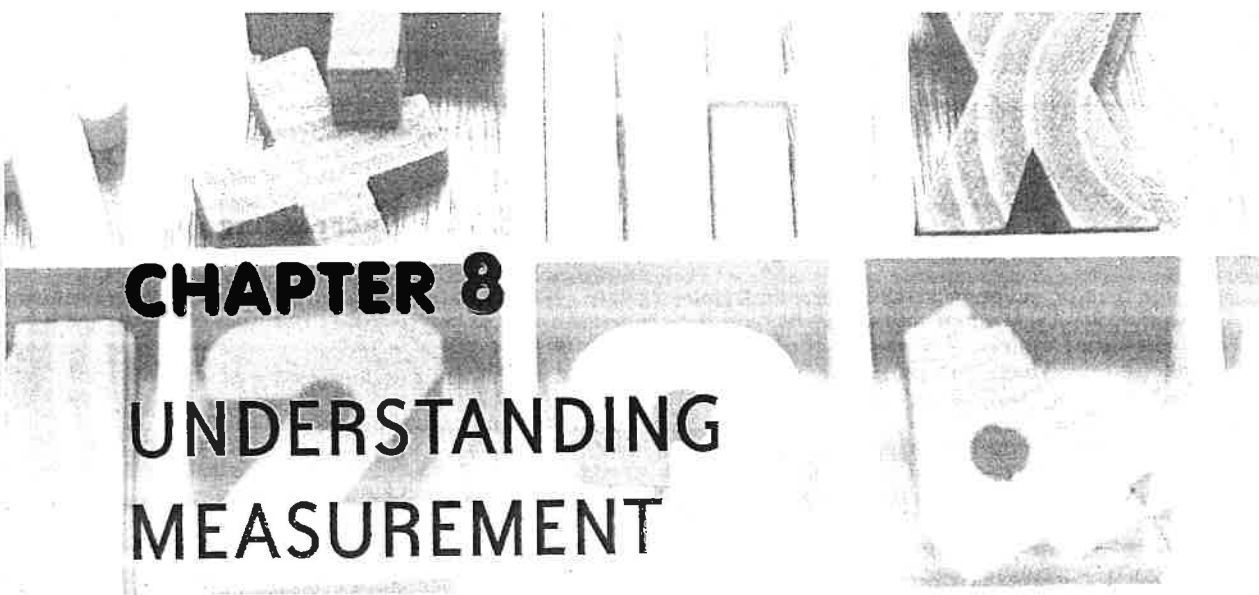




CHAPTER 8

UNDERSTANDING MEASUREMENT

A PERCEPTUAL MISUNDERSTANDING

Amy was trying unsuccessfully to lift a box full of wooden blocks in the nursery. She announced, 'I can't lift it because it's too big.' She then tipped all the blocks on the floor and carried the box to where she wanted it!

- What might be revealed by Amy's comment here?
- How might a teacher exploit this incident as a teaching opportunity?

In this chapter

In earlier chapters we have frequently found ourselves discussing operations with numbers in various measuring contexts. We now consider the mathematical ideas involved in measurement itself. Our aim is to help those who teach young children to have a secure understanding of the various attributes of objects and events that we measure and the general principles of measurement that underpin the curriculum.

Haylock, D. & Cockburn, A. (2013) *Understanding Mathematics for Young Children* (4th ed). London: Sage.

What do we measure?

Some of the comments that emerged when we raised this question with teachers set the agenda for the first part of our discussion of measurement:

- Length seems to be the most straightforward form of measurement, because the children can see what they're measuring.
- It's not quite the same when you're measuring the distance from one point to another. That's much more abstract, isn't it?
- My children do a lot with volume and capacity. But we normally only deal with measuring out stuff like water and sand, and pouring it into various containers.
- I'm never sure whether we're talking about the size of the container or the amount of water in it.
- What about mass and weight? Aren't they the same thing? Some books say you should talk about the mass and some say weight. That always confuses me, so I just ignore it and talk about weight.
- We measure time as well, but children find that very difficult. I think that's because you can't actually see what you're measuring.

In the age range that is the focus of this book, children's experiences of measuring are in the contexts of:

- length and distance,
- liquid volume and capacity,
- time of day and time intervals, and
- mass and weight.

Our experience of talking to teachers and children has made us aware that there are a number of possible confusions involved in each of these concepts.

Length and distance

To demonstrate that understanding of the concept of length is not as straightforward as might at first appear, consider some of the different types of situations to which, for example, the symbols '90 cm' might be connected.

First, we might connect '90 cm' to a straight part of some solid object, such as the edge of a table, and conclude that the table is 90 cm long, or 90 cm wide. Then we

might connect '90 cm' to a straight line drawn on the floor, on the board or on a piece of paper. We sometimes connect the same symbols to an imaginary straight line running through an irregular object, such as a child – for example, when measuring the height of the child as 90 cm. We could also connect '90 cm' to an imaginary straight line passing through the air, in order to talk about the height of a table. In the last two examples, the convention is that the imaginary line must be vertical. In other circumstances, such as finding the gap between two cupboards, the convention requires an imaginary horizontal line at right angles to the edges of the cupboards. Also we might connect the same symbols to an imaginary straight line when we talk about the *distance* from one point to another. To make this connection we may have to imagine a piece of string stretched between the two points. We might also connect the symbols to other than straight lines on various objects, for example when measuring a person's waist as 90 cm. And, finally, we might connect '90 cm' to paths that are not just a straight line, for example, when we add up the lengths of the four sides of a rectangle to find that the perimeter is 90 cm, or when we find the length of a curved path from A to B to be 90 cm.

This analysis helps us to understand some of the confusions that arise in children's minds because – once again – we connect the same symbols to such a variety of situations! In Figure 8.1, for example, the question might be posed as to whether A is longer than, shorter than or the same length as B. In one sense it is true to say that A and B are the same length, if we think of B as an object and the length of it as the imaginary horizontal line passing through it. But in another sense it is true to say that B is longer than A, if we focus on the lengths of the paths.

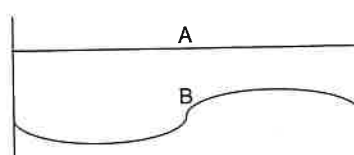


Figure 8.1 Are A and B the same length? Or is B longer?

Volume and capacity

These two words – volume and capacity – sometimes cause some confusion. This is not just because the word 'volume' is mostly associated nowadays with the level of sound on various electronic devices. The volume of something is the amount of three-dimensional space it occupies. Volume is normally measured in cubic units, such as cubic centimetres (symbol: cm^3). So we might say, for example, that the volume of a cuboid 3 cm long, 2 cm wide and 4 cm high is 24 cubic centimetres (24 cm^3), meaning that it occupies the same amount of space as 24 centimetre cubes.

Only *containers* have capacity. The capacity of a container is simply the volume of liquid that it will hold. Although capacity and liquid volume could be measured in the same units as volume in general, there are special units, such as litres and pints, which are usually used for measuring these aspects in particular.

Time

The importance of time in human experience is shown by the extensive language associated with the concept. Teachers of young children who recognize the importance of developing language in promoting understanding of mathematical and scientific ideas have a considerable agenda on their hands when they consider the everyday vocabulary that children have to understand and learn to use. The following list, while certainly not comprehensive, at least shows the extent of the teaching task:

how long, second, minute, hour, day, week, fortnight, month, quarter, year, leap year, decade, century, millennium, season, spring, summer, autumn, winter, weekend, term, half-term, lifetime, long time, short time, brief, temporary, for the time being, long-lasting, interval, pause, cycle, period, extra time, non-stop, never-ending, permanent, on and on, how old, age, age group, year group, teenager, young, old, elderly, middle-aged, under-age, ancient, modern, up to date, older, elder, oldest, younger, youngest, when, daytime, night-time, dawn, sunrise, morning, teatime, break-time, lunchtime (etc.), noon, midday, afternoon, dusk, sunset, evening, midnight, small hours, matinee, past, present, future, spell, then, now, before, after, next, previous, earlier, prior, following, later, afterwards, due, eventually, in the long run, in due course, never, always, once, once upon a time, one day, recent, soon, immediately, straight away, in a moment, instantly, in a jiffy, while, meanwhile, till, until, up to, not yet, in the meantime, during, nowadays, sometime, sooner or later, at the last minute, often, frequent, daily, hourly, weekly, monthly, annually, occasionally, regularly, now and again, every so often, from time to time, sometimes, hardly ever, once in a blue moon, clock, watch, sundial, timer, egg timer, meter, tick, tick-tock, dial, face, alarm, setting, hands, minute hand, hour hand, digital, (e.g.) three twenty-five, o'clock, a.m., p.m., half past, quarter to, quarter past, five past (etc.), twenty-five to (etc.), 24-hour clock, fourteen hours (etc.), timetable, zero hour, summer time, put clocks forward and back, overtime, half-time, slow, fast, on time, late, early, punctual, diary, calendar, date, Sunday, Monday, Tuesday, etc., yesterday, today, tomorrow, last week, January, February, etc., first, second, third, etc., nineteenth century, twentieth century, twenty-first century (etc.), ten-sixty-six, the year two thousand and thirteen (etc.), birthday, anniversary, mark time, beat time, keep time, rhythm, short notice, advance notice, afterthought, postpone, put off, waste time, ...

