

Resonance

James Gavin

2311749

ACCA4006

May 3, 2024

Abstract

A LC resonance circuit was analysed with simulated and physical performance being compared. The circuit was simulated using `ngspice` [1], whilst the physical circuit was built on a breadboard and tested with an oscilloscope. The quality factor (sharpness of the frequency response) was calculated. The peak power output of the physical circuit was much lower when compared to the simulated results and the quality factor was also much lower.

Contents

1	Introduction	4
2	Theory	4
2.1	AC Theory	4
2.2	Circuit Equation	5
2.3	Resonant Frequency	6
2.4	Quality Factor	7
3	Simulation Setup	10
4	Physical Setup	11
5	Results	14
6	Discussion	15
7	Conclusion	17

Figures

1	Diagram of the LC Resonant Circuit	4
2	Diagram of the LC resonance circuit, showing the potential divider	5
3	Theory Based Frequency Responce Graph for the LC Resonance Circuit	7
4	Theory Based Frequency Responce Graph for the LC Resonant Circuit with Q factor lines	9
5	resonance.cir , the file describing the resonance circuit analysis to be simulated	10
6	Netlist of the circuit described to ngspice	11
7	Output from running the command ngspice -b resonance.cir	11
8	Table of Results from the Simulated Circuit	11
9	Digital Breadboard Layout of the LC Resonance Circuit	12
10	Breadboard with the LC Resonance Circuit	12
11	The LC Resonant Circuit V_{in} and V_{out} when $f = 5032.92$ Hz	13
12	Table of Results from Physical Circuit	13
13	Simulated Frequency Responce Graph for the LC Resonance Circuit	15
14	Physical Frequency Responce Graph for the LC Resonance Circuit	16

1 Introduction

Figure 1 shows the LC resonant circuit that will be analysed in this report. This circuit produces output for a controllable frequency range, making it a key component in signal filtering and signal amplification.

An example use case for this circuit is in RF tuners like FM radios. This LC resonant circuit can be used to filter the signal received from the antenna to the required bandwidth for the target application.

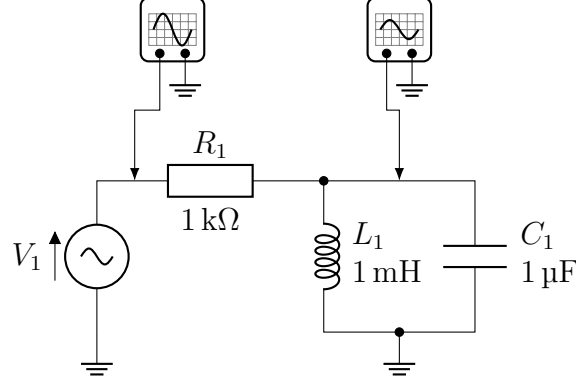


Figure 1: Diagram of the LC Resonant Circuit

2 Theory

2.1 AC Theory

AC circuits have an input sinusoidal alternating voltage that changes with time

$$V_{in}(t) = A \sin(\omega t)$$

where A is the amplitude (peak voltage) and $\omega = 2\pi f$ [2].

Reactance (X) opposes the flow of current through resistors, capacitors and inductors in an AC circuit and is measured in ohms (Ω). Ideal resistors have 0 reactance, whereas ideal capacitors and inductors have 0 resistance. Reactance is a consequence of the voltage and current being 90° out of phase through capacitors and inductors.

To represent this 90° shift in phase, it becomes necessary to use complex numbers. These are numbers in the form $a + bj$ where a and b are to be found and $j = \sqrt{-1}$ and have a conjugate pair $a - bj$.

For the capacitor the current leads the voltage by 90°

$$\text{phase}_C = \cos(-90) + j \sin(-90) = -j$$

For the inductor the voltage leads the current by 90°

$$\text{phase}_L = \cos(90) + j \sin(90) = j$$

The reactance for a resistor, inductor and capacitor

$$X_R = 0$$

$$X_C = -\frac{j}{\omega C}$$

$$X_L = j\omega L$$

Impedance (Z) is the sum of reactance and resistance for a component and is measured in ohms (Ω)

$$Z = X + R$$

$$Z_R = R$$

$$Z_C = -\frac{j}{\omega C}$$

$$Z_L = j\omega L$$

The impedance for both the capacitor and inductor are dependent on the frequency of the input, which makes them frequency dependent resistors.

The impedance of the capacitor is inversely proportional to the frequency of the input signal. This means that the capacitor will block lower frequencies from passing through it, but allows higher frequencies.

The impedance of the inductor is proportional to the frequency of the input signal. This means that the inductor will block higher frequencies from passing through it, but allows higher lower [3].

2.2 Circuit Equation

An equation that represents the relationship between the input and output voltage of the LC resonant circuit can be derived. A general equation that applies for all similar LC resonant circuits will be derived before introducing values specific to the circuit being analysed.

It is important to realise the limitations of such a model. It is based on the maths behind each component and will have performance characteristics unlike the hardware. It can be useful to derive such models, as they provide target values that are used to evaluate how well hardware is performing.

The C_1 and L_1 are in parallel to one another allowing their impedance to be combined [3]

$$\frac{1}{Z_{CL}} = \frac{1}{Z_C} + \frac{1}{Z_L}$$

$$Z_{CL} = \frac{j}{\frac{1}{\omega L} - \omega C}$$

which turns the resonant circuit into the potential divider in Figure 2

As frequency increases, $1/\omega L$ decreases and ωC increases. This causes the capacitor to filter out lower frequencies, whilst the inductor filters out higher frequencies.

