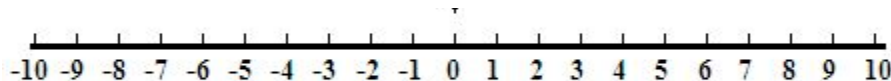


Understanding Directed Numbers Interactive Work Pack

Fill your answers in on each slide - would be useful to answer in a colour other than black



Your name here:

Year 8

Mrs Powell

Learning Objectives :

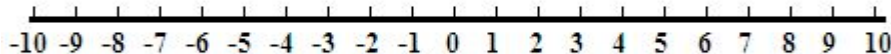
- To order negative and positive numbers
- To apply the four operations, including formal written methods, to integers, both positive and negative
- To use a number line to demonstrate addition and subtraction involving negative and positive numbers.
- To understand that adding a negative is equivalent to a subtraction. ($4 + -2 = 4 - 2$)
- To Understand why subtracting a negative results in an addition is vital to multiplying and dividing with negative numbers.

Common Misconceptions :

- Students often incorrectly consider negative numbers with a larger magnitude than positives to have a greater value. For example, $-3 < 2$.
- Trying to remember multiplication rules for when to leave the answer as a positive or negative often results in confusion when adding and subtracting. Use a number line to demonstrate $-3 \times 2 = -3 + -3 = -6$.

Ordering numbers

Rearrange these numbers so they are in order from smallest to biggest (click and drag)



10

-11

-6

2.5

$\frac{1}{2}$

-20

-3

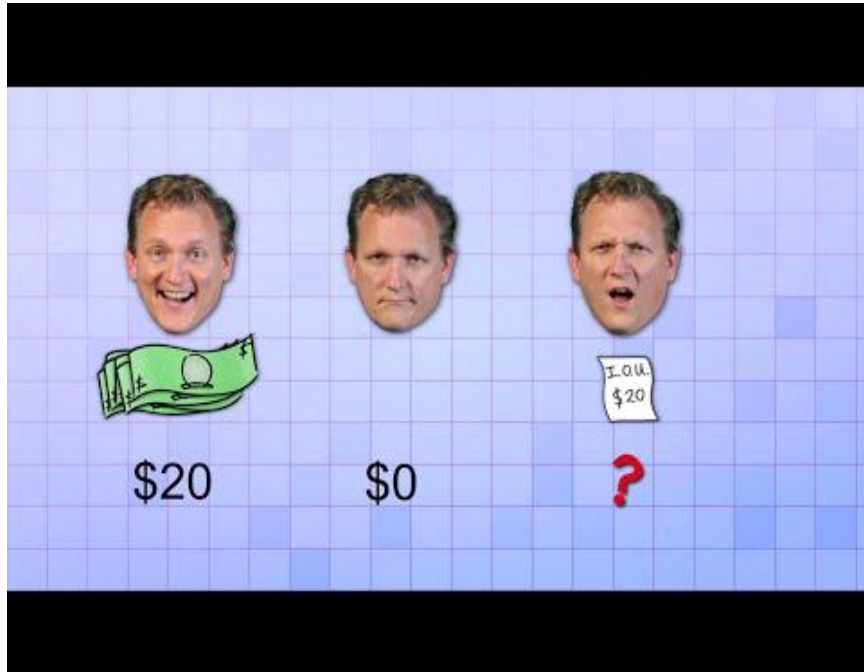
7

9.2

-1

5

Introducing negative numbers



Positive and Negative Numbers



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Negative numbers in everyday life : Temperature

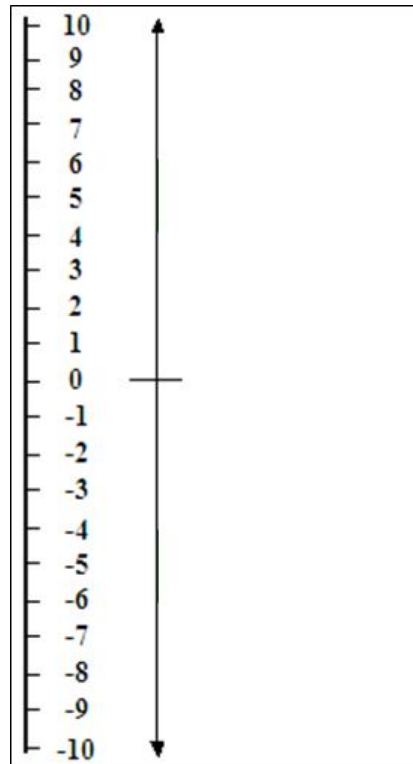
0° is the central point on a thermometer

The temperature is 0° .

If the temperature gets 7° warmer what would the temperature be?