There is one clear conclusion for the teacher: the most important activity in teaching the topic of time to young children is talking to them – and with them – aiming to focus on and to develop this extensive range of vocabulary and language patterns.

There are in fact two very different concepts associated with time that children will use and will learn to measure. First, there is the time at which something occurs. This begins as an awareness of the order in which things happen and eventually becomes the concept of *recorded time*. We use time in this sense when we consult our watch to find the time of day, or make an appointment for a particular day of the month, or recall the year in which something took place.

Second, there is the time that something takes to happen – its duration – or the time that passes between two events or between two moments of recorded time. This is the notion of a *time interval*: the time that passes from the bell signalling the start of playtime and the bell signalling its end; the time it takes to complete a train journey if the train leaves at 13:05 and arrives at 14:55. Time intervals might be measured in seconds, minutes, hours, days, weeks, months, years, decades, centuries or even millennia! Age, being the time interval from birth to the present, is one example of this rather abstract concept that children handle with surprising confidence. No doubt this is related to the fact that their age is very much part of their identity.

It suddenly struck us when we visited a school recently and spoke to some children, that the first two questions that people almost invariably ask a child are, 'What's your name?' and 'How old are you?' How old you are is as much part of who you are as a child as your name!

Teachers of young children should note that recorded time and time interval are two very different concepts. We show later in this chapter that – because they are two different types of measuring scales – they have quite different mathematical properties. We would suggest that many teachers focus too much on teaching children to tell the time – which is much better learnt in the context of real-life situations in which they need to tell the time – without giving children sufficient experience of measuring time intervals. The reader will find it a useful exercise now to go through the list of time language given above and decide which of it is associated with each of these two aspects of time.

Mass and weight

This is a tricky one. There is an important distinction in scientific terms between the mass and the weight of an object. Strictly speaking, units such as grams, kilograms, pounds and ounces are units of *mass*, not units of *weight*.

We will attempt to explain. First, we would refer to those sets of plastic shapes or pieces of metal that we use for weighing things as, for example, ten-gram masses, kilogram masses, four-ounce masses, and so on. This language convention associates the word 'mass' correctly with the units of mass, such as grams, kilograms and ounces. If an object balances against a kilogram mass, like the model car shown in Figure 8.2, then we deduce that it is also a kilogram mass.

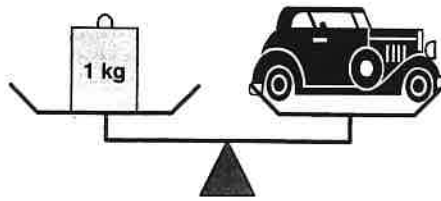


Figure 8.2 *An object weighing the same as a kilogram mass*

But what exactly is the mass of something? Well, in simple terms, the mass of an object is a measure of the amount of stuff making up the object – the quantity of matter in it – which is related to the number of atoms involved. Note that this is not the same thing as the *volume* (the amount of space it occupies), because in some objects the matter is more densely compressed together than in others.

However, when you hold an object in your hand you cannot actually experience its mass. You cannot perceive it, count it, smell it or feel it. It is the weight, not the mass, which you can feel and to which your muscles respond. This weight is the gravitational force pulling the object down towards the ground. Since weight is a force, it should be measured in the units used for measuring force. In the metric system, force – and therefore weight – is measured in *newtons*. Without being too technical, it may help the reader to know that a newton is about the weight of a small apple. On the earth's surface a mass of one kilogram has a weight of nearly 10 newtons. (More precisely, g newtons, where g is the acceleration due to gravity.)

Having seen film of astronauts on the moon, you may be familiar with the idea that if you were to take an object to the moon, where the gravitational force would be smaller, the weight of the object would be reduced. This is true – the weight would be reduced – although the object's mass is unchanged. Our one-kilogram mass would now weigh only about one and a half newtons. This is because the mass of the moon is much smaller than the mass of the earth, and therefore the gravitational pull is weaker.

This change in weight would show up if we were weighing our object on a spring-type weighing device, as shown in Figure 8.3. But it is interesting to observe that the situation in Figure 8.2 would not change if we took the balance to the moon. It appears, therefore, that the two weighing devices are measuring different things! The balance-type weighing device enables us to deduce the mass of an object, by balancing it against an equal mass. The spring-type weighing device clearly responds to weight. The former would give us the same result wherever we used it. The latter would give slightly different readings at the bottom of a coal mine or at the top of Mount Everest (due to a slight variation in the value of g as the distance from the centre of the earth changes), and a significantly different reading on the moon. In practice, of course, we will not often take our kitchen or bathroom scales to the moon.

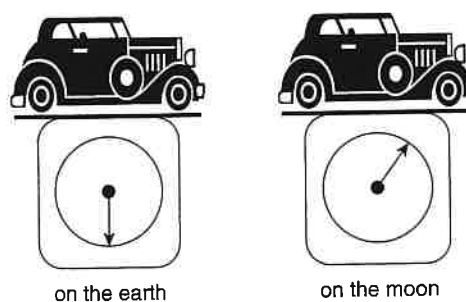


Figure 8.3 *Change in weight indicated on spring-type weighing device*

So the scales can safely be graduated in grams and kilograms, or pounds and ounces, and you can be confident that when you weigh something on the earth's surface and the arrow on the dial points to one kilogram then you do actually have a mass of about one kilogram.

Teachers may, therefore, be perplexed to be told that it is actually incorrect from a scientific point of view to say that the object weighs one kilogram or that the weight of the object is one kilogram. Strictly speaking, it is the mass of the object that is one kilogram. Unfortunately, from the point of view of the children's conceptual understanding, nearly everyone says that the weight of the object is one kilogram; and the idea of measuring weight in newtons will be introduced to children only in the later primary years.

Later on in secondary school students will need in their science lessons to distinguish between weight and mass, and the majority will find this very difficult. No doubt part of the difficulty will be unlearning the erroneous language of primary school and the marketplace. We offer some suggestions that might help with this problem. These are scientifically correct, mathematically sound and reasonably straightforward. First, be consistent in referring to those things you use to counterbalance the objects being weighed as masses, not weights. In the classroom cupboard you will have boxes of 1-gram masses, 10-gram masses, 100-gram masses, and so on. It seems a bit strange at first, but you soon get used to it! The word 'mass' is thus encountered when you shift from weighing with non-standard units to weighing with standard units.

Second, we suggest that young children's experience of weighing should be primarily with balance-type weighing devices. Third, when we have weighed something we say something along the lines of: 'The book weighs the same as twenty marbles' or 'The book weighs the same as a mass of 200 grams' or, simply, 'The book weighs the same as 200 grams'. The key phrase is 'weighs the same as', emphasizing the equivalence that has emerged. Finally, if an object weighs the same as, say, a mass of two hundred grams then we can also say that this object has a mass of two hundred grams.

Measurement in general

In this section we will consider what might be some of the fundamental mathematical ideas involved in the process of measuring. These ideas include the notion of comparison and ordering, the principles of transitivity and conservation, the idea of a unit, the use of non-standard and standard units, the importance of approximation and accuracy, and estimation.

Comparison

The primary purpose of making a measurement is to make a *comparison* between two items according to the magnitude of some attribute, such as length, weight, capacity, age, value, and so on, and thereby to put them in order.

Even the youngest children in my nursery class are able to make comparisons. The other day a 3-year-old said, 'My piece of cheese is the smallest, so I should get another piece!'