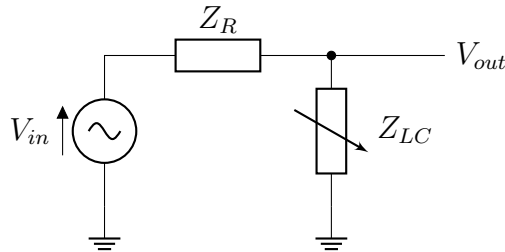


Figure 2: Diagram of the LC resonance circuit, showing the potential divider

The potential divider equation can now be used for V_{out}

$$V_{out} = V_{in} \frac{Z_{CL}}{Z_{CL} + Z_R}$$

Expanding Z_{LC} and Z_R

$$V_{out} = V_{in} \frac{1 + R \left(\frac{1}{wL} - wC \right) j}{1 + R^2 \left(\frac{1}{wL} - wC \right)^2}$$

Plotting imaginary numbers is not easy to visualise. The magnitude of V_{out} is preferred

$$\begin{aligned} |V_{out}| &= \sqrt{V_{out} \overline{V_{out}}} \\ |V_{out}| &= V_{in} \frac{\sqrt{1 + R^2 \left(\frac{1}{wL} - wC \right)^2}}{1 + R^2 \left(\frac{1}{wL} - wC \right)^2} \\ |V_{out}| &= \frac{V_{in}}{\sqrt{1 + R^2 \left(\frac{1}{wL} - wC \right)^2}} \\ |V_{out}| &= \frac{V_{in}}{\sqrt{1 + R^2 \left(\frac{1}{2\pi fL} - 2\pi fC \right)^2}} \end{aligned}$$

Substituting $L = L_1 = 1 \text{ mH}$, $R = R_1 = 1 \text{ k}\Omega$, $C = C_1 = 1 \text{ }\mu\text{F}$ and $V_{in} = 1 \text{ V}$

$$|V_{out}| = \frac{1}{\sqrt{1 + 1000000 \left(\frac{1}{0.002\pi f} - 0.000002\pi f \right)^2}}$$

produces the equation of the LC resonant circuit. The plot of this equation can be found in Figure 3.

2.3 Resonant Frequency

There is a frequency at which $(1/wL) - wC = 0$. At this frequency, Z_{LC} has infinite impedance since the denominator is 0. This is known as the resonant frequency.

The infinite impedance makes $V_{out} = V_{in}$. The resonant frequency equation can be found by rearranging the Z_{LC} denominator.

$$\begin{aligned} 0 &= \frac{1}{wL} - wC \\ w^2 LC &= 1 \\ w &= \frac{1}{\sqrt{LC}} \\ f &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}$$

This shows that the resonant frequency is independent of the impedance value for Z_R . Substituting $L = L_1 = 1 \text{ mH}$ and $C = C_1 = 1 \text{ }\mu\text{F}$ to calculate the resonant frequency for the circuit to be analysed, gets a frequency of 5032.921 Hz. $x = 5032.92$ is plotted on Figure 3.

The attenuation is calculated using [4]

$$\text{Attenuation} = 20 \log \left(\frac{V_{out}}{V_{in}} \right)$$

This sets up a logarithmic scale for the amplitude that is measured in decibels (dB). By finding the ratio V_{out}/V_{in} , the graph shows V_{out} relative to V_{in} , which is the line $y = 0$.

The scale of the attenuation is such that for an increase of 20 dB the actual amplitude has increased 10-fold. This method of viewing the amplitude is useful for resonant circuits as it shows a higher level of detail for the amplitude when the frequency is far from the resonant frequency.