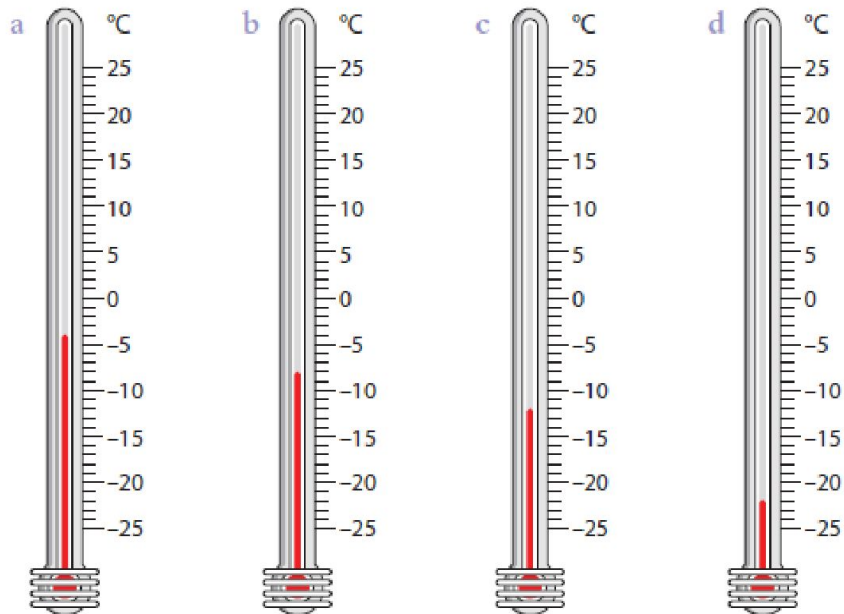
The temperature is 5° .

If the temperature got 9° colder what would the temperature be?



Negative Numbers - Temperature

1: What is the temperature on each of these thermometers?



a b c d

2. The table below gives the morning temperature at 8am and 4pm for five days in January.

Date	8 a.m.	4 p.m.
3 January	-4 °C	9 °C
4 January	-3 °C	7 °C
5 January	0 °C	11 °C
6 January	-3 °C	10 °C
7 January	-2 °C	9 °C

- a) Which day and time had the coldest temperature?
- b) What day and time was the temperature warmest?
- c) What morning was the temperature warmest?
- d) Which day had the biggest range in temperature?

Negative numbers in everyday life : Money

Explain how you think bank accounts use negative numbers?
(Type your answer in this box)

Any money you spend that you don't have (below £0.00) is money that you are borrowing from the bank.

This money is called your 'overdraft' and is represented using negative numbers,

Paid out (£)	Paid in (£)	Balance (£)
		20.03
	80.00	100.03
20.00		80.03
25.00		55.03
40.00		15.03
		15.03

If you were to buy something that cost £10 and you didn't have the money in your account your balance would read - £10.00

If you paid that £10.00 back to the bank what would your bank balance read as?
(Type your answer here)

Look at the bank account balances of these people then answer the questions



Lord Blue



Mr Red



Sir White



Mrs Orange



Lady Pink

They all have negative balances and cannot spend any money until their overdrafts are cleared.

1. How much does Lord Blue need to pay off so that his bank balance is back at zero?
2. Mr. Red wants to buy a guitar which costs £300. What's the difference between the cost of the guitar and his current balance?
3. Sir White needs £10,000 (what for? It's a secret). How much more money does he need?
4. Mrs. Orange wants to buy a car which costs £750. How much does she need to earn to be able to afford it?
5. Lady Pink wants to buy a drink for £2.50. How much more money would she need in her bank account to be able to buy it?

Negative numbers in everyday life : Time zones

When it is early morning in London, it is late evening in Sydney.

All over the world, time zones are fixed according to the time at Greenwich, in London.

When calculating the time differences of different countries we may need to use negative numbers depending on the country we are looking at.

West is earlier and East is later.

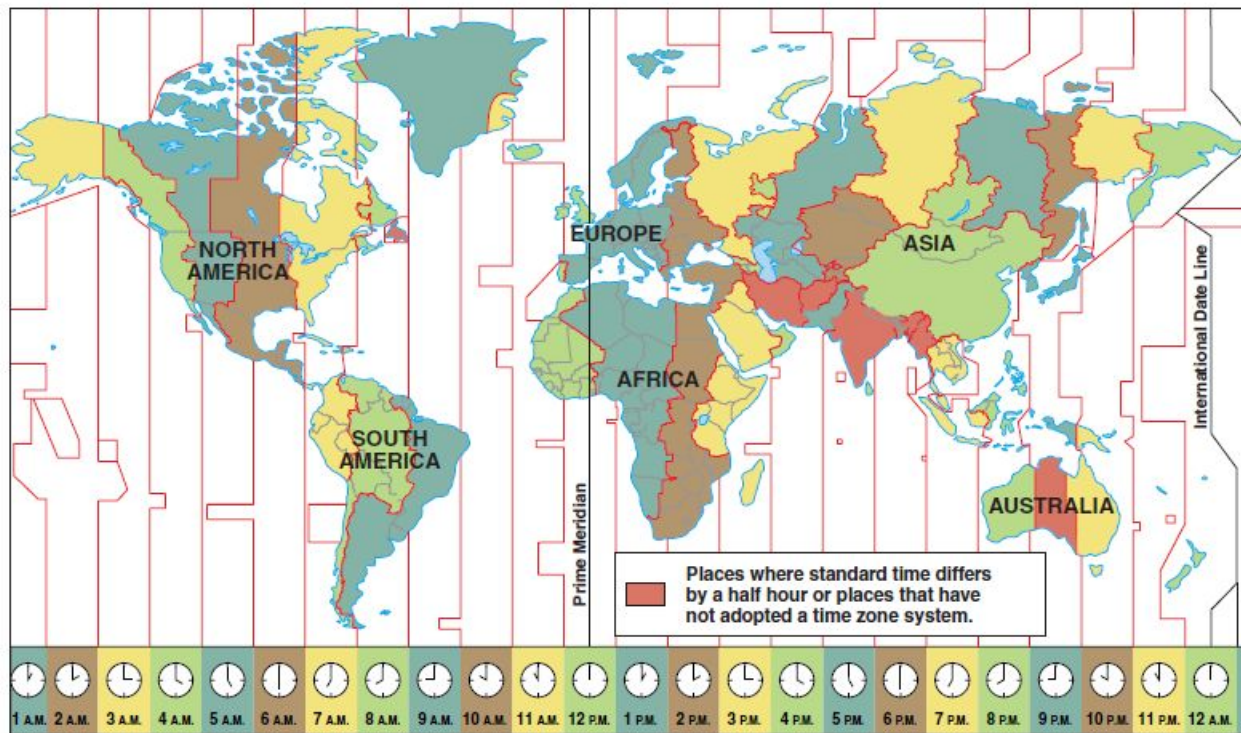
Sam would like to call Sophie at 8pm Friday in London. At what time should he call from Sydney?

