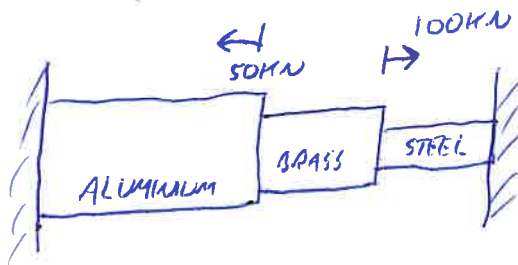


I FEM (1)

①



AL: $A = 400 \text{ mm}^2$
 $L = 280 \text{ mm}$
 $E = 70 \text{ GPa}$

$\rho = 2800 \text{ kg/m}^3$

BRASS: $A = 200 \text{ mm}^2$
 $L = 100 \text{ mm}$
 $E = 100 \text{ GPa}$

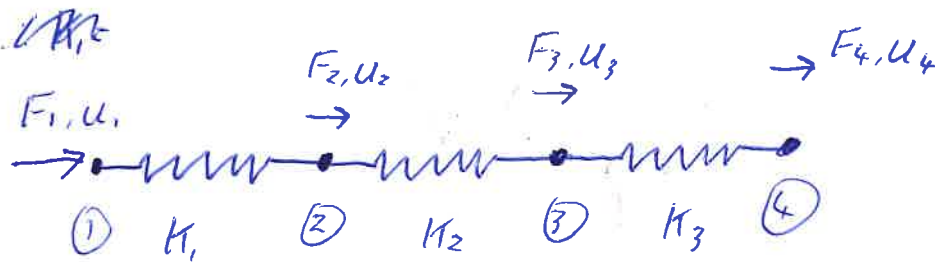
$\rho = 8250 \text{ kg/m}^3$

STEEL: $A = 70 \text{ mm}^2$
 $L = 100 \text{ mm}$
 $E = 200 \text{ GPa}$

$\rho = 7700 \text{ kg/m}^3$

$\rho = \text{AL} =$

~~K2, K3~~



$K = \frac{AE}{L} = \text{STIFFNESS OF EACH ELEMENT}$

NOTE WE ARE WORKING IN KN & mm !

$K_1 = \frac{400 \times 10^{-6} \times 70 \times 10^9}{280 \times 10^{-3}} = 100 \times 10^6 \text{ N/m} = 100 \text{ kN/mm}$

$K_2 = \frac{200 \times 10^{-6} \times 100 \times 10^9}{100 \times 10^{-3}} = 200 \times 10^6 \text{ N/m} = 200 \text{ kN/mm}$

$K_3 = \frac{70 \times 10^{-6} \times 200 \times 10^9}{100 \times 10^{-3}} = 140 \times 10^6 \text{ N/m} = 140 \text{ kN/mm}$

SPRING ONE $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$ $F = Kx$

SPRING TWO $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$

SPRING THREE $\begin{Bmatrix} F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 140 & -140 \\ -140 & 140 \end{bmatrix} \begin{Bmatrix} u_3 \\ u_4 \end{Bmatrix}$

① $\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$

② $\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 200 & -200 & 0 \\ 0 & -200 & 200 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$

③ $\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 140 & -140 \\ 0 & 0 & -140 & 140 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$

STIFFNESS MATRIX

$$\begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & (100+200) & -200 & 0 \\ 0 & -200 & (200+140) & -140 \\ 0 & 0 & -140 & 140 \end{bmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{pmatrix}$$