Associated with this idea is the extensive set of language of comparison given in Table 3.6 and discussed in Chapter 3: longer, shorter, heavier, lighter, younger, older, and so on.

We should note that, particularly in early experiences of measuring, comparisons of this sort are often made directly without the use of any measuring units. So two children can stand next to each other and determine who is the taller and who is the shorter. Two objects can be put on either end of a simple see-saw balance and a deduction made about which is the heavier and which is the lighter. It is helpful at this stage of the child's development of understanding of the particular measuring concept that they experience objects that exaggerate the attribute in question. A large, light object can be balanced against a small, heavy object to focus the attention on the weight rather than the size. Water can be poured from a tall jar with a small capacity into a short, squat jar with a large capacity. Comparing by weight pairs of identical-looking sealed yoghurt cartons filled with materials of different density – sand, sawdust, ball bearings – again focuses the attention on the one way in which they are different, namely, in terms of their heaviness. Amy, for example, in the anecdote at the head of this chapter, would benefit from discovering and talking about the fact that two boxes might be as big as each other, but that one can be too heavy to lift and the other can be lifted easily: because one is heavier – not bigger – than the other.

Incidentally, it is often surprisingly difficult to compare by hand the weights of two objects and decide which is the heavier and which the lighter. This is probably because the pressure on your hand is dependent on the shape of the object and the area in contact with your hand. For example, put this book in one hand and a 500-gram mass

in the other and try to decide which is the heavier. However, if you put the two objects into plastic carrier-bags and hold the bags rather than the objects you will find that it is markedly easier to make the comparison. This is a good tip for teachers wanting to give children practical experience of comparing weights by hand.

We should remind ourselves of the fundamental idea that every comparison statement has an alternative equivalent form (see Table 3.6 in Chapter 3), and that children should be encouraged always to make both statements. This is such a key aspect of language and mathematical development that it cannot be overemphasized. Table 8.1 provides some examples of pairs of such equivalent statements, demonstrating that this is a key principle that pervades all areas of measurement.

Table 8.1 *Equivalent statements of comparison*

The table is higher than the desk.	The desk is lower than the table.
Breakfast is earlier than dinner.	Dinner is later than breakfast.
The book is heavier than the pencil.	The pencil is lighter than the book.
John is older than Meg.	Meg is younger than John.
London is bigger than Norwich.	Norwich is smaller than London.
A motorway is wider than my street.	My street is narrower than a motorway.
Your car is faster than my car.	My car is slower than your car.
My teddy is taller than yours.	Your teddy is shorter than mine.

In making comparisons of this sort we are attending to what is different about two objects, in other words to a transformation that occurs when we shift our attention from some attribute of one to the same attribute of the other, such as height, weight, and so on. As in the example of the identical yoghurt cartons, the two objects may in all other outward respects be the same.

But there is, of course, another possibility when two objects are compared for some particular attribute: we may decide that they are equal. So two objects, such as a book and a box, may be very different in a number of respects, such as their size or their shape, but be equivalent in another. For example, they may weigh the same. So we see that what have been referred to in Chapter 1 as the concepts of transformation and equivalence are here associated with the most fundamental idea of measuring. These two ideas appear to permeate all mathematical thinking. The equivalence notion, 'is the same as', will turn up frequently in measuring experiences, in such language forms as 'is the same length as', 'weighs the same as', 'happens at the same time as', and so on. The photograph shows a child in a nursery class with balance scales trying to make the two sides weigh the same – which she achieved successfully and described with appropriate language. What she did not yet get was that when the two piles of bricks did not balance it was the one that went down that was the heavier!



PAUSE TO REFLECT Moonstruck

Looking out at the moon the other night it struck me that it felt very near. Indeed it appeared so close that it gave the impression that you could get to it in no time at all and certainly a lot faster than you might reach Australia! Why do you think that might be?

There was a very funny episode of the TV show 'Father Ted' in which Father Ted was using some toy cows and some cows in a field to explain to Father Dougal – unsuccessfully – the difference between 'small' and 'far away'. If you haven't seen it, find it on YouTube! Is this a difficulty for young children when they are comparing sizes and learning about measurement?

We asked some young children, which is bigger, the moon or an elephant? They chose the elephant. But when we asked them, which is bigger, this van parked here or the house a long way away at the end of the road, they all chose the house. What do you make of that?

Ordering and transitivity

An important principle common to any ordering relationship is that of *transitivity*. This principle is evident when three or more objects are being ordered according to a particular attribute, such as their length, their weight, their time of day, their capacity,

and so on. In Figure 8.4(a) the arrow could represent any one of our comparison statements, such as 'is longer than', 'is lighter than', 'is earlier than', 'holds more than', and so on. The objects A, B and C have been ordered using such a relationship, by comparing A with B and then B with C. The principle of transitivity allows us to make the logical deduction about the relationship between A and C, shown in Figure 8.4(b): A is longer than C, or A is lighter than C, and so on. The relation is said to be transitive when the following is necessarily the case: if it occurs between A and B and between B and C, then it can be 'carried across' directly from A to C.

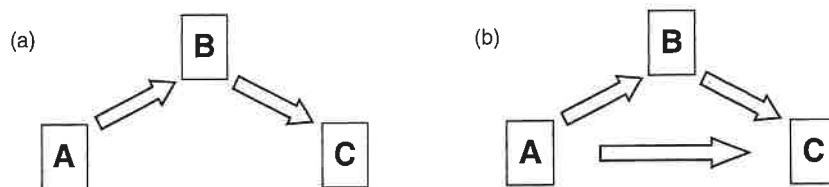


Figure 8.4 The principle of transitivity

Formally, this principle is expressed in one of two ways, as follows:

- a using the symbol $>$ to stand for any 'greater than' relationship
 if $A > B$ and $B > C$ then $A > C$
 (If A is greater than B and B is greater than C then A is greater than C.)
- b using the symbol $<$ to stand for any 'less than' relationship
 if $A < B$ and $B < C$ then $A < C$
 (If A is less than B and B is less than C then A is less than C.)

All the different ways in which we measure things satisfy these properties – because putting things in order is a fundamental property of any form of measurement. In addition to the 'greater than' and 'less than' transitivity properties, there is a third form of transitivity in the context of measurement. Clearly the arrows in Figure 8.4 could also represent *equivalence*. For example, they could stand for statements about *equal* length or *equal* weight, such as 'is the same length as', 'weighs the same as', and so on. It is not a difficult logical deduction that, if A is equal to B and B is equal to C, then A is equal to C. So, we should note therefore that equivalence also satisfies transitivity. This example of transitivity is expressed in symbols as follows: if $A = B$ and $B = C$ then $A = C$. Although this might appear to be a rather obvious statement, in the context of measurement it is actually very significant. It is this property that allows us to make repeated copies of standard measurements, such as the kilogram or the metre, without always going back to the originals! In the context of the classroom children will experience this, for example, by making their own 100-gram mass, by

balancing play-dough against a standard 100-gram mass, and then using their play-dough version to generate further copies.

To understand the concept of transitivity it might be helpful for the reader to experience some non-exemplars as well as exemplars. We have seen how the arrows in Figure 8.4 might represent any relationship of the form 'is greater than', 'is less than', or 'is the same as'. But not all mathematical relationships are transitive. In fact, part of our understanding of any relationship must be an implicit awareness of whether it is or is not transitive. To demonstrate this, readers may care to assess their understanding of the concept of 'factor' by asking this question: if the arrows in Figure 8.4(a) represent the relationship 'is a factor of', where A, B and C are three natural numbers (positive whole numbers), does the relationship between A and C shown in Figure 8.4(b) necessarily follow? We posed this question to some of the teachers we worked with:

- S. *Remind me again – what's a factor?*
- D. *A number that divides exactly into another number. For example, four is a factor of twelve, because four divides into twelve exactly.*
- E. *So, if A is a factor of B and B is a factor of C we want to know if A is a factor of C.*
- S. *I haven't the foggiest idea. It might be, I suppose.*
- E. *Four is a factor of twelve and twelve is a factor of 24. That works. Four is a factor of 24.*
- D. *Will it always work?*

Try the same exercise with other mathematical relationships, such as 'is one more than', 'is half of', 'is a multiple of'. The facility with which you make the deduction about whether or not the fact that A is related to B and B is related to C necessarily implies that A must be related to C is a good indicator of your understanding of the mathematical relationship. If you do not find it immediately obvious then you are in a similar situation to young children learning about measurement. Having compared A with B and B with C they will not automatically make the deduction about the relationship between A and C.

