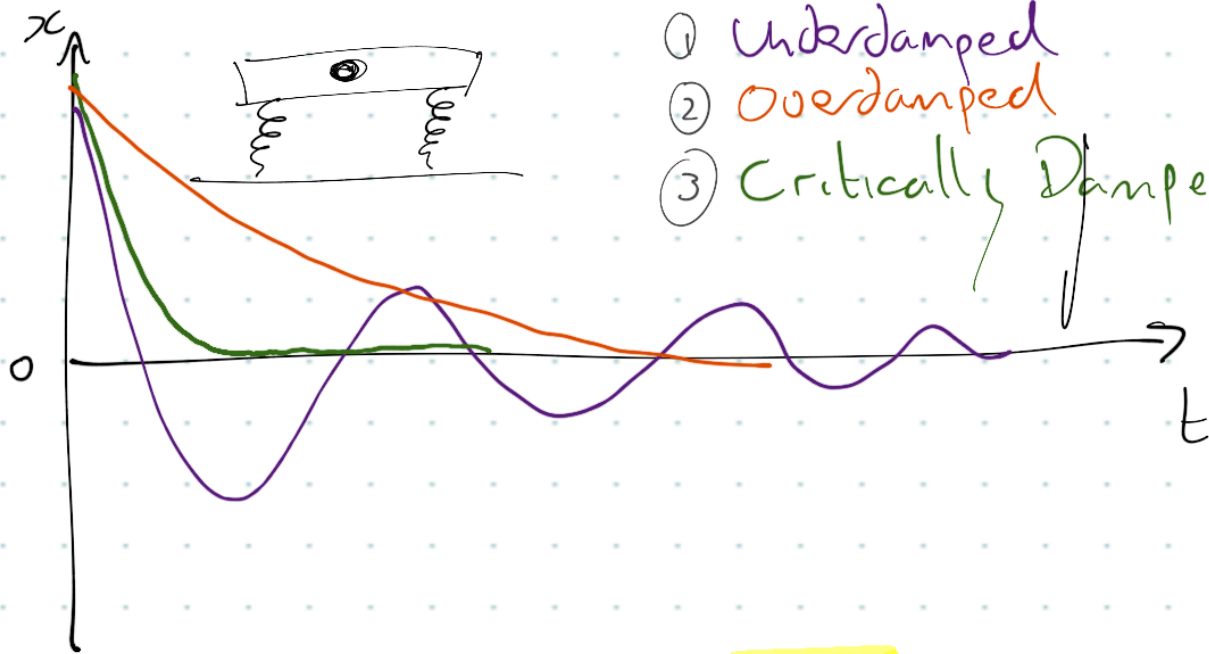


Suspension and Mathematics



Recap: Springs: force = kx ←

Ride frequency $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

Dampers:

force = cV ←

$\left(\frac{N}{m/s}\right) \left(\frac{Ns}{m}\right)$ ←

= (Damping coefficient $\times$ Velocity)

Critical Damping = $2\sqrt{km}$

Damping Ratio = $\frac{c}{c_{crit}} = \zeta$

$\frac{\sqrt{m}}{k \zeta} c$

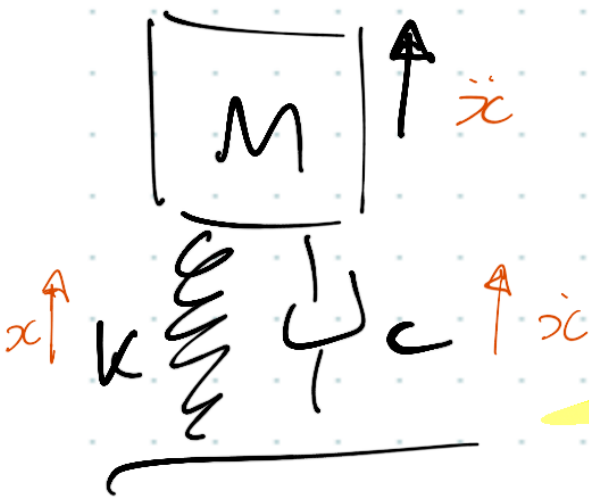
Also Remember : Force = ma 

Now then :

Altogether

$$F = ma + cV + kx$$

This takes into account ALL the forces



$$\text{So } ma + cV + kx = 0$$

We can also write this as

$$M \frac{d^2x}{dt^2} + C \frac{dx}{dt} + kx = 0$$

acceleration velocity position

This is a second order differential equation

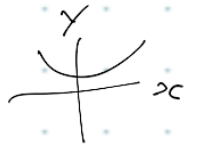
$\Rightarrow$ Suspension modelling

So Mathematics!

$$a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$$

eg.

$$3 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 10y = 0$$



How do we deal with those things ... and why.

How can we make it easier to solve?

we need to make a clever substitution:

lets use $y = e^{mx}$

$$\begin{array}{l} y = x^2 \\ \frac{dy}{dx} = 2x \end{array}$$

we need to differentiate ... so $\frac{dy}{dx} = Me^{mx}$

and again $\frac{d^2 y}{dx^2} = M^2 e^{mx}$

So what Now?

well $a \frac{\partial^2 y}{\partial x^2} + b \frac{\partial y}{\partial x} + cy = 0$

becomes $am^2 e^{mx} + bme^{mx} + ce^{mx} = 0$

Now, this is much better!

factorise out e^{mx}

So $(am^2 + bm + c) e^{mx} = 0$
why???

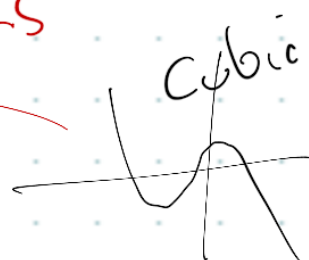
Now $a \times b = 0$

either $(am^2 + bm + c) = 0$ or $e^{mx} = 0$

We know $e^x \neq 0$ so $am^2 + bm + c = 0$

So we have a quadratic

So we get 2 solutions



So, there are two solutions to a quadratic
So how many different 'type' of roots are there
Three!

① Real and equal

$$M_1 = M_2 = M$$

② Real and Different

$$M_1 \text{ and } M_2$$

③ Complex

$$M = \alpha \pm i\beta$$

Remember Complex numbers?

$$\sqrt{-1} = ?$$

$$\sqrt{4} = ? \pm 2$$

$$2^2 = 4$$

$$-2^2 = 4 \quad ? \quad ?$$

We made this split up...

$$\sqrt{-4} = ?$$

So?

$$M = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow M = 3 + 5i$$

if this is -ve

① For the case of real and equal roots we get:

Critically Damped $y = (A + Bx)e^{mx}$ $m_1 = m_2$

② For the case of real and Different $\rightarrow$

Overdamped $y = Ae^{m_1x} + Be^{m_2x}$ $m = m_1 \text{ and } m_2$

③ And for complex .. we get

Underdamped $y = e^{\alpha x} [A \cos(\beta x) + B \sin(\beta x)]$ $(u = \alpha + i\beta)$

Example

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} - 3y = 0$$

$\Rightarrow$ quadratic

$$m^2 + 2m - 3 = 0$$

$$(m+3)(m-1) = 0$$

$$\text{So } m_1 = -3 \text{ and } m_2 = 1$$

We have 2 solutions $\rightarrow$ Real + Different

$$y = A e^{m_1 x} + B e^{m_2 x}$$

$$y = A e^{-3x} + B e^x$$

This is the 'general solution'