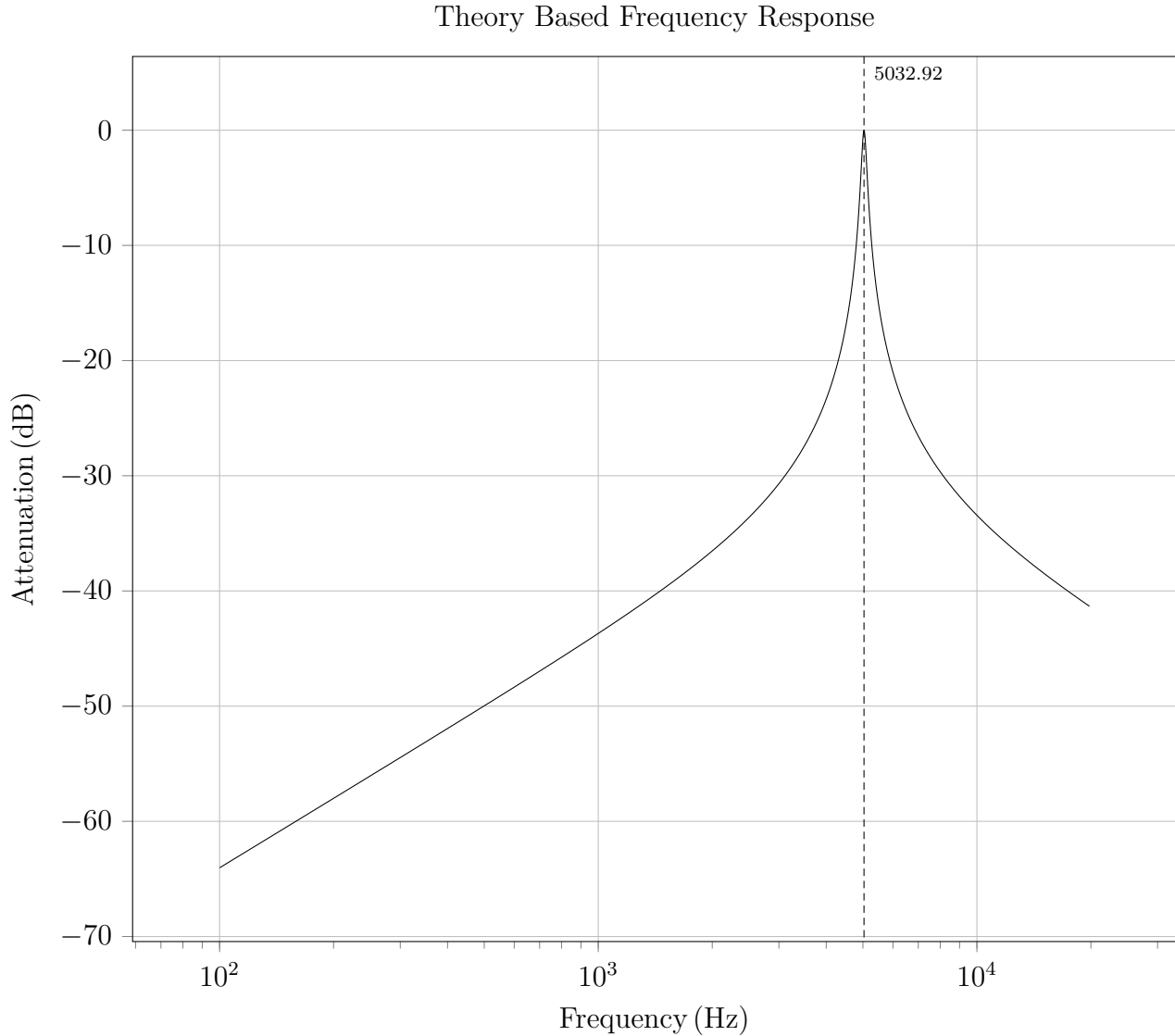


Figure 3: Theory Based Frequency Response Graph for the LC Resonance Circuit

2.4 Quality Factor

An important characteristic for a resonant circuit is its quality factor Q . This value represents the sharpness of the peak, with higher Q values being sharper peaks [3].

To calculate the quality factor, the frequencies that are 3 dB below the resonant frequency (f_r) need to be found (f_u , f_l). The Q factor is then as follows

$$Q = \frac{f_r}{f_h - f_l}$$

The solutions for f_h and f_l can be found by solving the resonant circuit equation when the

attenuation is -3 dB

$$\begin{aligned}
 -3 &= 20 \log \frac{1}{\sqrt{1 + R^2 \left(\frac{1}{wL} - wC \right)^2}} \\
 \left(\sqrt[20]{1000} \right)^2 &= 1 + R^2 \left(\frac{1}{wL} - wC \right)^2 \\
 \pm \sqrt{\frac{\left(\sqrt[20]{1000} \right)^2 - 1}{R^2}} &= \frac{1}{wL} - wC \\
 Cw^2 \pm \sqrt{\frac{\left(\sqrt[20]{1000} \right)^2 - 1}{R^2}}w - \frac{1}{L} &= 0
 \end{aligned}$$

Solving the two quadratic equations, with $L = L_1 = 1\text{ mH}$, $R = R_1 = 1\text{ k}\Omega$ and $C = C_1 = 1\text{ }\mu\text{F}$ gives the following solutions

$$\begin{aligned}
 0.000001w^2 + \sqrt{\frac{\left(\sqrt[20]{1000} \right)^2 - 1}{1000000}}w - 1000 &= 0 \\
 w &= 31127.90, -32125.52 \\
 0.000001w^2 - \sqrt{\frac{\left(\sqrt[20]{1000} \right)^2 - 1}{1000000}}w - 1000 &= 0 \\
 w &= 32125.52, -31127.90
 \end{aligned}$$

Given that negative frequency is not possible

$$\begin{aligned}
 w &= 31127.90, 32125.52 \\
 f &= 4954.16, 5112.78
 \end{aligned}$$

Finally calculate the Q factor

$$Q = \frac{5032.92}{5112.78 - 4954.16} = 31.73$$

The Q factor can also be calculated by drawing lines on a graph. By drawing a parallel line at -3 dB from the peak, with a pair of perpendicular lines that intersect with the plot and the first line. This setup can be seen in Figure 4.