Transitivity is one of the logical principles that Piaget identified as a key indicator of children's development in their understanding of measurement. Whether the difficulty for children who appear not to grasp the principle is a failure in logic or simply an inability to recall and process at the same time all the details in the situation is a matter of some debate. But, whatever the correct interpretation of children's responses to tasks assessing their grasp of this principle, it would seem to us to be self-evident that – since transitivity is such a fundamental property of measurement – children learning about measuring should simply have a considerable amount of experience of putting three (or more) objects in order according to their length, their weight, their capacity, and so on. It would appear likely that through practical experience of this

kind of ordering activity – supported by well-focused questions and discussion with their teachers – they will be helped to get this mathematical structure into their thinking about measurement in general.

Conservation

Another fundamental and crucial principle that applies to many aspects of measurement is that known as *conservation*. This refers, for example, to the fact that a ball of play-dough will still weigh the same when it has been squashed into a sausage shape or broken up into a number of smaller pieces; that the volume of water in a tall, thin flask does not change when it is poured into a short, squat flask; that the length of a pencil remains the same when it is displaced a little to the right. We have already noted in Chapter 2 that grasping the conservation of number is a key component of learning to count: children have to learn that changing the arrangement of the objects being counted does not change the number. Figure 8.5 provides some examples of tasks that have sometimes been used to assess children's grasp of this principle in various measuring contexts. (See also the Research Focus in Chapter 2.)

Demonstrating a grasp of this principle of conservation, first of all with number, then with length, weight and volume, has also been identified by Piaget as one of the key indicators of a child's intellectual development. However, many of the tasks used to assess the child's grasp of conservation, such as those in Figure 8.5, have been justifiably criticized as not being embedded in a context that has any meaning or

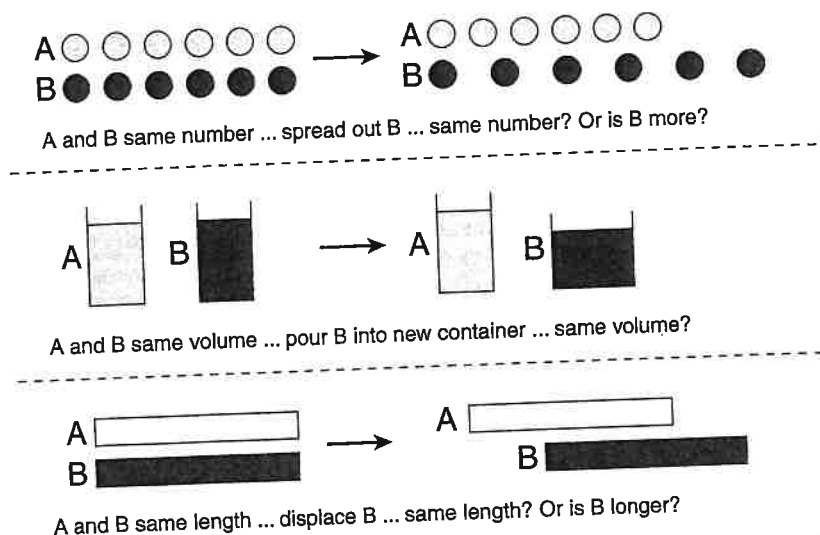


Figure 8.5 Assessing understanding of conservation

purpose for a child. It appears to us to be the case that quite young children seem to use the principle of conservation of measurement quite happily when engaged in a purposeful task in a meaningful context. When, for example, they are cooking, they will measure out quantities of water and flour, and then they will proceed to transfer these from one container to another without any apparent concern that these activities might alter the quantities they measured out in the first place!

In spite of such reservations as these about some of the methods used to assess children's understanding of conservation, we should stress that this is nevertheless a key component of understanding measurement. Children will certainly benefit from activities and discussion designed to focus their attention upon this principle. In fact, we would argue that the thinking process involved is an instance of fundamental mathematical reasoning, because the principle of conservation provides yet another experience of the concepts of transformation and equivalence that we are highlighting throughout this book. What the child has to grasp, for example, is that if you take a quantity of water in a container and apply to it certain transformations, such as pouring it into a different shaped container, the volume of water stays the same. In other words, there is an equivalence, something the same about the two situations, in spite of the transformation that has occurred to, say, the height of the water level, or the shape of the water. In fact, we could say that what the child has to learn is which of the transformations that can be applied in a range of measurement contexts preserve which equivalences and which transformations destroy which equivalences. If you take a square piece of paper and transform it by moving it to a different position or by cutting it up into two triangles, the area is preserved: it will still cover the same amount of the table surface. If, on the other hand, you screw the paper up into a tight little ball, or set light to it, these are transformations that destroy this equivalence. The reader may find it a challenging, but useful exercise to analyse, in terms of which equivalences are preserved or destroyed, transformations such as rearranging some building blocks, boiling some water, rolling up a tape measure, taking an object to the moon, or putting the clocks forward for summer time.

Units

Common to all aspects of measurement is the idea of a *unit*. Instead of using measurement concepts just to order two or more objects, we now move to comparing some attribute of a single object with a number of equal units. So a child might use, for example, a set of unused pencils as units for measuring. They would already be confident with statements such as 'the book is longer than a pencil'. But now they would move on, for example, to laying out a line of pencils along the edge of the table and saying 'the table is about ten pencils long'.

It is conventional to introduce children to the idea of a unit through the use of non-standard units – for example, measuring lengths in pencils, as above, or weighing

objects with marbles – before moving on to standard units such as centimetres and grams. Some of the teachers we talked to were very uncertain about the point of getting children to use non-standard units for measuring, especially since children will often have met the standard units from shopping. We outline below a number of reasons that can be put forward for the value of incorporating experience of non-standard units.

First, by using non-standard units in the early stages, children are introduced to the idea of measuring in units through familiar objects – such as pencils, marbles, hand spans, feet, and so on – rather than going straight into those mysterious things called centimetres and grams. In this way they are not required to handle a new piece of vocabulary for the unit at the same time as meeting a new measuring experience. As far as the mathematical ideas involved are concerned, there is no essential difference between measuring a table in pencils and measuring it in centimetres, so if children find a familiar non-standard unit less threatening then all well and good.

Second, it is suggested that the non-standard unit can be a more appropriate size of unit for the first practical measuring tasks that young children will undertake. Centimetres may be too small and therefore too numerous for your first experience of measuring the length of the desk top, or the height of a friend – and metres are clearly too large a unit for this purpose. Grams are very small units for weighing most of the objects around the classroom, such as books, scissors, shoes, and so on, and a kilogram of iron will put the child in hospital if dropped on their foot. Something like a Lego brick is likely to be a more appropriate unit to begin with, because it will result in measurements of just a handful of units, rather than hundreds of them.

Third, we should note that adults do not always use standard units when engaged in practical measuring tasks. Whether or not they do might depend on the accuracy required by the task. For example, most of us are happy to measure out fertilizer for our lawns by the handful, and we will often use spans and paces for certain stages of practical jobs around the home.

Finally, it is argued that through using non-standard units children will become aware of the need for a standard unit – for example, when they discover that the classroom is twenty paces wide when they do the measuring, but only eleven paces wide when the teacher does it.

- When my 6-year-olds were measuring lengths in spans and feet they made me do them all as well. They seemed fascinated by the idea that my span or my feet always gave a smaller number.
- I find that a lot of useful mathematical talk takes place when a group of children are deciding what to use to measure something – like whether to use spans or cubits to measure the length of their table.
- A favourite activity in my nursery class is to count how many bricks tall they are.
- When you talk to them about measuring with non-standard units they do seem to pick up the idea of the need for a standard unit quite quickly.

- My 3- and 4-year-olds enjoy measuring their heights in centimetres using the height chart on the wall. They get very excited when they are more than 100! They seem to get the idea that the taller you are the bigger the number is.

One teacher had serious doubts about using units like spans and paces with young children:

I always find an element of competition creeps in when they start measuring in hand spans or paces, and the maths goes out of the window. One girl refused to admit that her span was shorter than someone else's and got quite upset. Then when they were pacing out the length of the corridor all the boys started taking enormous leaps, rather than strides, to see who could do it in the smallest number!