Use the information on the map to complete the sentences below:

Sydney is 10 hours ahead of Greenwich.

Hong Kong is _____ hours _____ Greenwich

New York is _____ hours _____ Greenwich



Tokyo is _____ hours _____ Greenwich

Tokyo is _____ hours _____ New York

London is _____ hours _____ Los Angeles

Negative numbers - Magic Squares

Complete each of these magic squares so that each row, column and diagonal adds up to the same total. The last grid is blank for you to make your own.

-7		
-3	-8	-1

		-1
2	0	
1	-4	

		-4
	-1	1
2		

-6		
-7	-5	-3
		-4

-3		
	-2	0
1		-1

		2
5	1	
0		

Can you work out the missing numbers?

$$a + 6 + 3 = 7$$

$$2 + b + 4 = 1$$

$$3 + 6 + c = 4$$

$$1 + d + 5 = 2$$

$$e + 9 + 1 = 0$$

$$f + 4 + -3 = 2$$

$$8 + g + 2 = 5$$

$$-3 + 5 + 2 = h$$

$$i + 3 = -1$$

$$j + 2 + 6 = -4$$

$$2 + k = -1$$

$$4 + -1 = L$$

$$m + -3 = -5$$

$$n + -1 = 4$$

$$o + 2 + -3 = 1$$

$$3 - p = -2$$

$$7 - q = 9$$

$$2 - r = 4$$

a=

b=

c=

d=

e=

f=

g=

h=

i=

j=

k=

l=

m=

n=

o=

p=

q=

r=

Directed Numbers

Directed numbers are numbers that have a direction and size.

We have looked at examples in temperature reading and bank accounts already, now we will look at them using operations.

Continue through your workbook - watch the videos and work through the powerpoints. Be sure to fill in the questions as you go along.

Take your time and message me if you need any extra help.

What is an integer?

An integer (pronounced IN-tuh-jer) is a whole number (not a fractional number) that can be positive, negative, or zero.

Examples of integers are: -5, 1, 5, 8, 97, and 3,043.

Examples of numbers that are not integers are: -1.43, $1\frac{3}{4}$, 3.14, .09, and 5,643.1.

This song is a little cheesy but worth a listen :



Adding and Subtracting Integers



Answers these questions without a calculator

(a) $+6 - +4 =$

(d) $-6 + -4 =$

(b) $+4 - +6 =$

(e) $+6 - -4 =$

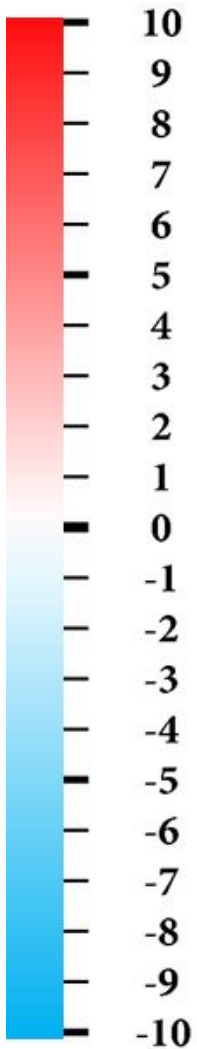
(c) $-4 + +6 =$

(f) $-6 - -4 =$

Choose **two** of the questions and explain your answers below using words or diagrams or both.

Explanation for question

Explanation for question



Directed numbers

Adding and subtracting
positive and negative numbers
using an analogy

Think of the temperature of the room you are in

What would happen to the temperature if:

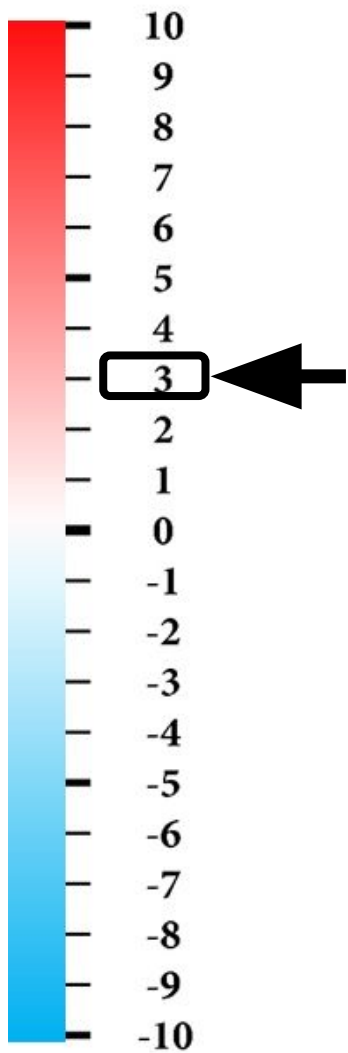
- 1) I added hot air to it? **It gets hotter!**
- 2) I took the hot air out? **It gets colder!**
- 3) I added cold air to it? **It gets colder!**
- 4) I took the cold air out? **It gets hotter!**

Hot and cold numbers...