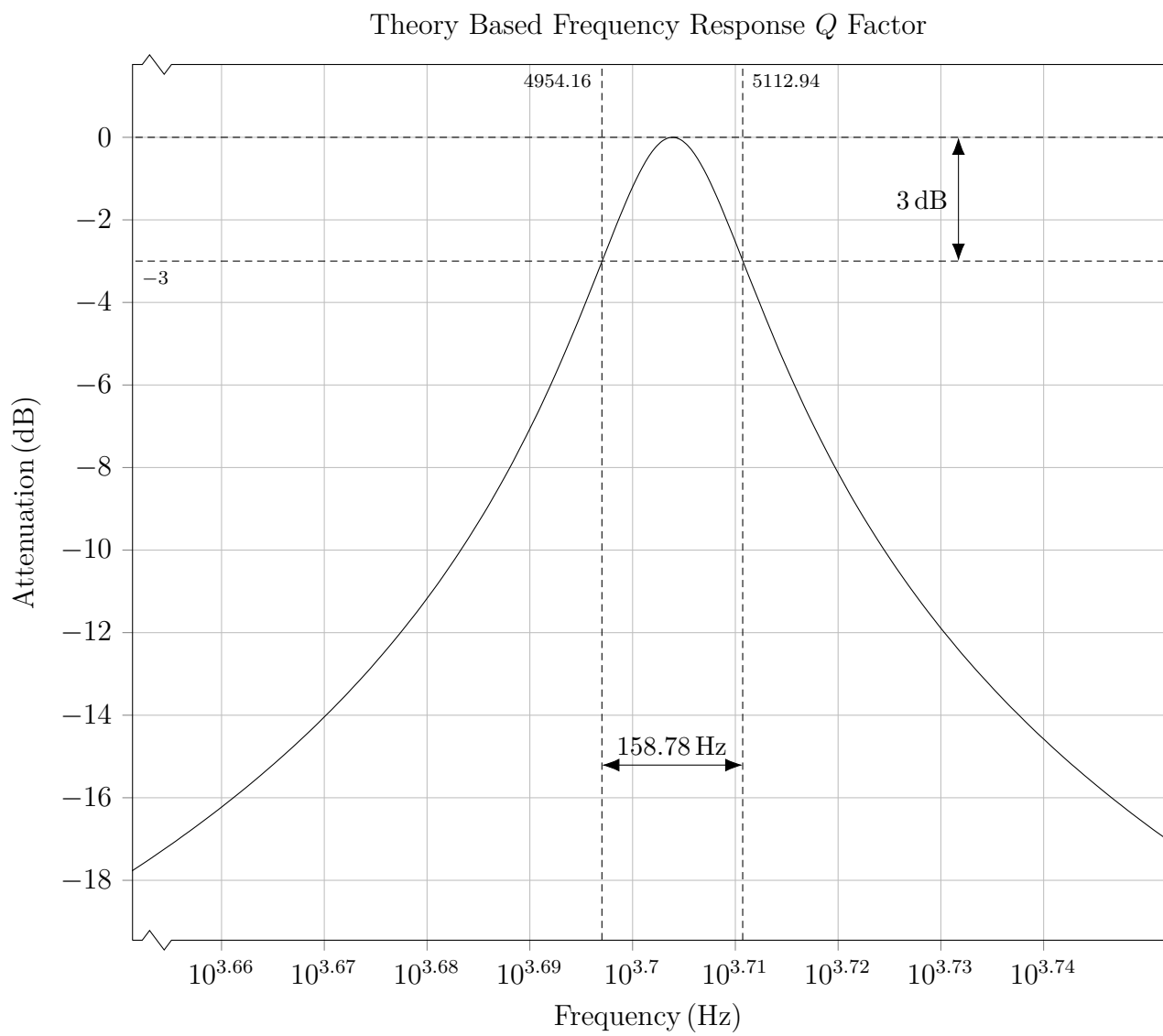


Figure 4: Theory Based Frequency Response Graph for the LC Resonant Circuit with Q factor lines

3 Simulation Setup

ngspice [1] was used to simulate the resonance circuit.

The circuit to be analysed is described in an input file along with commands that control the analysis. The input file for this resonance circuit can be seen in Figure 5.

```
1 .title resonance netlist
2
3 V1 1 0 ac 1
4
5 R1 1 2 1k
6 C1 2 0 1uF
7 L1 2 0 1mH
8
9 .control
10 ac dec 400 100 20k
11
12 set wr_vecnames
13 option numdgt=4
14 wrdata simulated.plt db(v(2))
15 .endc
16 .end
```

Figure 5: *resonance.cir*, the file describing the resonance circuit analysis to be simulated

A breakdown of Figure 5:

3-7 The netlist of the circuit (See Figure 6 for visualisation).

- 3 Voltage source [5] V1 with the +ve node connected to wire 1 and the -ve node connected to the reserved wire 0 that represents GND. ``ac 1`` configures this voltage source as an Alternative Current signal source with a amplitude of 1 V.
- 5 Resistor [6] R1 with the +ve node connected to wire 1 and the -ve node to 2. ``1k`` sets the resistance to 1 k Ω .
- 6 Capacitor [7] C1 with the +ve node connected to wire 2 and the -ve node to 0. ``1u`` sets the capacitance to 1 μ F.
- 7 Inductor [8] L1 with the +ve node connected to wire 2 and the -ve node to 0. ``1m`` sets the reactance to 1 mH.

10-16 The actions to perform on the netlist.

- 11 Setup an AC frequency analysis [8]. ``dec 100 100 20k`` configures the analysis to simulate 400 frequencies per decade through the range 100 Hz to 20 kHz.
- 13-15 Write the results of the analysis to a file in table form [9].
 - 13 Write the name of each column to the top of the file.
 - 14 Set the number of significant figures of the output to 4.
 - 15 Write the vector ``db(v(2))`` to a table in file *simulated.plt*. The vector holding the voltages on wire 2 from the frequency analysis (``v(2)``), is put through the `db` function to compute the attenuation.

The simulation is executed from a command line via `ngspice -b resonance.cir` producing the table of results in *simulated.plt*. The output from running the above command is shown in Figure 7.

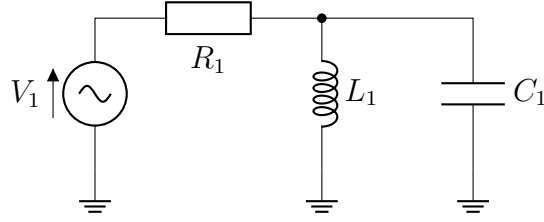


Figure 6: Netlist of the circuit described to `ngspice`

```

1
2 Note: No compatibility mode selected!
3
4
5 Circuit: resonance netlist
6
7 Doing analysis at TEMP = 27.000000 and TNOM = 27.000000
8
9 Note: v1: has no value, DC 0 assumed
10
11 No. of Data Rows : 921
12 Note: Simulation executed from .control section

```

Figure 7: Output from running the command `ngspice -b resonance.cir`

Line 11 in Figure 7 shows that 921 values of ``db(v(2))`` were written to *simulated.plt*. The frequency analysis was setup to capture 400 points per decade through the range 100 to 20 000. Therefore:

$$920.412 = 400 (\log 20000 - \log 100)$$

This number of points would be impractical to put into a table for this report, so the frequencies closest to those in the physical results have been selected for Figure 8.

Frequency (Hz)	Attenuation (dB)
100	-64.03
199.59	-58.02
498.67	-50.00
1,001	-43.68
1,997.9	-36.54
4,010.7	-23.24
4,906.3	-5.56
4,991.8	-1.03
5,020.7	-0.10
5,197.2	-7.10
6,002	-21.02
10,021	-33.46
14,996	-38.45
20,000	-41.42

Figure 8: Table of Results from the Simulated Circuit

4 Physical Setup

`ngspice` produces output based on rules. These rules assume that all components are working perfectly. In the real world, connections between components may be less than ideal and

tolerances on these components can have a large impact on actual performance.