We take this teacher's point, and it may be that there are occasions when impersonal spoons and sticks might be preferable as units to individuals' spans and strides. But we wonder whether it is really the case that the mathematics has gone 'out of the window' in these examples. These children seem very aware of the significance of the size of the units being used, and the experience has clearly provoked a competition and discussion based on mathematical criteria!

SI base units and other metric units

Teachers may come across references to the SI (International System/Système International) version of metric units. This is an internationally accepted convention used in trade and industry, and in technological and scientific work. The main feature of this system is that each aspect of measurement should have just one base unit. Some of these are shown in Table 8.2 for aspects of measurement likely to concern teachers, together with the appropriate symbol. It is an accident of history that the base unit for mass is the kilogram (kg) not the gram (g).

Table 8.2 *Some SI units*

Measure	SI Unit	Symbol
length	metre	m
mass	kilogram	kg
time	second	s
volume	cubic metre	m ³

According to the SI convention, all lengths, for example, would be measured in metres. This would mean that the width of a piece of A4 paper (about 21 cm) would be given as about 0.21 m. If other units are to be used, by the use of various prefixes, then there is a further convention that as far as possible only those representing powers of a thousand should be employed. Some of these 'preferred prefixes' are shown in Table 8.3.

Table 8.3 *Preferred prefixes*

Prefix	Meaning	Symbol	Example
mega	a million	M	1 Mg = one million grams
kilo	a thousand	k	1 km = one thousand metres
milli	a thousandth	m	1 mm = one thousandth of a metre
micro	a millionth	μ	1 μ g = one millionth of a gram

Using a preferred prefix (milli) the width of the piece of A4 paper would then be given as about 210 millimetres (210 mm). However, for practical purposes other prefixes are required because the base unit and the base unit with a preferred prefix turn out to be inappropriate sizes of unit for the task in hand. For example, for many tasks that involve measuring length the millimetre is too small and the metre too large. This is why the *centimetre* is used so frequently, even though 'centi' is not a preferred prefix. So, for example, it is much more convenient to give the width of a piece of A4 paper as about 21 cm. Some of these other prefixes are shown in Table 8.4. Of those shown here the two we are most likely to meet are *centi* and *deci*.

Table 8.4 *Some other prefixes*

Prefix	Meaning	Symbol	Example
centi	a hundredth	c	1 cm = one hundredth of a metre
deci	a tenth	d	1 dm = one tenth of a metre
deca	ten	da	1 dag = ten grams
hecto	a hundred	h	1 hg = one hundred grams

There are some other metric units that are derived from SI units. The commonest of these, encountered in everyday experience, is the litre (l), used for measuring liquid volume or capacity. A litre is the same volume as one thousand cubic centimetres of solid volume. Based on the litre are the millilitre (1 ml = one

thousandth of a litre), the centilitre (1 cl = one hundredth of a litre) and the decilitre (1 dl = one tenth of a litre). We have noticed that bottles of wine are variously labelled as 0.75 litre, 7.5 dl, 75 cl and 750 ml. Those who purchase solid fuel, for example, may also encounter the tonne, referred to in speech as 'the metric tonne'. This is a thousand kilograms (1000 kg) or a million grams (that is, one megagram, 1 Mg).

We conclude this section with an assortment of facts and conventions about metric units that teachers will find helpful.

- One litre is the same volume as 1000 cm³.
- 1 ml is the same volume as 1 cm³.
- A medicine spoon holds 5 ml.
- Base-ten materials for number work are often based on centimetres; if the unit is a centimetre cube then the 'ten' is a decimetre long, the 'hundred' is a decimetre square, and the 'thousand' is a decimetre cube.
- A decimetre cube is 1000 cm³, the same volume as a litre.
- A litre of water has a mass of 1 kg.
- Hence 1 ml of water has a mass of 1 g.
- A cubic metre of water has a mass of one tonne (1000 kg).
- The distance from the North Pole to the Equator is approximately ten thousand kilometres (10 000 km).
- A simple pendulum of length 1 m ticks once approximately every second.
- The symbol for litre is l (el), but because this can be confused with 1 (one) we recommend writing the word 'litre' in full.
- Note the correct spelling of 'gram'.
- The symbol for gram is not gm but g.
- Symbols for units do not have plural forms, so do not write, for example, 5 cms, but 5 cm.
- The symbol for kilo is a lower case k, not a capital K; so we write km, not Km.
- English readers should note that the stress in the word 'kilometre' should be on the first syllable, not the second!
- No full stops should be used after a symbol for a unit, such as 5 cm or 10 kg, except at the end of a sentence.

Approximation and accuracy

An interesting feature of measurement is that nearly all measurement is *approximate*. Except when the measurement is simply a form of counting, such as when 'measuring' the value of the coins in my pocket, we always measure to the nearest something. It is not possible, for example, to make an exact measurement of the length of anything. All we can do is to measure the length of the table to the nearest centimetre,

or nearest millimetre, and so on. Even with the most refined measuring device in the world we would still be making an approximate measurement. This is a difficult idea for us to incorporate into our thinking about measurement, because we tend to think of mathematics as providing exact results.

Even in the early stages of measuring in non-standard units, young children will encounter this problem and have to find a way of expressing their observations. So they might be helped to formulate language patterns like:

- The book weighs more than 8 blocks but less than 9 blocks.
- The book weighs a bit more than 8 blocks.
- The book weighs between 8 and 9 blocks.

The limits of 8 blocks and 9 blocks here are examples of what are known technically as the lower and upper bounds of the measurement. If you have carried out a very careful measurement of the width of a piece of A4 paper to, say, the nearest millimetre, you will only be able to assert, for example, that the width of the paper lies between a lower bound of 210 mm and an upper bound of 211 mm.

We also have to learn the importance of measuring to an appropriate level of *accuracy*. In measuring a person's age, for example, it is often appropriate to do this to the year below or the year above. So a child might say 'I'm seven' or 'I'm eight next birthday'. For many purposes we measure time of day to the nearest five minutes, so that we would be amused by someone who said, 'I'll meet you at 4:23 this afternoon' – because they appear to be employing an inappropriate level of accuracy. But we would not be amused if a train timetable advertised a departure time of 16:25 and we missed the train because it had left at nearer 16:23.

Consequently, all that we have said earlier about equivalence should be understood in these terms. Phrases such as 'is the same length as' and 'weighs the same as' must be interpreted as implying that the comparison has been made to an appropriate level of accuracy. We might actually mean something like 'is the same length to the nearest millimetre' or 'weighs the same as far as we can judge using this balance'. There is no way in which we can assert that two lines are *exactly* the same length or that two objects have exactly the same mass.

To sum up, the notions of approximation, upper and lower bounds, and the appropriate level of accuracy are important ideas that are fundamental to most aspects of measurement. Experience and discussion of all these aspects of measurement are often insufficiently emphasized in work with children.

Estimation

Children's understanding of measurement is developed significantly through experience of making estimates. When children estimate which is longer or shorter,

which holds more or holds less, which weighs more or weighs less, whether they have enough of some materials, and so on, they are engaging with the attribute of the measurement in a way that makes sense to them and which uses the key principles of comparison, ordering and approximation. Even in the earliest stages of their developing awareness of the size of things or the amount of something, young children will make estimates. This became clear when we asked some pre-school teachers about 3-year-olds' early experiences of informal measurement.

- Some of the children were playing in a tent. One of the girls said to one of the boys, 'You can't fit in, because you're too big!'
- Emily looked at a large area she wanted to paint green and her supply of green paint and told me there wasn't enough paint to do it.
- The children like guessing where someone will come to before we mark their height on the wall; they do this by comparing different children's heights.
- Harry wanted some water to put in a bowl to float a plastic boat. When the class assistant gave him a bottle of water to use he looked at it and said, 'I need more than that!'

PAUSE TO REFLECT Reasonable estimates

It will help enormously to develop your confidence with units of measurement if you get into the habit of making estimations of length, mass, time and capacity, using standard metric units. To do this you need to have at your finger tips some reference items. A good start is to memorize your own measurements, such as: height, waist and pace measured in centimetres; weight (mass) in kilograms.

Here are some questions we think it would be useful if you were able to answer without much thought. How many kilometres is it approximately from where you live to where you work? About how many seconds would it take you to walk 100 metres? How many litres of water does your watering can hold? About how many millilitres are there in a small glass of wine? What would be an average birth weight in kilograms for a baby? About how many grams would you expect to be the mass of this book?