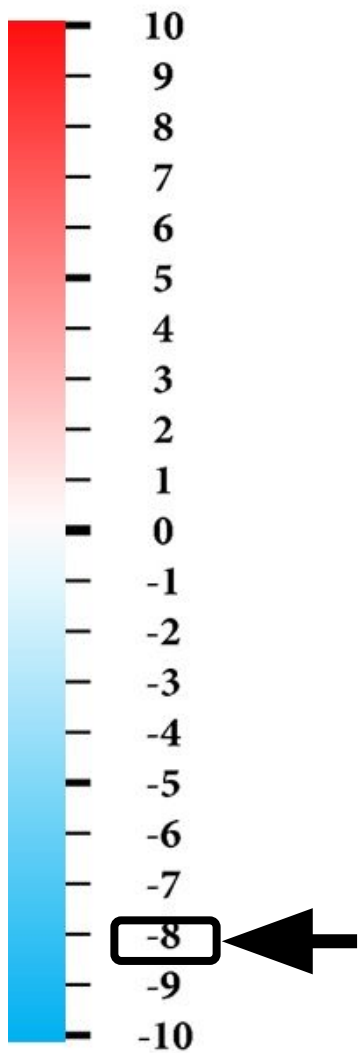
Imagine **positive numbers are hot** and **negative numbers are cold**

Thinking back to the temperature analogy, what would happen if:

- | | | |
|----|-------------------------------|-----------------|
| 1) | I add a positive number | It gets hotter! |
| 2) | I take away a positive number | It gets colder! |
| 3) | I add a negative number | It gets colder! |
| 4) | I take away a negative number | It gets hotter! |



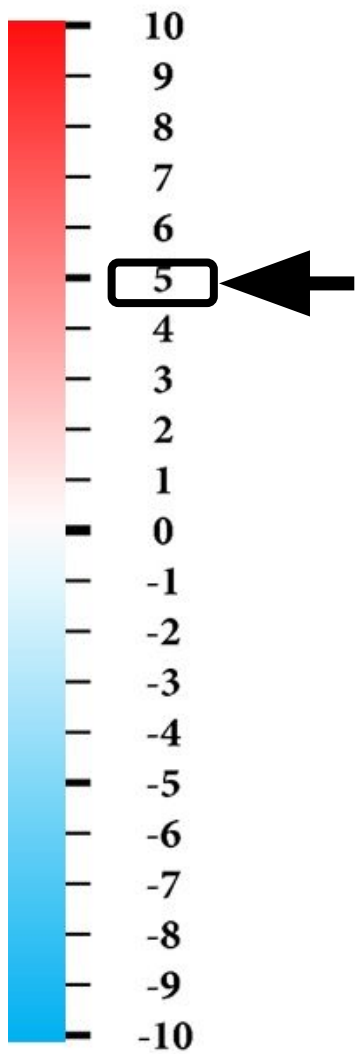
$$\text{Starting point } 3 + \text{Adding hot } 2 = 5$$