$$F_1 = 100U_1 - 100U_2 \quad (1)$$

$$F_2 = -100U_1 + 300U_2 - 200U_3 \quad (2)$$

$$F_3 = -200U_2 + 340U_3 - 140U_4 \quad (3)$$

$$F_4 = -140U_3 + 140U_4 \quad (4)$$

BOUNDARY CONDITIONS

ONCE STIFFNESS MATRIX IS FOUND BOUNDARY CONDITIONS ARE APPLIED AS PER CASE

$$U_1 = 0 \quad \therefore \text{FROM (1)} \quad F_1 = -100U_2 \quad (4a)$$

$$U_4 = 0 \quad \therefore \text{FROM (4)} \quad F_4 = -140U_3 \quad (4b)$$

$$F_2 = -50 \text{ kN} \quad \therefore \text{FROM (2)}$$

$$-50 = 0 + 300U_2 - 200U_3 \quad (5)$$

$$F_3 = 100 \text{ kN} \quad \therefore \text{FROM (3)}$$

$$100 = -200U_2 + 340U_3 + 0 \quad (6)$$

SOLVING (5) & (6) SIMULTANEOUSLY:

$$\begin{array}{l} \text{FROM (5)} \\ (+2) \end{array} \quad -100 = 600U_2 - 400U_3 \quad (7)$$

$$\begin{array}{l} \text{FROM (6)} \\ (+3) \end{array} \quad 300 = -600U_2 + 1020U_3 \quad (8)$$

I FEM (6)

(7)

ADANG

(7) & (8)

$$200 = 620 u_3$$

$$u_3 = 0.323 \text{ mm}$$

FROM (5) $-50 = 300 u_2 - 200 u_3$

$$u_2 = \frac{-50 + 200 u_3}{300}$$

$$= \frac{-50 + 200 \times 0.323}{300}$$

$$= 0.049 \text{ mm}$$

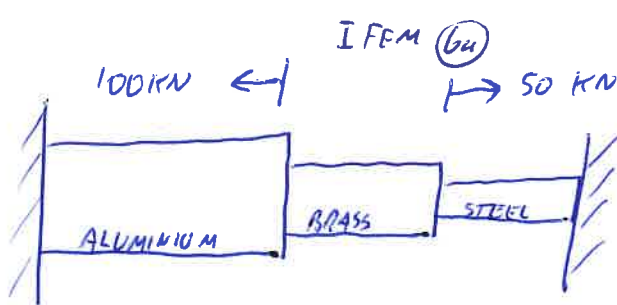
FROM (4a) $F_1 = -100 u_2$
 $= -4.9 \text{ kN}$

FROM (4b) $F_4 = -140 u_3$
 $= -45.22 \text{ kN}$

NEGATIVE
BECAUSE THEY
ACT TO THE
LEFT.

-50 kN

(2)



① WITH REVERSE FORCES

(8)

$$AL : A = 400 \text{ mm}^2$$

$$L = 280 \text{ mm}$$

$$E = 70 \text{ GN/m}^2$$

$$BRASS : A = 200 \text{ mm}^2$$

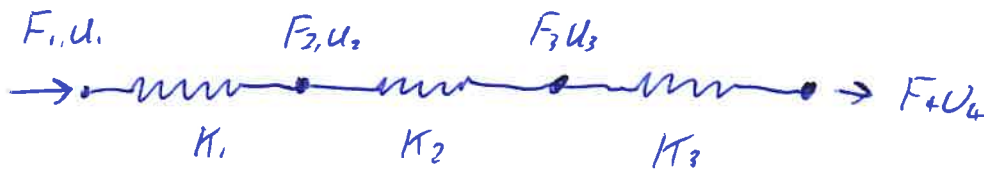
$$L = 100 \text{ mm}$$

$$E = 100 \text{ GN/m}^2$$

$$STEEL : A = 70 \text{ mm}^2$$

$$L = 100 \text{ mm}$$

$$E = 200 \text{ GN/m}^2$$



$$K_{STEEL} \quad K = \frac{AE}{L}$$

~~$$K_1 = 100 \text{ kN/mm}$$

$$K_2 = 200 \text{ kN/mm}$$

$$K_3 = 40 \text{ kN/mm}$$~~

STIFFNESS MATRIX IS THE SAME !