Do you have your own favourite reference items that you use for comparison when making measurements? Share your ideas with a colleague. You might have a competition to see who is better at estimating time intervals in seconds.

Types of measuring scales

In the final section of this chapter we discuss from a mathematical perspective three different kinds of measuring scales. We do this to raise the awareness of teachers of young children to some fundamental mathematical principles of measurement, examples of all of which are encountered in the earliest experiences of measurement in the classroom. The easiest kind of measuring scale is what is called an *ordinal scale*. More sophisticated is an *interval scale* and the most sophisticated is a *ratio scale*. For ease of explanation it is best to consider these in reverse order. We aim to show how these different kinds of measurement scales provide contexts for the different aspects of number and number operations outlined in Chapters 2, 3 and 4.

Ratio scales

Length, mass, capacity, time interval, and so on, are examples of what is called a *ratio scale*. They are called ratio scales because the ratio of two measurements has a real meaning. So, for example, we can compare a length of 90 cm with one of 30 cm, not just by the difference (60 cm), but also by the ratio (3). Hence, using the ratio comparison, we would observe that a length of 90 cm is three times as long as a length of 30 cm; or that 90 cm can be made up by combining three lengths of 30 cm. Similarly, with measurements of mass, we can observe that a mass of 100 g is ten times as heavy as a mass of 10 g; or that a mass of 100 g is equivalent to ten masses of 10 g. Likewise, with measurements of time, we can assert that a time interval of 240 minutes is 6 times longer than a time interval of 40 minutes; or that a period of 240 minutes can be made up of 6 periods of 40 minutes. Notice that the two central ideas in these comparisons are ratio and scaling – two ideas that are central in division and multiplication, as explained in Chapter 4.

In fact, this basic property of ratio scales implies that these are the only kind of measurements on which it is usually legitimate to perform many aspects of arithmetic involving multiplication or division. Some of the structures of multiplication and division discussed in Chapter 4 have meaning only in those measurement contexts that are ratio scales, such as measurements of length, mass, volume and capacity, and time intervals – especially the scaling structure of multiplication and the ratio structure of division.

Hence, it is legitimate and meaningful to make statements such as these, only because in each case the measurements involved are taken from a ratio scale:

- Three 25-kg sacks of potatoes have a total mass of 75 kg. (The repeated addition structure of multiplication.)
- The walk to school takes 15 minutes. There and back will be double this, that is, 30 minutes. (The scaling structure of multiplication.)

- A length of 30 m of ribbon shared between three people gives a length of 10 m each. (The equal sharing structure of division.)
- From a jug of 250-ml capacity we can fill five containers of 50-ml capacity. (The inverse of multiplication structure of division.)
- This stick of length 80 cm is four times the length of that stick of 20 cm. (The ratio structure of division.)

We should note that 'monetary value' is also essentially a ratio scale, because we can compare two amounts of money or prices by ratio. We can, for example, make meaningful statements about how many times more expensive or less expensive one thing is than another, by what scale factor a salary might be increased or decreased, rates of inflation, percentage rises in the cost of living, and so on. These are all employing the ideas of scaling and ratio that are central to the notion of a ratio scale.

Interval scales

Ratio scales can be contrasted first with what are called *interval scales*. In an interval scale two measurements can be compared only by their difference, not by their ratio. The two most familiar examples of interval scales are (a) measurements of temperature in degrees Celsius, and (b) the time of day. They are called interval scales because the steps from one mark on the measurement scale to the next are equal intervals. This means that the interval between 1 and 2 is the same as that between 2 and 3, and between 3 and 4, and so on. For example, it takes the same amount of time to move from 2 p.m. to 3 p.m. as it does to move from 3 p.m. to 4 p.m. It takes the same amount of heat energy to raise the temperature of something from 2 °C to 3 °C as it does to raise it from 3 °C to 4 °C. Of course, what is true about interval scales is also true of ratio scales. But there are more restrictions on what can be said and done meaningfully with measurements on an interval scale.

For example, to compare a temperature of 30 °C with one of 10 °C you can use only the difference (20 °C), observing that one temperature is 20 °C hotter or colder than the other. It makes no sense at all to talk about one temperature being 'three times hotter' than the other. You cannot make up a temperature of 30 °C from three temperatures of 10 °C! Similarly, 6 p.m. is not in any sense three times 2 p.m. To compare 6 p.m. with 2 p.m. we can make use only of the idea of difference (it is four hours later), not the idea of ratio.

This marked contrast between the two types of measuring scale is related to the fact that in a ratio scale the 'zero' is actually 'nothing' or 'an absence of something' (see the discussion of zero in Chapter 2). So, a length of 0 cm or a mass of 0 kg really is nothing. But in an interval scale the zero is arbitrary. For example, it is quite arbitrary that in the Celsius temperature scale the freezing point of water should be taken as

0 °C. Similarly, it is clearly an arbitrary decision that 'zero hours' (midnight) comes when it does – otherwise we would not be able to put the clocks forward and back for summer time, or to have different time zones for various sectors of the globe. So, when the temperature falls to zero it is not the case that there is now no temperature or that the temperature has become 'nothing' – there is a very definite temperature and we can certainly feel it! And time does not disappear at midnight when your watch indicates zero hours (00:00).

The nature of interval scales means that – in contrast to ratio scales – we cannot meaningfully do any arithmetic involving many aspects of multiplication and division, particularly those related to the ideas of ratio and scaling. There are no meaningful statements in the contexts of time of day or temperature that have the same structure as the examples of multiplication and division statements given in the previous section for ratio scales. For example, three people would not share a temperature of 30 °C and have 10 °C each! The addition and subtraction structures discussed in Chapter 3 that have most application in these contexts are those of increasing, comparison and difference, and reduction. These allow us to make statements in the contexts of measurement of temperature and time of day such as these:

- Her temperature was 38 °C yesterday, but it rose 2 °C during the night and is now 40 °C. (The increasing structure of addition.)
- The train arriving at 16:30 gets to London 2 hours later than the one arriving at 14:30. (The comparison structure of subtraction.)
- The temperature was 30 °C at midday, but it has fallen by 10 °C, so it is now 20 °C. (The reduction structure of subtraction.)

Of course, statements with the same structures as these could also be made in the context of ratio scales, such as measurements of length and mass. We should also emphasize the distinction made earlier in this chapter between the two aspects of time: time intervals and recorded time. We now see that the essential difference in the way we handle these two concepts is that the first is a ratio scale and the second is an interval scale.

We will mention one further legitimate and meaningful arithmetic process that can be applied to measurements on interval scales and ratio scales. This is the process of finding an average (strictly called an arithmetic mean). So, for example, given three lengths of 30 cm, 20 cm and 16 cm (a ratio scale), we can legitimately add them up and divide by 3 and assert that the average length is 22 cm. Similarly, given maximum temperatures on three consecutive days of 30 °C, 20 °C and 16 °C (an interval scale), we can add these up and divide by 3 and make the meaningful statement that the average maximum temperature was 22 °C. This is permissible only because the steps between measurements on an interval scale are equal intervals. Without this property the process of finding an average (arithmetic mean) would be entirely bogus.

Ordinal scales

Finally, there is what is called an *ordinal scale*. This is a measuring scale that makes use of no more than the basic ordinal aspect of number. We show in Chapter 2 that sometimes numbers used in an ordinal sense label things and put them in order. It is in this way that numbers are used in ordinal scales – for labelling and putting in order. For example, we are using an ordinal scale when measuring how good a hotel is by a system of star ratings; or when determining children's achievement using, say, levels 1, 2, 3, 4, 5 or 6. Sometimes letters rather than numbers are used for an ordinal scale – for example, performance in a public examination is often measured by a system of grades, such as A, B, C, D, E, F, making use of the alphabetical ordering associated with letters. These are all ordinal measuring scales – because they do no more than put things in order according to specified criteria. No one could ever assert that the steps between the measurements are equal intervals, so they do not qualify as interval scales. We can assume only that a four-star hotel – according to some stated criteria – is better than a three-star hotel, that a grade B result is better than a grade C, that a child at level 3 has attained more in mathematics than a child at level 2, and so on.