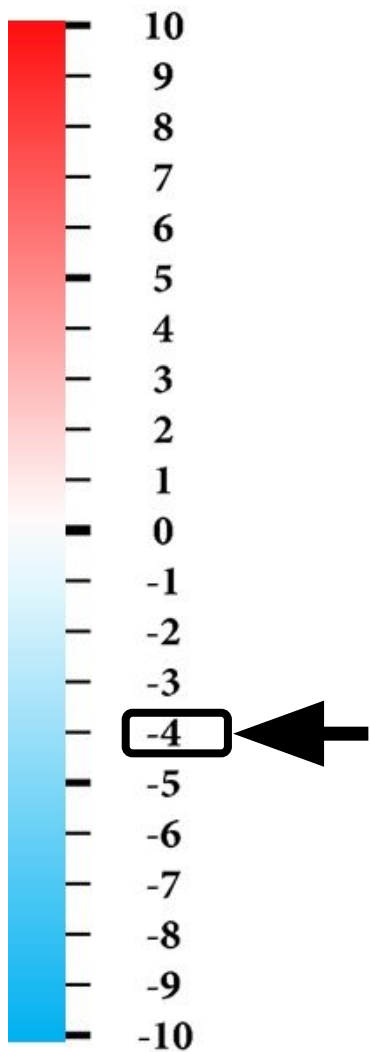
Starting point

Adding hot

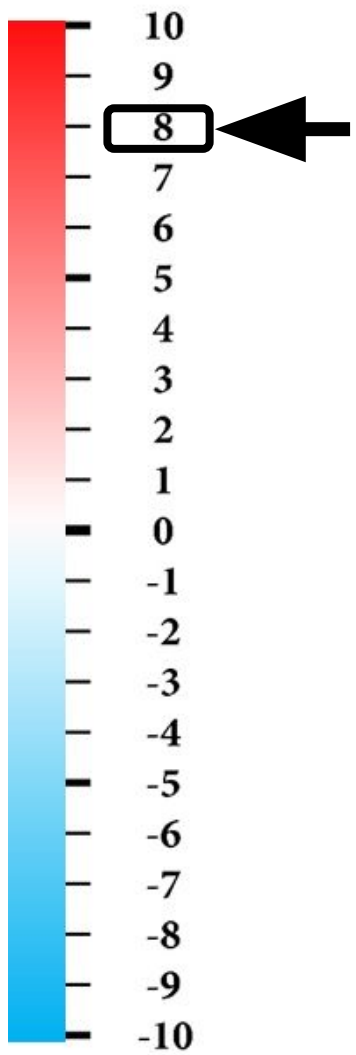
$$-8 + 7 = -1$$



$$\text{Starting point } 5 + \text{Adding cold } -3 = 2$$



$$\begin{array}{c} \text{Starting point} \nearrow \text{ } \textcircled{-4} \end{array} \quad \begin{array}{c} \text{Adding cold} \nearrow \text{ } \textcolor{lightblue}{+ -2} \end{array} = -6$$



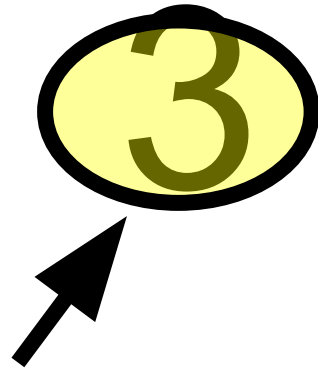
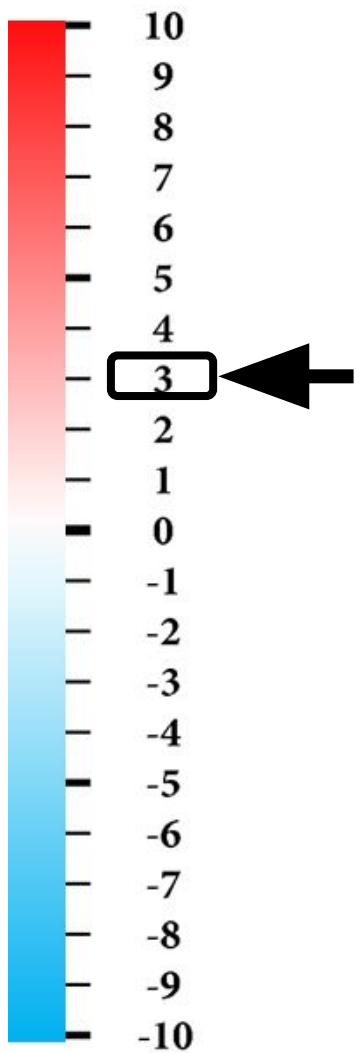
8

Starting point

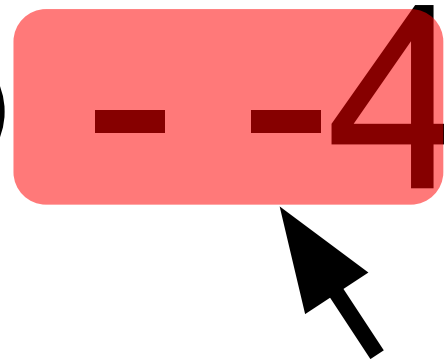
- 5

Taking away hot

= 3

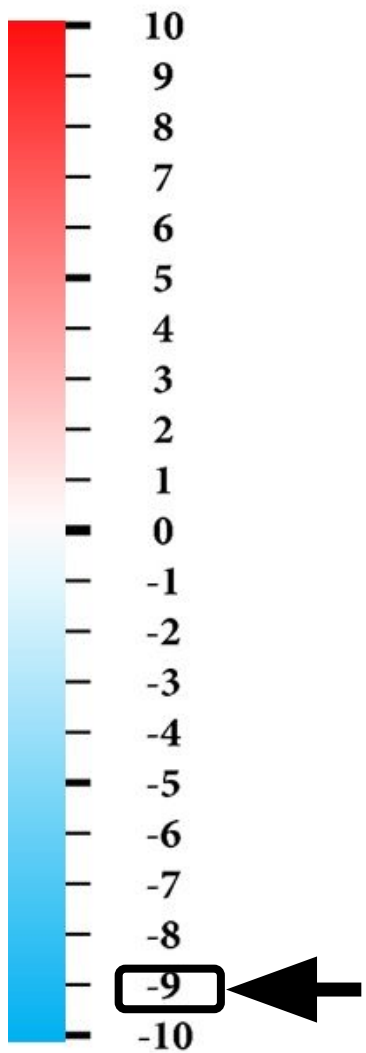


Starting point

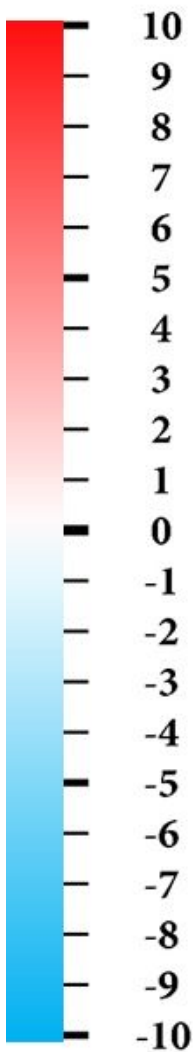


Taking away cold

$$- 4 = 7$$



$$\begin{array}{c} \text{Starting point} \nearrow \text{ } \textcircled{-9} \text{ } \textcolor{red}{-7} \text{ } = -2 \\ \text{Taking away cold} \nearrow \end{array}$$



Now try these :

1) $2 + 7 =$

2) $-8 + 10 =$

3) $7 + -4 =$

4) $-2 + -5 =$

5) $6 - 8 =$

6) $-4 - 3 =$

7) $3 - -5 =$

8) $-5 - -9 =$

9) $-6 - -3 =$

10) $12 + 5 - 6 + -9 - -4 =$

Handy hint!