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 300 & -200 & 0 \\ 0 & -200 & 340 & -140 \\ 0 & 0 & -140 & 140 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix}$$

$$F_1 = 100U_1 - 100U_2 \quad (1)$$

$$F_2 = -100U_1 + 300U_2 - 200U_3 \quad (2)$$

$$F_3 = -200U_2 + 340U_3 - 140U_4 \quad (3)$$

$$F_4 = -140U_3 + 140U_4 \quad (4)$$

NEW BOUNDARY CONDITIONS: I FEM (6b)

(9)

$$U_1 = 0 \quad \therefore \text{FROM (1)} \quad F_1 = -100U_2 \quad (4a)$$

$$U_4 = 0 \quad \therefore \text{FROM (4)} \quad F_4 = -140U_3 \quad (4b)$$

$$F_2 = -100 \text{ kN} \quad \therefore \text{FROM (2)} \quad -100 = 0 + 300U_2 - 200U_3 \quad (5)$$

$$F_3 = 50 \text{ kN} \quad \therefore \text{FROM (3)} \quad 50 = -200U_2 + 340U_3 + 0 \quad (6)$$

$$\text{FROM (5)} \quad -200 = 600U_2 - 400U_3 \quad (7)$$

$$\text{FROM (6)} \quad 150 = -600U_2 + 1020U_3 \quad (8)$$

$$\text{ADDING (7) \& (8)} \quad -50 = 620U_3$$
$$\therefore U_3 = \underline{\underline{-0.081 \text{ mm}}}$$

$$\text{FROM (5)} \quad -100 = 300U_2 - 200U_3$$
$$U_2 = \frac{-100 + 200U_3}{300}$$
$$= \frac{-100 + 200 \times -0.081}{300}$$

$$U_2 = \underline{\underline{-0.387 \text{ mm}}}$$

$$\text{FROM (4a)} \quad F_1 = -100 \times U_2$$
$$= \underline{\underline{38.7 \text{ kN}}}$$

$$(4b) \quad F_4 = -140U_3$$
$$= \underline{\underline{11.34 \text{ kN}}}$$

$$\underline{\underline{50 \text{ kN}}}$$

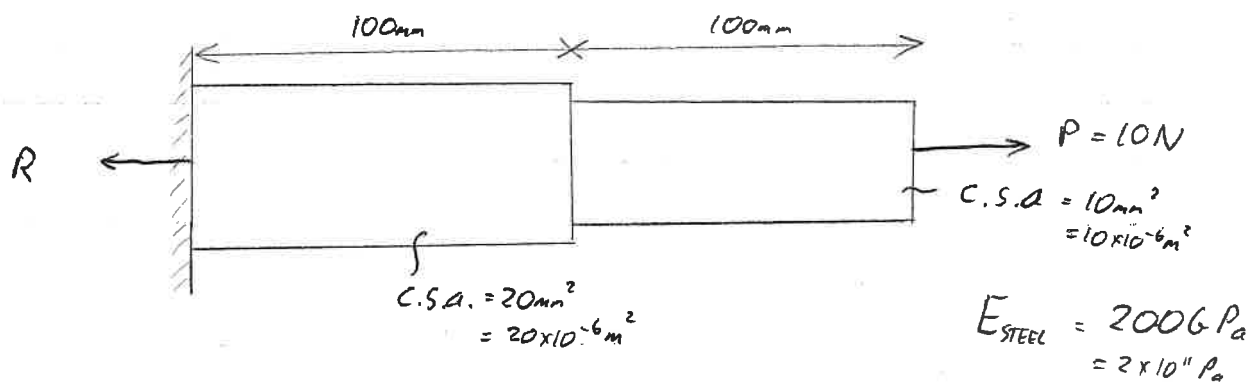
(3)