Of course, ratio scales and interval scales also share the same properties as ordinal scales, in that they can also be used to put things in order. But, in an ordinal scale, the measurements can be used *only* for ranking and ordering and in no other way. It certainly makes no sense to use ratios for comparison of measurements of something that is only an ordinal scale, as we would with the numbers used for measurements in ratio scales. A four-star hotel is not twice as good as a two-star hotel. A level 4 child has not achieved twice as much as a level 2 child. It also makes no sense to talk about the intervals between various measurements, as we would with the numbers used for measurements in interval scales. Progression from level 2 to level 3 in a National Curriculum subject is in no sense equal to progression from level 3 to level 4 – although numbers are used for these levels we should not assume that there is an equal amount of learning involved! All we can deduce is that level 4 is a higher level of achievement than level 3, and so on.

The consequence of this analysis is that any attempt to do arithmetic with measurements on an ordinal scale is invalid and will lead to meaningless statements. For example, any aggregation of levels of achievement – such as calculations of averages – is invalid, simply because the intervals between different levels cannot be assumed to be equal steps. You cannot equate the achievement of three children gaining levels 1, 2 and 3 on an ordinal scale with three other children each gaining level 2, on the basis that the 'average' is the same for the two groups – because getting from level 1 to level 2 is not the same thing as getting from level 2 to level 3. This is the problem with using numbers for ordinal scales – as soon as people see numbers they start doing arithmetic with them, regardless of whether their conclusions have any mathematical validity or meaning.

In conclusion, we note, therefore, that we have different types of measuring scales, characterized by different mathematical properties, with different degrees of validity for the application of various arithmetic processes. But what remains throughout, even with ordinal scales, is the ever-present transitive property. We can still conclude that if A is better than B and B is better than C, then A is better than C. For without this property we could not put things in order, and without ordering there would be no measurement.

RESEARCH FOCUS Time to understand

Teaching young children to tell the time can be a very frustrating business as, almost invariably, they find it harder than you might anticipate. Having said that, do you know any adults who *cannot* tell the time? We don't. Generally, people learn to tell the time in practical real-life contexts where they need to do this.

A quick on-line search of *Google Scholar* suggests that there has been little recent research on the subject of learning about time. But, if you delve more deeply, you will find there was quite a bit in the 30-year period, 1946–76. For example, in 1946 Ames reported on a large-scale study she undertook with American children between the ages of 18 months and 8 years who were deemed to be of higher than average intelligence. She noted that it was not until they were 6 years of age that these children showed an understanding of the four seasons. This is hardly surprising, given that, in their relatively short lives, they are unlikely to recall more than three or four winters or springs, and so on. What is even more relevant to this discussion, however, is that the majority of these able children were unable to tell the time (from an analogue clock) until they were 7 years old. Discussing Ames's work, Lovell (1966) pointed out that being able to tell the time does not necessarily mean that a child has a well-developed concept of time, for reading a clock only demonstrates that someone can read a dial.

Why might learning to tell the time be tricky? Hopkins, Pope and Pepperell (2004) point out that the face of an analogue clock is a linear scale presented as a circle, rather than as it usually appears as a horizontal number line. This in itself may perplex some children. But consider too, for example, the fact that '1' on an analogue clock can signify 1 o'clock but it can also represent 5 past the hour: in essence the same numeral has at least two meanings depending on which hand is pointing to it (and we haven't even touched on seconds). Added to that, unlike other measures children learn about, telling the time involves focusing on not one but two pieces of information – for example, the information provided by the two hands on a clock. Dudgeon and Hansen (in Hansen, 2011, p. 115) remind us that the same time of day can be described in a number of ways, making life even more confusing. They illustrate this idea with:

Twenty-five to nine

Eight thirty-five

Twenty-five to nine in the evening

Eight thirty-five in the evening

Eight thirty-five p.m.

Twenty thirty-five

Returning to the idea of understanding time as a concept, we note that Piaget (1969) concluded that it was necessary for children: (1) to appreciate that events occur in a temporal order; and (2) to have a sense of the duration of time between these events. The first of these is the ordering stage of the development of the concept of measuring time in the sense of 'the time at which something happens' that we have identified earlier in the chapter. The second is the idea of a time interval. Research by Kerslake (1975) suggests that – certainly in the past and we suspect this may still be the case in many classrooms – the idea of time intervals may be particularly difficult to acquire if, for example, lessons on telling the time involve a teacher setting the hands of a clock to one time, asking for the identification of the time depicted, and then swiftly moving the hands to another time, and then another.

Our own experience as practitioners suggests that more important than focusing on digital or analogue clocks at any early age is simply talking with young children about the order in which events occur. Shatz, Tare, Nguyen and Young (2010) investigated pre-schoolers' understanding of 'time-duration words', namely, minutes, hours, days and years, in relation to the duration of various events in the child's life. They concluded that, 'Like Piaget, we believe that children's early knowledge of time is very likely grounded in their personal experiences of activity and that it gradually becomes conventionalized through language' (p. 34). They then go on to recommend that, 'We situate time-word learning within the larger context of a general strategy for language learners to use conversations as an entrée into building a lexicon of abstract-word domains even before they know the meanings of individual abstract words' (p. 34). They also warn, as does Cockburn (1999), that adults have a tendency to use idioms such as 'just a minute' in a very loose sense. This may explain to some extent why, on coming to school, children's notion of time duration is weak. If you stop to think about it though, even in our more mature years, an hour doing something you enjoy can 'pass in a flash' whereas other, less enjoyable, activities of the same duration may feel as if they are lasting forever.

Long and Kamii (2001) have done some ingenious research into children's understanding of the principle of transitivity in the context of time, using extracts of music and various bits of equipment that the children could use to record informally how long these extracts lasted (water bottles and marker pens, and so on). They found that by the age of 7 years about half the children they studied could reason that, if extract A takes the same time as event X and extract B takes the same time as event Y, then they could compare the time taken for extracts A and B by comparing events X and Y. When you think about it, that's very sophisticated reasoning! But by the age of 9 years, 90% of children could reason in this way.

Some activities to use with children

Children will best develop their understanding of measurement concepts through a great deal of practical experience of measuring and talking with each other and with the teacher. The more purposeful this is – such as solving real problems, planning events, constructing timetables, cookery, shopping, science experiments, and so on – the better. Teachers in our group have noted that the automation of weighing, pricing and payment in supermarkets makes the need for practical experience in the classroom even greater. We provide here just a few examples of activities that might be used to focus on some of the central ideas in this chapter.



ACTIVITY 8.1 Measurement Role Play

3-4	4-5	5-6	6-7	7-8
☐				

Objective

To develop young children's awareness that the process of measuring is part of real life.

Materials

Dressing-up clothes and imitation equipment such as watches, tape measures, thermometers, weighing scales.

Method

Encourage children to include in their role play various situations from real life that involve measuring. For example, the children driving round the playground on their toy cars have to stop at the garage every now and then to fill up with fuel. The child playing the part of a doctor or nurse pretends to take patients' temperatures. One child giving out dressing-up clothes has to measure the other children first to decide which clothes will fit. Children pretending to be bakers measure out their ingredients on a weighing scale. None of these are real measurements, of course. The children are simply beginning to learn that measuring is something people do.



ACTIVITY 8.2 Children as Non-Standard Units

3-4	4-5	5-6	6-7	7-8
	☐	☐		

Objective

To give children experience of estimating and measuring lengths and distance using non-standard units, and to explore the effects of using different-sized units.

Method

A group of children stands shoulder to shoulder along one of the classroom walls, from one corner to the other, and another child counts how many children there are. The children are then asked how many would be needed to fill up the opposite wall. A group of children line up shoulder to shoulder to check this. Next they estimate how many children are needed for the length of the other two walls. This is then done and the results recorded. This is then repeated with the children standing with arms outstretched touching fingertips, and then again standing in a queue one behind the other as close as they can get. Each time the children are invited beforehand to guess how many will be required.

This experience is then extended to estimating distances (from one point to another). For example, they might estimate the number of children required to stretch diagonally across the room from one corner to the other, again using the three different units (shoulder to shoulder, fingertip to fingertip, and queuing). They might then go into the hall or into the playground and try this with the distances between two chairs placed in various positions. All this should be followed by discussion about what is discovered.



ACTIVITY 8.3 Estimation Challenge

3-4	4-5	5-6	6-7	7-8
			□	□

Objective

To familiarize children with the sizes of some standard units of measurement.

Materials

A 30-cm rule; scrap paper; something to act as a small screen (such as a large book).

