consider a four-component series system involving an owner, a main/head contractor, a subcontractor and a supplier. The probability that the supplier gets its claimed payment is of interest. This probability may depend on the probabilities of all three upstream parties (owner, contractor, subcontractor) being paid. The series system is shown in Figure 9.

The probability that the supplier gets its claim paid from the subcontractor is an extension of equation (14) and becomes:

$$\begin{aligned}\rho_{s'}(t) &= \rho_1(t)\rho_2(t)\rho_3(t) = \beta_1 e^{-(k_1 t^2/2)} \beta_2 e^{-(k_2 t^2/2)} \beta_3 e^{-(k_3 t^2/2)} \\ &= \beta_1 \beta_2 \beta_3 e^{-((k_1 + k_2 + k_3) t^2)/2}\end{aligned}\quad (19)$$

where the subscript s' refers to the supplier, and $\rho_3(t)$ is the characteristic payment behaviour of the subcontractor, assumed to be probabilistically independent from $\rho_1(t)$ and $\rho_2(t)$.

Similarly the rate of non-payment probability against time is given as:

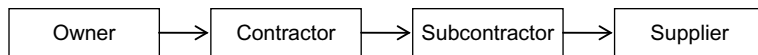
$$\lambda_{s'}(t) = \beta_{s'} k_{s'} t e^{-(k_{s'} t^2/2)} \quad (20)$$

where:

$$\beta_{s'} = \beta_1 \beta_2 \beta_3 \quad (21)$$

$$k_{s'} = k_1 + k_2 + k_3 \quad (22)$$

Figure 9.
Four-component payment
series system



A generalisation

The model can be extended to an n-component series system, which might be the case for large and complicated projects. The contractual links are from owner to contractor to subcontractor to sub-subcontractor, and so on. The probability of being paid of the party at the end of the links is given as:

$$\rho_n(t) = \prod_{i=1}^n P_i(t) = \prod_{i=1}^n \beta_i e^{-(k_i t^2/2)} = e^{-(t^2/2) \sum_{i=1}^n k_i} \prod_{i=1}^n \beta_i \quad (23)$$

where the subscript n refers to the nth party, and k_i and β_i are the shape parameters characterising the payment behaviour of the ith participant in the links.

Alternatively, for projects involving multiple contractors, subcontractors, sub-subcontractors, etc. the system can be modelled as a combined series-parallel system. The analysis is also valid for a subcontractor working for two contractors at the same time, or for a contractor working for two owners at the same time. Knowing the payment behaviour of each upstream payer, a party can estimate the likelihood of recovering its outstanding money as a function of time.

The probability of non-payment for an n-component parallel system, denoted as $\varphi_p(t)$, is as follows:

$$\varphi_p(t) = \prod_{i=1}^n \varphi_i(t) = \prod_{i=1}^n (1 - \rho_i(t)) \quad \rho_p(t) = 1 - \prod_{i=1}^n (1 - \rho_i(t)) \quad (24)$$

The general probability of payment to the party at the end of an n-component parallel system is:

$$\rho_p(t) = 1 - \varphi_p(t) = 1 - \prod_{i=1}^n [1 - \rho_i(t)] = 1 - \prod_{i=1}^n [1 - \beta_i e^{-(k_i t^2/2)}] \quad (25)$$

Summary approach for a subcontractor

The analysis proposed in this paper can be performed by a subcontractor at the pre-tender stage of a project. It will also find use during contract administration where amounts are being withheld by the contractor. The analysis requires no more than some experience about past payment behaviour of the owner and the contractor on the project. The following summary steps are given for a subcontractor to follow, in estimating the likelihood of receiving an outstanding amount against the age of that amount:

- (1) Obtain information about late and incomplete payment behaviour of the owner and the contractor using past projects similar to the upcoming project. Summarise the claim-payment data in the form of outstanding project money against days/weeks/months following claim submission. Perform a Markov chain analysis as in Carmichael and Balatbat (2010) to obtain the entries of the NR matrices for the owner and the contractor.
- (2) Estimate β and k parameters representing owner and contractor payment behaviour. This is done by fitting a Rayleigh distribution curve to the first column of the NR matrix using equation (2), for the owner and contractor in turn. Alternatively if past payment data are not available, take the values for β and k from the typical owners (Table IV).

- (3) Express the probability of being paid $\rho_1(t)$ of the owner and $\rho_2(t)$ of the contractor as a function of a claim's age using the estimated β and k parameters in Step 2 and equations (12) and (13).
- (4) Calculate the minimum possible value of the conditional probability that the contractor will pay a claim, given that the money for that work item has been paid to the contractor by the owner. $P(B|A)$ has a lower bound of $\rho_2(t)$, which is the probability of payment by the contractor, and an upper bound of 1.
- (5) Calculate the unconditional probability $\rho_s(t)$ that a claim aged t will be paid or not paid using equations (14) and (15). For the sub-subcontractor and supplier case, the unconditional probability of payment is calculated using equation (23).

The process involves relatively straightforward calculations. The estimation of the parameters of the probability of payment functions can be performed in a spreadsheet or other available curve fitting software.

Conclusion

Subcontract work contains uncertainties with regard to recovering payments from contractors. The uncertainty in payment is increased for subcontractors and suppliers over contractors because of the combined uncertainty in payment behaviour of the owner and the contractor. The paper presents a useful analysis tool for subcontractors and suppliers to approach new business with foreknowledge of the likelihood of getting paid. The calculation of the likelihood of payment to a subcontractor takes into account the uncertainties associated with both owner and contractor payment practices. Such likelihood information is crucial for the management of a subcontractor's financial affairs. It feeds into the subcontractor's cash planning, bid price adjustment and strategies in pursuing claims, thus improving the operation of the subcontractor's business. The analysis is primarily for subcontractors but can also be looked at from the point of view of all project stakeholders, making them aware of late and incomplete payments which could possibly affect overall project performance.

The analysis utilises historical payment data across projects to shape the payment characteristics of owners and contractors. Multiple data sets from multiple projects, if available, should be used to examine various payment patterns that can be made by the owner and the contractor. Data can also be processed according to different claim types (for example, progress claims and variation claims) to give better estimates. The analysis is performed on specific data sets and based on a specific choice of time unit and duration. However, the analysis framework has flexibility and adaptability across all projects.

The subcontractor's likelihood of payment is largely insensitive to assumptions on the parameters of the distribution functions representing owner and contractor payment behaviour. The most accurate result will be achieved by using available data that are detailed and as close as possible to the circumstances of the project under consideration. Nevertheless, the analysis results are still acceptable when performed using predefined typical payment behaviour.

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