Having your own number line would be really helpful and is easy to make!

Some people find drawing one in the margin of their workbook or on a test paper is really useful!

Always, Sometimes, Never True?

Thinking of what you have learnt so far : Highlight the statements below -

Always Green, Sometimes Orange and Never True Red

Subtraction always leaves you with a negative number

A negative and a positive is a negative

A calculation involving adding always gives you a positive answer

Subtracting a negative number is the same as adding

A positive and a negative is a negative

Adding a negative number is the same as subtracting

Two negatives make a positive

Two positives make a positive

An integer can be a fractional number

Spot the misconception!

What is the value of $-8+6$?

The correct answer is :

Look at the boxes below - what did these students do to arrive at their answers? Write the misconception in the box

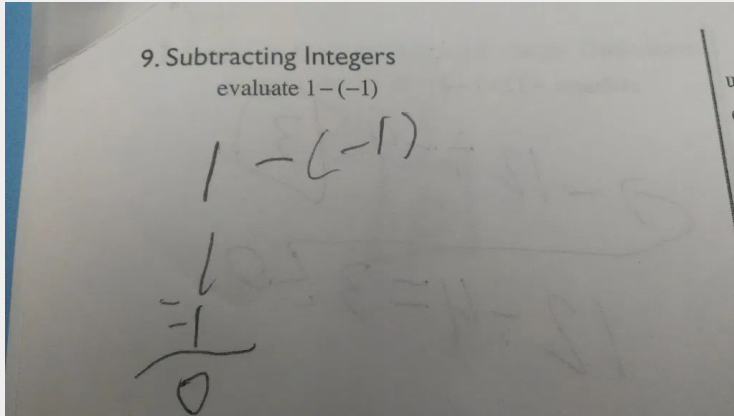
Student 1 gave the answer $-8 + 6 = 2$

How did they arrive at this answer?

One person gave the answer $-8 + 6 = -14$

How did they arrive at this answer?

Correct or Incorrect? And why ...



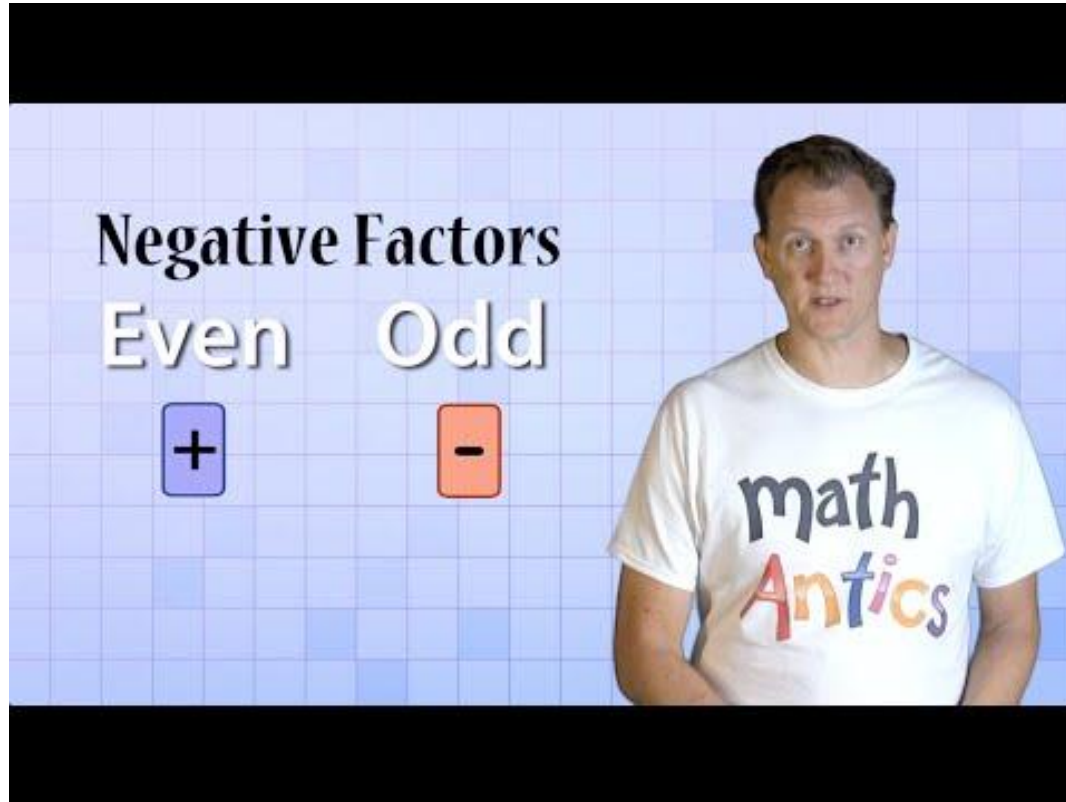
This answer is :

$$-2-6 = 8$$

When asked how the child answered this question they said a - and a - make a positive

This answer is :

Multiplying and Dividing Integers



Negative Factors

Even	Odd
+	-

The image shows a man in a white t-shirt with the text "math Antics" (where "Antics" is in multi-colored letters) standing in front of a blue grid background. To his left, the text "Negative Factors" is displayed above a table. The table has two columns: "Even" and "Odd". Under "Even" is a blue square containing a white plus sign (+). Under "Odd" is an orange square containing a white minus sign (-). The entire scene is framed by black bars at the top and bottom.