Method

This is a simple but effective game for a small group of children. Players take turns to draw on a piece of scrap paper a line with a length of any number of centimetres they wish, using the ruler. They do this behind the screen, so that the others cannot see them doing it. The line is then shown and each of the other players estimates how long it is. One player should write down all the estimates. They then check the length of the line with the ruler. The player (or players) getting nearest wins a point. They win a bonus point if they get the length spot on.

Variations of this game can be devised using other aspects of measurement. For example, children could use a balance to weigh out quantities of sand into a yoghurt pot (using 10-g masses only), again behind a screen – the others then have to estimate the mass.



ACTIVITY 8.4 Conservation Game (Volume and Mass)

3-4	4-5	5-6	6-7	7-8
			□	□

Objective

To give children experience related to the concept of conservation of mass and volume.

Materials

A number of different-shaped clear plastic or glass containers and measuring jars; a small cup; a supply of water; three identical non-transparent containers with lids (such as yoghurt pots); a supply of sand; a balance; a ball bearing.

Method

This is a game for four players, in two teams of two. Team A measures out the same quantity of water into three clear containers, and then adds one extra cupful of water to one of them. While they are doing this, team B measures out the same mass of sand into the three pots, buries the ball bearing in one of them and replaces the lids. They then challenge each other to identify the odd one out. Team B is allowed only to look at the containers of water. Team A is allowed to pick up – but not to rattle vigorously – the pots of sand. After they have made their judgements each team can check to see if they are right, by pouring water or by balancing pots. They then swap over and repeat the experiment with team A doing mass and team B doing volume.



ACTIVITY 8.5 Investigating Ordering

3-4	4-5	5-6	6-7	7-8
			□	□

Objective

To give children experience of transitivity and ordering.

Materials

(1) A set of cards with events of the day written or depicted on them – alarm goes off, get up, have breakfast, and so on. (2) A collection of containers, such as bottles and jars. (3) A balance and a set of identical-looking sealed parcels, filled with a variety of materials, such as sand, sawdust, ball bearings. (4) A list of tasks taking various amounts of time (count to a hundred, tie up your shoe laces, do ten press-ups, and so on).

Method

These materials provide four different opportunities for children to develop a strategy for ordering three or more items (events, containers, parcels, tasks). In each case they should start with three items and order these. Then extend to four, five, six or more. The four activities are as follows:

- 1 The children have to put the events of the day in order from earliest to latest. To start with, any three cards can be used. A nice touch is to have letters written on the backs of the cards, so that when the whole set is finally ordered they can be turned over to reveal a message such as 'well done!'
- 2 By pouring water from one to another the children should order three containers by capacity. Gradually extend this by adding a fourth container, a fifth, and so on.
- 3 Using the balance and the parcels, the children explore the same problem with mass.
- 4 Two children perform two of the tasks starting at the same time, and discover which takes the longer and which takes the shorter time. Eventually they must order the whole list from the one taking the least amount of time to the one taking the greatest.

**ACTIVITY 8.6 Talk about Time**

3-4	4-5	5-6	6-7	7-8
			☐	☐

Objective

To develop children's understanding of the language of time.

Materials

The list of words and phrases associated with time given earlier in this chapter.

Method

Write on the board a small collection of words or phrases from the list of time language. Seat the class on the carpet and simply hold a discussion with them incorporating the words on the board. For example, you might ask the children to think of a sentence that includes one of the words or phrases. Here are two examples of suitable collections of words and phrases: (1) day, weekend, week, lifetime, long time, short time, brief, temporary, for the time being, non-stop, never-ending, permanent; (2) how old, age, age group, year group, teenager, young, old, elderly, middle-aged, underage, older, elder, oldest, younger, youngest.



ACTIVITY 8.7 Ordering Packets

3-4	4-5	5-6	6-7	7-8

Objective

To provide more experience of ordering, and to heighten children's awareness of measurement in the world outside school.

Materials

Children are asked to bring in from home as many different empty packets as possible marked in grams or kilograms.

Method

Along the whole length of a wall, make a number-line frieze, marked from 0 to 2000 g. As packets are collected, children display them on the appropriate part of the number line. Write the number of grams in large numerals underneath. The children should be challenged to seek out at home packets with different numbers of grams from those already displayed.



ACTIVITY 8.8 Recording with Arrows

3-4	4-5	5-6	6-7	7-8

Objective

To develop understanding of the principle of transitivity in measurement.

Materials

Three objects labelled A, B and C; three cards labelled A, B and C; three arrows cut from card, labelled 'is heavier than'; three further cards labelled 'is lighter than'; a balance for weighing.

Method

The layout of arrows shown earlier in the chapter in Figure 8.4 provides a useful way for a small group of children to record their comparisons of three objects. They are given the three objects, the three cards labelled A, B and C, and the three arrows with 'is heavier than' written on them. Their task is then to compare A with B, B with C, and A with C, using the balance, to construct an arrangement of the arrows and cards as shown in Figure 8.4, and then to discuss the ordering of the three objects that emerges. The same task with the same objects should then be undertaken with the arrows labelled 'is lighter than'.

Variations on this basic activity can be devised in any measuring context, using appropriate objects and labels, with the arrows indicating various aspects of the language of comparison.

Summary of key ideas

- 1 Measurements in units of length and distance might refer to a wide variety of situations, such as the straight or curved parts of objects, lines drawn on paper, imaginary lines drawn through the air or through objects, straight or curved paths.
- 2 Volume is the amount of three-dimensional space occupied by an object.
- 3 Only containers have capacity. The capacity of a container is the volume of liquid it can hold. In the metric system, capacity and liquid volume are often measured in litres.
- 4 Two meanings of time are: the time at which an event occurs (recorded time); and the time that something takes to happen (time interval).
- 5 Kilograms and grams are units of mass, not weight. A key phrase for describing the results of weighing is 'weighs the same as' (for example, the book weighs the same as 200 grams).
- 6 A basic purpose of measurement is for comparison and ordering.
- 7 The principle of transitivity is that if A is greater than (less than/equal to) B, and B is greater than (less than/equal to) C, then A must be greater than (less than/equal to) C.
- 8 Conservation of quantity involves recognizing which transformations (such as pouring a quantity of water into a different-shaped container) preserve which equivalences (e.g. the volume of water stays the same).
- 9 There are good arguments for introducing children to measuring in units by means of experience with non-standard units.
- 10 The metre, kilogram and second are examples of base units in the SI version of metric units.
- 11 Prefixes for deriving other units include: mega (million), kilo (thousand), hecto (hundred), deci (tenth), centi (hundredth), milli (thousandth) and micro (millionth).
- 12 Nearly all measurement is approximate – normally you can only ever measure to the nearest something.
- 13 Estimation is an important experience for developing understanding of measurement concepts.
- 14 Three different kinds of measuring scales are: ratio scales, such as length and mass, in which measurement can be compared by ratio as well as by difference; interval scales, such as temperature or time of day, in which measurements can be compared by their difference, but not by their ratio; and ordinal scales, such as examination grades and levels of achievement, in which measurements can only be compared by ordering.
- 15 Transitivity applies in all three types of measuring scale.



Suggestions for further reading

Fenna (2002) is an extensive dictionary of measurement that will provide authoritative answers to any questions about units, prefixes and styles for recording measurements, as well as some fascinating material about the historical and scientific background to the measurements we use.

Chapter 22 of Haylock (2010) provides further detailed coverage of the mathematics of measurement.

Chapter 4 on young children's responses to measurement systems in Nunes and Bryant (1996) is well worth reading.

Those who work with younger children should read Margaret Donaldson's classic book on children's learning (Donaldson, 1978), in which she examines critically some of Piaget's theories. In particular, she identifies the differences in children's performance when they are engaged in disembedded tasks and those that are embedded in contexts that have meaning for them.

Also highly recommended is section 2 of Dickson, Brown and Gibson (1984), which provides a thorough and fascinating survey of what research shows us about children's understanding of measurement concepts; this includes consideration of Piaget's work, fundamental ideas such as conservation and transitivity, and the difference between counting and measuring.

Some of the potential difficulties young children face with time concepts and when reading analogue and digital clocks are discussed in Chapters 7 and 8 of Cockburn (1999).

The entries on 'Conservation' and 'Transitivity' in Haylock with Thangata (2007) provide further discussion of these key concepts.

Leather (2000) is a very practical book full of activities and strategies for teaching measurement.

Finally, we would recommend Chapter 10 of Gifford (2005), which is entitled 'Measuring'.