EXAMPLE

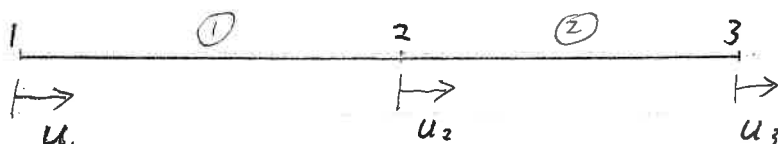
$$u_2 = ? \quad \epsilon_1 = ?$$

$$u_3 = ? \quad \sigma_1 = ?$$

(10)



$$(1 \times 10^{-3})(1 \times 10^{-3})$$



For element 1 : $K^e = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$= \frac{(20 \times 10^{-6})(2 \times 10^{11})}{100 \times 10^{-3}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= 4 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \times 10^7 \text{ N/m}$$

$$K^{e_1} = \begin{matrix} u_1 & u_2 \\ \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{matrix} \times 10^7 \text{ N/m}$$

For element 2 : $K^e = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$$= \frac{(10 \times 10^{-6})(2 \times 10^{11})}{100 \times 10^{-3}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$K^{e_2} = \begin{matrix} u_2 & u_3 \\ \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \end{matrix} \times 10^7 \text{ N/m}$$

11

$$\therefore [K] = \begin{matrix} & u_1 & u_2 & u_3 \\ \begin{matrix} u_1 \\ u_2 \\ u_3 \end{matrix} & \begin{bmatrix} 4 & -4 & 0 \\ -4 & 4+2 & -2 \\ 0 & -2 & 2 \end{bmatrix} \end{matrix} \times 10^7 \text{ N/m}$$

$$= \begin{bmatrix} 4 & -4 & 0 \\ -4 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix} \times 10^7 \text{ N/m}$$

$$\{F\} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 10 \end{Bmatrix}$$

$$\{F\} = [K]\{U\}$$

$$\begin{Bmatrix} 0 \\ 0 \\ 10 \end{Bmatrix} = \begin{bmatrix} 4 & -4 & 0 \\ -4 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix} \times 10^7 \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ 0 \\ 10 \end{Bmatrix} = 10^7 \begin{bmatrix} 4 & -4 & 0 \\ -4 & 6 & -2 \\ 0 & -2 & 2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \text{APPLYING B.C.'S}$$

$$\textcircled{1} \therefore -4u_2 \times 10^7 = R_1$$

$$\textcircled{2} (6u_2 - 2u_3) \times 10^7 = 0$$

$$6u_2 = 2u_3 \quad u_2 = \frac{1}{3}u_3$$

$$\textcircled{3} (-2u_2 + 2u_3) \times 10^7 = 10$$

$$\text{From } \textcircled{3} : -2u_2 \times 10^7 + 2u_3 \times 10^7 = 10$$

$$-\frac{2}{3}u_3 + 2u_3 = 10 \times 10^7$$

$$\frac{4}{3}u_3 = 10 \times 10^7$$

$$u_3 = 7.5 \times 10^{-7} \text{ m}$$

$$u_2 = 2.5 \times 10^{-7} \text{ m}$$

$$R = -10 \text{ N}$$

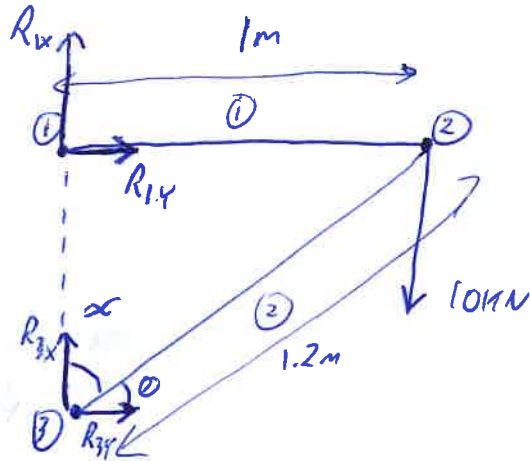
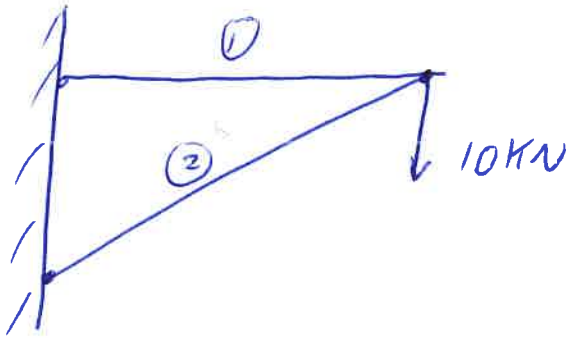
$$E_0 = \frac{-u_1 + u_2}{L} = \frac{-0 + 2.5 \times 10^{-7}}{100 \times 10^{-3}} = 2.5 \times 10^{-6}$$