Multiplying and dividing with negative numbers

On the next slide you will find a multiplication grid - one quadrant has been completed for you containing the **products** you get from multiplying the two numbers - can you complete the rest of the grid

Think carefully about rules you may have to apply

(remember - in mathematics a **product** is the result of multiplying)

					5	5	10	15	20	25
					4	4	8	12	16	20
					3	3	6	9	12	15
					2	2	4	6	8	10
					1	1	2	3	4	5
-5	-4	-3	-2	-1	x	1	2	3	4	5
					-1					
					-2					
					-3					
					-4					
					-5					

What did the grid show you?

- Positive x Positive =
- Positive x Negative =
- Negative x Positive =
- Negative x Negative =

Can you use logic to complete these sentences?

- Positive $\div$ Positive =
- Positive $\div$ Negative =
- Negative $\div$ Positive =
- Negative $\div$ Negative =

Let's think ...

Have a look at this thinking board - consider each statement - decide if it is true or false and give an example

Statement	Agree / Disagree	Prove it !
<i>"I think multiplying always makes a number bigger"</i>		
<i>"If you multiply by zero the product is zero"</i>		
<i>"Dividing a number always makes it smaller"</i>		
<i>"Multiplying by a negative number makes a smaller number"</i>		

Try these questions - The answers are on the next slide for you to check you are on the right track

Questions

1) $3 \times -4 =$

2) $-7 \times -2 =$

3) $9 \times 9 =$

4) $6 \times -6 =$

5) $4 \times -1 \times 3 =$

6) $-20 \div 5 =$

7) $-100 \div -10 =$

8) $-10 \times 2 =$

9) $-11 \times -19 =$

10) $-450 \div -5 =$

11) $36 \div -6 =$

12) $-7 \times 11 =$

13) $-12 \times 11 =$

14) $-3 \times -20 =$

15) $-6 \times -3 \div 2 =$

Answers

1) -12

2) 14

3) 81

4) -36

5) -12

6) -4

7) 10

8) -20

9) 209

10) 90

11) -6

12) -77

13) -132

14) 60

15) 9

Match the answers to the questions. Which one is the odd one out?

$$-5 + 10$$

$$8 \times -2$$

$$8 - -10 - 4$$

$$100 \div -5$$

$$8 - -7$$

$$-7 - +10$$

$$-11 - 2$$

$$-44 \div -11$$

$$-30 \div 5$$

$$-6 \times 5$$

$$60 \div -15$$

$$8 + 11 - -6 - 16$$

$$-15 - -21$$

$$-5 + -10$$

$$3 \times -4$$

12

-13

5

4

15

6

9

-12

-30

-17

14

-4

-6

-20

-16

-15

Drag
And
drop!!!

Odd one
out

< Lesser than or Greater than >

Add in the correct symbol to make the sentence right

a)	-3		5		f)	2×-3		-2×-3
b)	9		-9		g)	-21		-2×-6
c)	$1 + 2$		$-1 + 3$		h)	$9/3$		$-9/3$
d)	$2 + 5$		$-2 - 5$		i)	1.2		$6/7$
e)	-7		$6 - 1$		j)	563		25×-3



This is how the Premier League stands at the moment.
Calculate the goal difference for the top 6 teams.
GD = For - Against
Use the table to answer the questions below.

	Team	Played	Won	Drawn	Lost	For	Against
1	Liverpool	29	27	1	1	66	21
2	Manchester City	28	18	3	7	68	31
3	Leicester City	29	16	5	8	58	28
4	Chelsea	29	14	6	9	51	39
5	Manchester United	29	12	9	8	44	30
6	Wolverhampton Wanderers	29	10	13	6	41	34

Goal difference

$$\begin{aligned} &\text{GD(Liverpool)} + \text{GD(Man City)} - \text{GD(Leicester)} = \\ &\text{GD(Chelsea)} \times \text{GD(Man Utd)} \div \text{GD (Wolves)} = \\ &\text{GD (Leicester)} - \text{GD(Liverpool)} \times \text{GD (Man City)} = \\ &\text{GD(?)} + \text{GD (?)} \times \text{GD (?)} = 630 \end{aligned}$$

Problem solving and Investigating



Temperatures and Problem solving

In each of these problems there are 5 cities: London, Paris, Oslo, Moscow and Rome. Use logic to work out the temperatures of the cities on the different days.

Day 1.

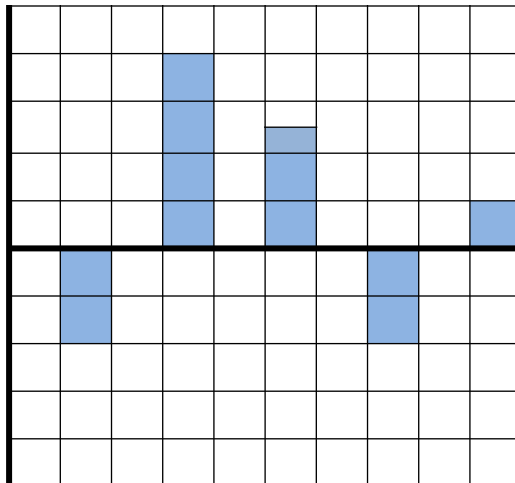
- Paris is 10°C warmer than Oslo.
- London is 3°C colder than Paris.
- Rome is twice the temperature of London.
- Moscow is twice as cold as Oslo.
- Oslo is -5°C.