The physical resonance circuit was created on a breadboard. This allows the circuit to be easily created and modified for adding components and probes where required. Figure 9 and Figure 10 show the digital and actual breadboard circuit respectively.

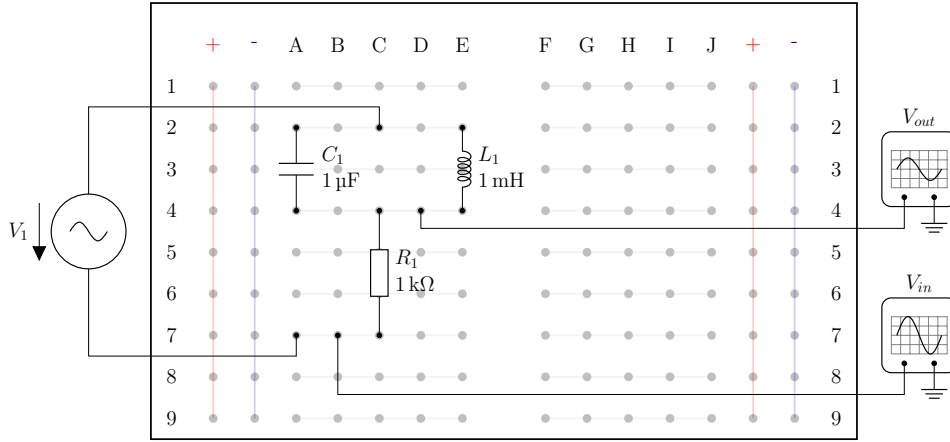


Figure 9: Digital Breadboard Layout of the LC Resonance Circuit

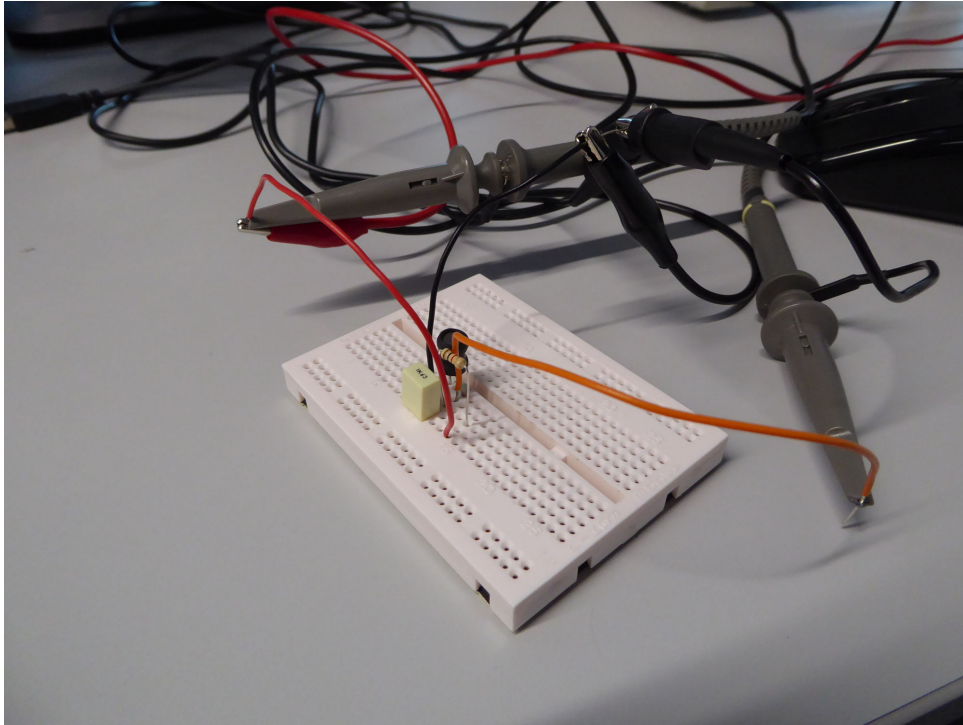


Figure 10: Breadboard with the LC Resonance Circuit

An oscilloscope was used to measure the input (V_{in}) and output (V_{out}) signal. While the function generator V_1 , was setup to always provide a 2 V AC signal, small discrepancies in the hardware meant that when reading the input signal, the voltage was rarely 2 V. Figure 11 shows the circuit output at the theoretical resonant frequency 5032.92 Hz.

Figure 12 shows the table of results collected from this testing. More samples were collected around the resonating frequency so the peak would properly form when plotting the data on a graph.