$$\sigma_1 = E E_0 = 2.5 \times 10^{-6} \times 2 \times 10^{11} \\ = 500 \times 10^3 \text{ N/m}^2$$

4

1 FEM (13)

18



$$x^2 + 1^2 = 1.2^2$$

$$\therefore x = \underline{\underline{0.663m}}$$

$$\sin \alpha = \frac{o}{H} \quad \therefore \alpha = \sin^{-1} \frac{1}{1.2}$$

$$\alpha = 56.44^\circ$$

$$\therefore \theta = \underline{\underline{33.56^\circ}}$$

ELEMENT (1)

$$[K_e]_b = \frac{AE}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & +c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$\theta = 0$$

$$\therefore \cos \theta = 1$$

$$\sin \theta = 0$$

$$= \frac{40 \times 10^6 \times 200 \times 10^9}{1} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} & U_{1x} & U_{1y} & U_{2x} & U_{2y} \\ \begin{matrix} U_{1x} \\ -U_{1y} \\ U_{2x} \\ U_{2y} \end{matrix} & \begin{bmatrix} 8 & 0 & -8 & 0 \\ 0 & 0 & 0 & 0 \\ -8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \times 10^6 \text{ N/m}$$

ELEMENT (2)

1 FEM (14)

(19)

$$\theta = 33.56$$

$$\cos \theta = 0.833$$

$$\sin \theta = 0.553$$

$$\cos \theta \sin \theta = 0.461$$

$$\cos^2 \theta = 0.694$$

$$\sin^2 \theta = 0.306$$

$$[K_2^e]_G = \frac{40 \times 10^6 \times 200 \times 10^9}{1.2} \begin{bmatrix} 0.694 & 0.461 & -0.694 & -0.461 \\ 0.461 & 0.306 & -0.461 & -0.306 \\ -0.694 & -0.461 & 0.694 & 0.461 \\ -0.461 & -0.306 & 0.461 & 0.306 \end{bmatrix}$$

$$= \begin{matrix} & U_{2x} & U_{2y} & U_{3x} & U_{3y} \\ \begin{matrix} U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{matrix} & \begin{bmatrix} 4.627 & 3.073 & -4.627 & -3.073 \\ 3.073 & 2.040 & -3.073 & -2.040 \\ -4.627 & -3.073 & 4.627 & 3.073 \\ -3.073 & -2.040 & 3.073 & 2.040 \end{bmatrix} \end{matrix} \times 10^6 \text{ N/m}$$

$$\therefore [K^e]_L = \begin{matrix} & U_{1x} & U_{1y} & U_{2x} & U_{2y} & U_{3x} & U_{3y} \\ \begin{matrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{matrix} & \begin{bmatrix} 8 & 0 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -8 & 0 & 8+4.627 & 3.073 & -4.627 & -3.073 \\ 0 & 0 & 3.073 & 2.040 & -3.073 & -2.040 \\ 0 & 0 & -4.627 & -3.073 & 4.627 & 3.073 \\ 0 & 0 & -3.073 & -2.040 & 3.073 & 2.040 \end{bmatrix} \end{matrix} \times 10^6 \text{ N/m}$$

$$\{F\} = [K] \{U\}$$

$$\begin{pmatrix} R_{1x} \\ R_{1y} \\ R_{2x} \\ R_{2y} \\ R_{3x} \\ R_{3y} \end{pmatrix} = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{pmatrix} U_{1x} \\ U_{1y} \\ U_{2x} \\ U_{2y} \\ U_{3x} \\ U_{3y} \end{pmatrix}$$