Write the cities and their temperatures in
ascending order.

City	Temperature

Day 2.

- Oslo and Moscow are the same temperature.
- London is 6°C warmer than Rome.
- The difference in the temperatures of Rome and Paris is 3°C .



City	Temperature

Use the graph to help you calculate the temperatures of the cities. Write your answer in the table in **ascending** order of temperature.

Day 3.

- The difference between the temperatures in Paris and Oslo is 6°C.
- The temperature in London is double the temperature of Paris.
- Moscow is twice the temperature of Oslo.
- Rome is 4°C warmer than London.
- Moscow is currently the coldest city.
- The temperatures of the 5 cities are 2°C, -4°C, -8°C, 8°C, 4°C.

City	Temperature

Write the cities and their temperatures in **ascending** order.

Day 4.

- The difference between the temperatures in London and Oslo is 12°C .
- The temperature in Moscow is half that in Paris.
- The difference between the temperatures in London and Paris is 12°C .
- Moscow is warmer than Paris.
- The temperatures of the 5 cities are 4°C , -8°C , 16°C , -4°C , 8°C .

City	Temperature

Write the cities and their temperatures in **descending** order.

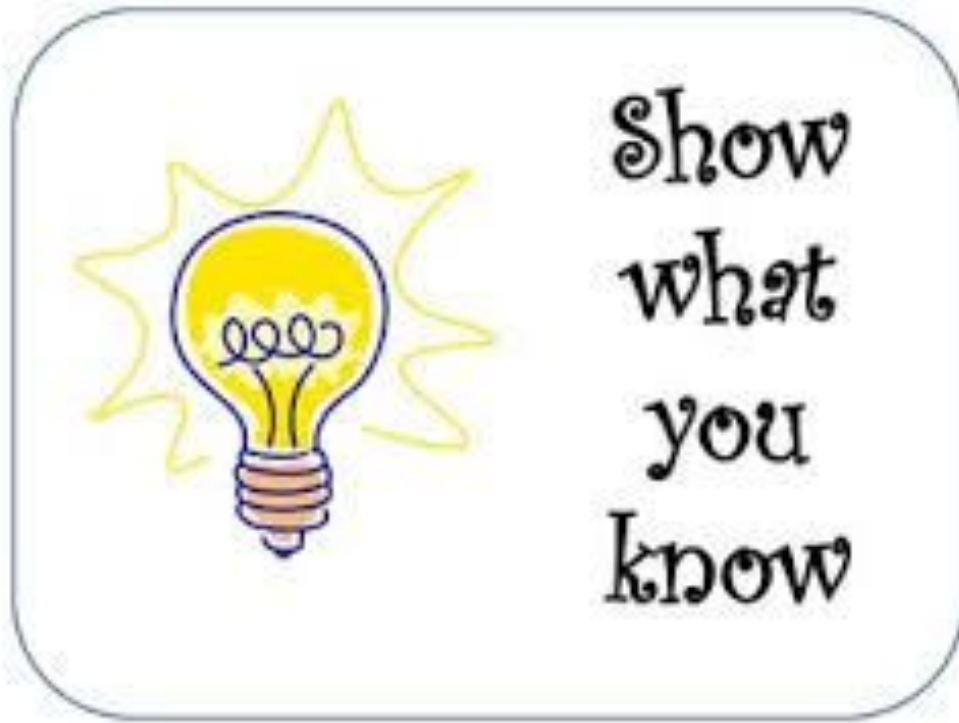
Day 5.

- Moscow is currently the coldest city.
- Rome is 3°C warmer than London.
- The difference in temperatures between Rome and Moscow is 10°C.
- Paris is 8°C warmer than London.
- The temperature in Paris is double that in Rome.
- The difference in temperatures between London and Oslo is 5°C.
- The difference in temperatures between Moscow and Oslo is 2°C.

Write the cities and their temperatures in **descending** order.

City	Temperature

Look at the temperatures of the cities over the course of the five days. Write a paragraph about the results summarising the hottest, coldest cities and anything else you noticed in the results that maybe of interest.



Complete the next slides using your knowledge of directed numbers.

Curious Cassie has made these observations.
What is your opinion?

Question	Curious Cassie's Answer	Your Answer	Does Curious Cassie have a misconception - if so what is it?
$54 \div -6 =$	9		
$-3 + -2 = 5$	5		
$? = -3 \times -2$	-6		
When $n = -5$ $3n > 10$ True or false	True		
$-3 \times 1.5 = 4.5$	4.5		

Design your own model ...

Earlier I showed you a model I like to use for adding and subtracting directed numbers.

Now I would like you to create your own model (or analogy) for directed numbers to pass on to Year 7 - Be creative, but remember it must make sense and is used to encourage misconceptions not to happen, so try to make it user friendly. Please feel free to add slides with your work.

Be creative - use pictures, colours and make it memorable.

Pupil feedback on this workbook

Did you like working like this? What was your favourite part? Any suggestions?

Please use this box to comment!!