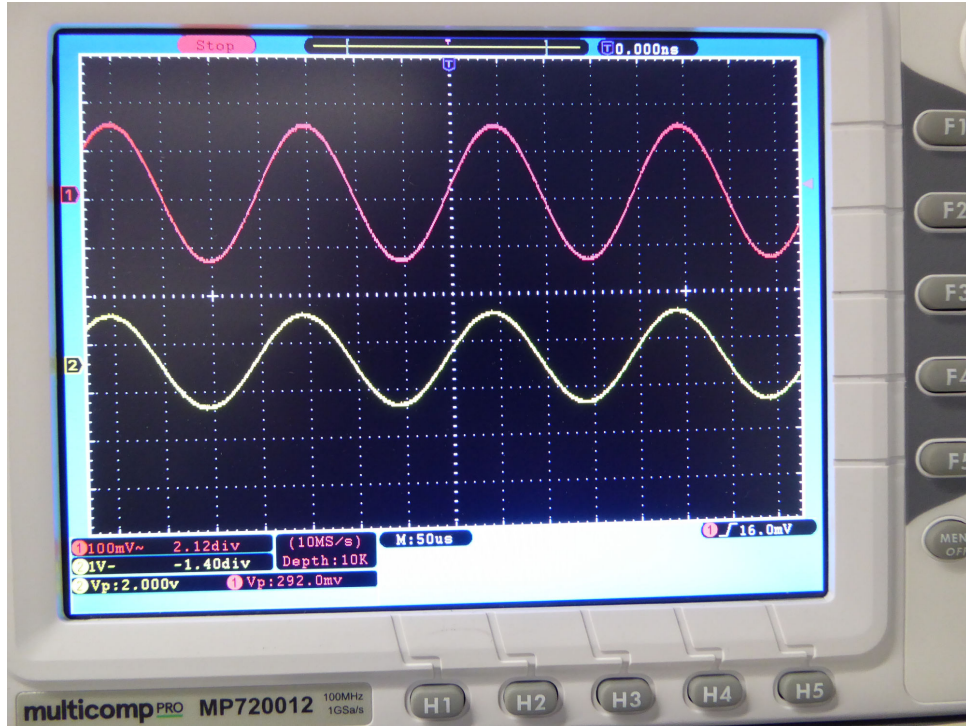
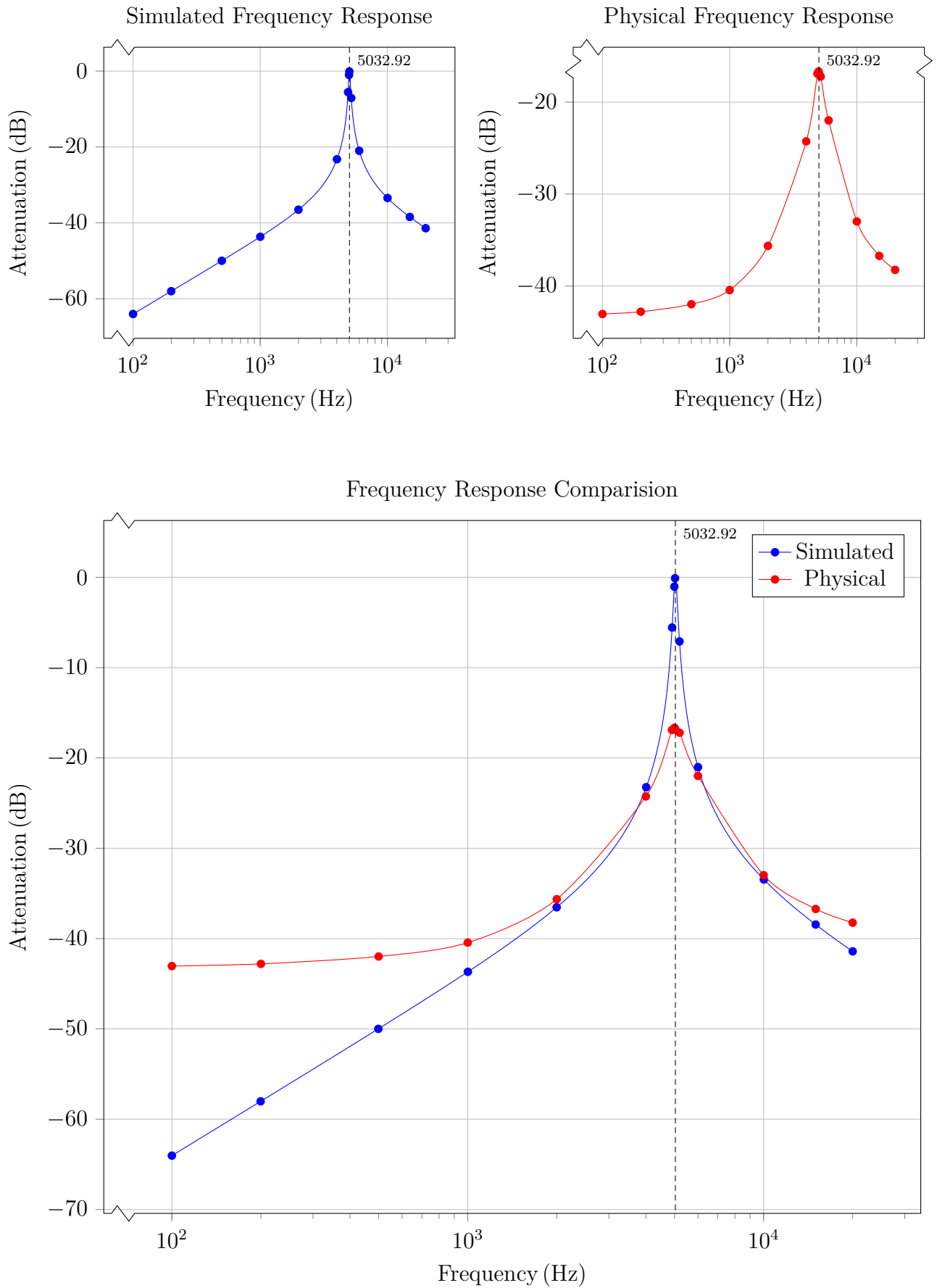


Figure 11: The LC Resonant Circuit V_{in} and V_{out} when $f = 5032.92$ Hz

Frequency (Hz)	V_{in} (V)	V_{out} (V)	Attenuation (dB)
100	1.96	$1.38 \cdot 10^{-2}$	-43.05
200	1.96	$1.42 \cdot 10^{-2}$	-42.80
500	1.96	$1.56 \cdot 10^{-2}$	-41.98
1,000	2.00	$1.9 \cdot 10^{-2}$	-40.45
2,000	1.96	$3.24 \cdot 10^{-2}$	-35.63
4,000	1.96	$1.2 \cdot 10^{-1}$	-24.26
4,900	1.96	$2.8 \cdot 10^{-1}$	-16.90
5,000	1.96	$2.88 \cdot 10^{-1}$	-16.66
5,032.92	2.00	$2.88 \cdot 10^{-1}$	-16.83
5,200	2.00	$2.76 \cdot 10^{-1}$	-17.20
6,000	1.96	$1.56 \cdot 10^{-1}$	-21.98
10,000	1.96	$4.4 \cdot 10^{-2}$	-32.98
15,000	1.92	$2.8 \cdot 10^{-2}$	-36.72
20,000	1.96	$2.4 \cdot 10^{-2}$	-38.24

Figure 12: Table of Results from Physical Circuit

5 Results



6 Discussion

To calculate the Q factor, the -3 dB frequencies were found by drawing on the necessary lines as can be seen in Figure 13 and 14.

From the data collected for the simulated circuit, the peak V_{out} collected was -0.102 dB away from V_{in} at 5020.7 Hz, which is the resonant frequency. If more data points had been collected during the simulation, the resonant frequency would have been closer to 5032.92 Hz.

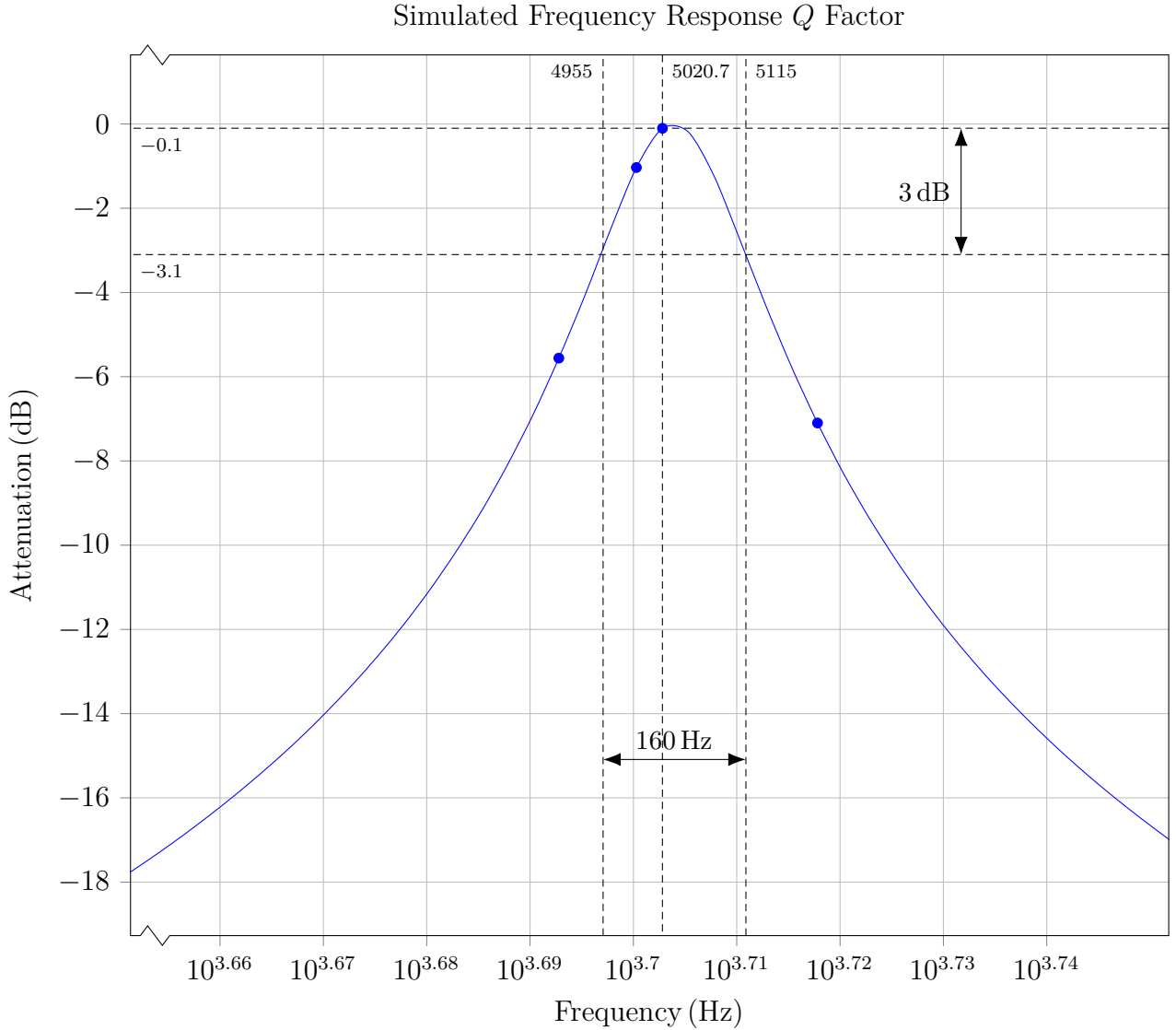


Figure 13: Simulated Frequency Response Graph for the LC Resonance Circuit

From the data collected for the physical circuit, the peak V_{out} was -16.66 dB away from V_{in} at 5000 Hz, which is the resonant frequency. A sample was taken at the predicted resonant frequency of 5032.92 Hz with a V_{out} of 0.288 V. This is the same V_{out} as at 5000 Hz, but the attenuation is greater due to V_{in} increasing for this reading from 1.96 V to 2 V.

The -3 dB frequency width for the simulated resonance circuit is 160 Hz which gives a Q factor of $5020.7/160 = 31.38$, which is comparable to the theoretical Q factor which was 31.73 . Small discrepancies and simplifications in the simulation of the circuit can mostly explain this small discrepancy.

However, for the physical resonance circuit is 920 Hz which gives a Q factor of $5000/920 = 5.43$. This is very large reduction in sharpness when compared to the simulated circuit.

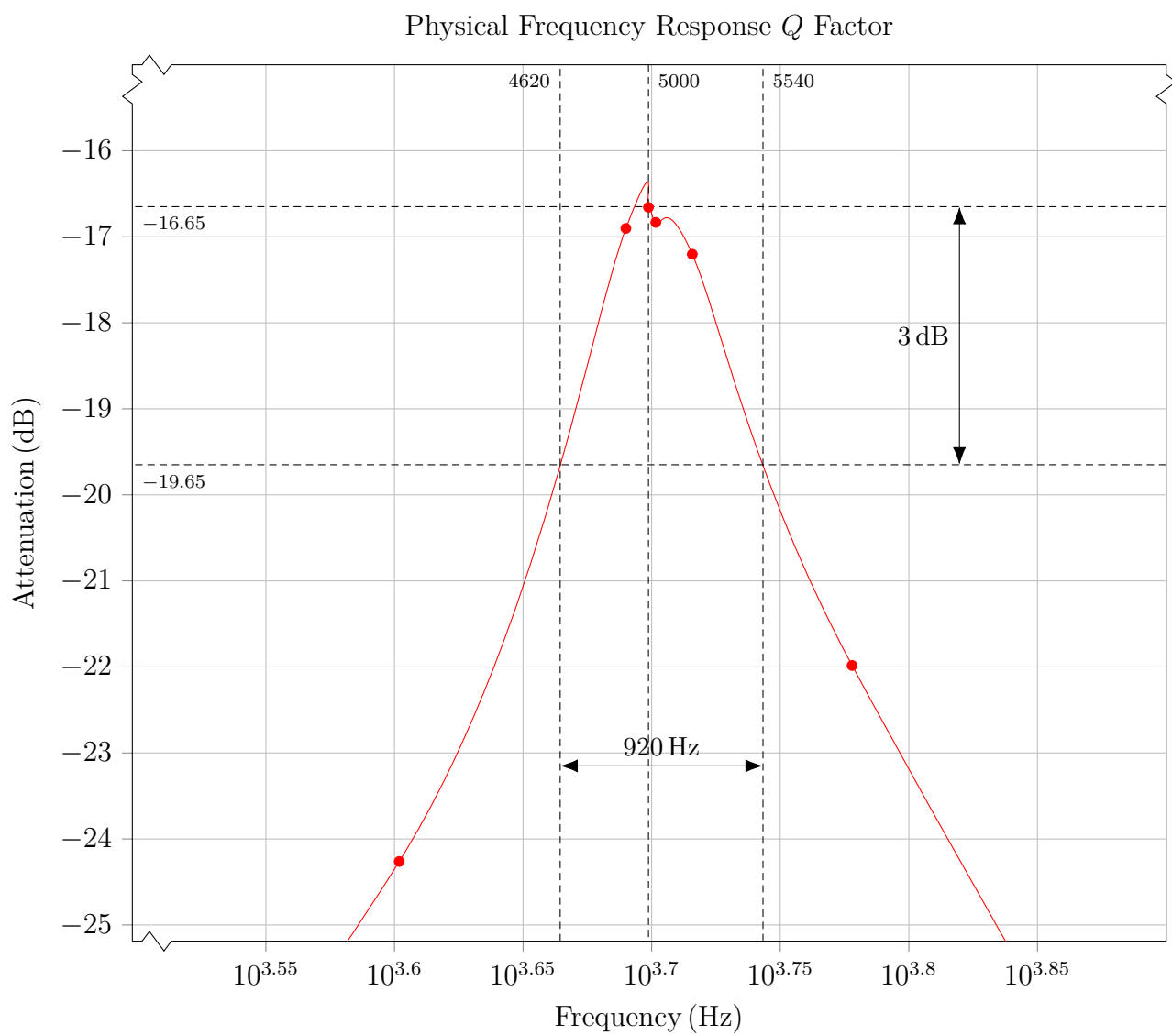


Figure 14: Physical Frequency Response Graph for the LC Resonance Circuit

There are losses in the inductor and capacitor that dampens the peak output voltage and stops it from being a perfect system. This factor can be minimised by choosing components that have better performance characteristics and lower tolerances.

The sharpness of the peak is greatly effected by V_{in} itself. If the input source has a high impedance, this gets combined in series with Z_R , which increases the impedance Z_{LC} needs to overcome the potential divider. The closer the input source comes to being a current source, the sharper the peak will be. Increasing the impedance of the resistor also has the same effect.

7 Conclusion

In this experiment it has been determined that the Q factor of the physical circuit is alot smaller than that of the simulated circuit. The physical circuit has a large amount of voltage loss across the resonant circuit that results in a large power loss.

Increasing the impedance in the input source or the resistor would increase the sharpness of the peak and therefore increase the Q factor.

Components with better performance characteristics that reduce the loss of voltage, would get the power output of the physical circuit closer to the simulated circuit.

References

- [1] “Ngspice, the open source Spice circuit simulator,” ngspice. (2024), [Online]. Available: <https://ngspice.sourceforge.io/>.
- [2] P. Horowitz and W. Hill, *The Art of Electronics*, 3rd ed., 2015, p. 14.
- [3] P. Horowitz and W. Hill, *The Art of Electronics*, 3rd ed., 2015, pp. 40–55.
- [4] P. Horowitz and W. Hill, *The Art of Electronics*, 3rd ed., 2015, p. 15.
- [5] H. Vogt and G. A. adn Paolo Nenzi, *Ngspice User’s Manual Version 42 plus*, Jan. 20, 2024, pp. 93–94.
- [6] H. Vogt and G. A. adn Paolo Nenzi, *Ngspice User’s Manual Version 42 plus*, Jan. 20, 2024, pp. 76–77.
- [7] H. Vogt and G. A. adn Paolo Nenzi, *Ngspice User’s Manual Version 42 plus*, Jan. 20, 2024, pp. 81–82.
- [8] H. Vogt and G. A. adn Paolo Nenzi, *Ngspice User’s Manual Version 42 plus*, Jan. 20, 2024, pp. 85–86.
- [9] H. Vogt and G. A. adn Paolo Nenzi, *Ngspice User’s Manual Version 42 plus*, Jan. 20, 2024, pp. 428–429.