APPLYING BOUNDARY CONDITIONS.

$$\begin{pmatrix} R_{1x} \\ R_{1y} \\ 0 \\ -10,000 \\ R_{3x} \\ R_{3y} \end{pmatrix} = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ U_{2x} \\ U_{2y} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} R_{1x} \\ R_{1y} \\ 0 \\ -10,000 \\ R_{3x} \\ R_{3y} \end{pmatrix} = \begin{bmatrix} 8 & 0 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -8 & 0 & 12.623 & 3.073 & -4.627 & -3.073 \\ 0 & 0 & 3.073 & 2.040 & -3.073 & -2.040 \\ 0 & 0 & -4.627 & 3.073 & 4.627 & 3.073 \\ 0 & 0 & -3.073 & 2.040 & 3.073 & 2.040 \end{bmatrix} \times 10^6 \begin{pmatrix} 0 \\ 0 \\ U_{2x} \\ U_{2y} \\ 0 \\ 0 \end{pmatrix}$$

$$R_{1x} = -8 \times 10^6 U_{2x} \quad (1)$$

$$R_{1y} = 0 \quad (2)$$

$$0 = (12.623 U_{2x} + 3.073 U_{2y}) \times 10^6 \quad (3)$$

$$-10000 = (3.073 U_{2x} + 2.040 U_{2y}) \times 10^6 \quad (4)$$

$$R_{3x} = (-4.627 U_{2x} - 3.073 U_{2y}) \times 10^6 \quad (5)$$

$$R_{3y} = (-3.073 U_{2x} - 2.040 U_{2y}) \times 10^6 \quad (6)$$

From (3) $U_{2x} = \frac{-3.073 U_{2y}}{12.623} \therefore U_{2x} = -0.243 U_{2y}$

From (4) $\frac{-10000}{1 \times 10^6} = 3.073 U_{2x} + 2.040 U_{2y}$

$$-0.01 = 3.073(-0.243 U_{2y}) + 2.040 U_{2y}$$

$$\therefore U_{2y} = \underline{\underline{-7.73 \times 10^{-3} \text{ m}}}$$

$$\therefore U_{2x} = -0.243 \times -7.73 \times 10^{-3}$$

$$= \underline{\underline{1.88 \times 10^{-3} \text{ m}}}$$

FORCE IN ①

$$E = \frac{\sigma}{\epsilon}$$

$$\therefore = \frac{F}{AE}$$

$$\therefore F = EAE$$

$$= EA \frac{\Delta L}{L}$$

$$= \frac{200 \times 10^9 \times 40 \times 10^{-6}}{1} \times 1.88 \times 10^{-3}$$

$$= \underline{\underline{15.04 \text{ kN}}} \quad \text{TENSILE FORCE IN ①}$$

FORCE IN ②

TO CONVERT FROM THE DISPLACEMENTS FROM GLOBAL TO LOCAL

$$\{u\}_L = [T] \{u\}_G$$

$$\begin{Bmatrix} u_{1x} \\ u_{1y} \end{Bmatrix} = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{Bmatrix} u_{2x} \\ u_{2y} \end{Bmatrix}$$

$$= \begin{bmatrix} 0.833 & 0.553 \\ -0.553 & 0.833 \end{bmatrix} \begin{Bmatrix} 1.88 \times 10^{-3} \\ -7.73 \times 10^{-3} \end{Bmatrix}$$

$$= \begin{bmatrix} 1.566 \times 10^{-3} - 4.275 \times 10^{-3} \\ -1.040 \times 10^{-3} - 6.439 \times 10^{-3} \end{bmatrix}$$

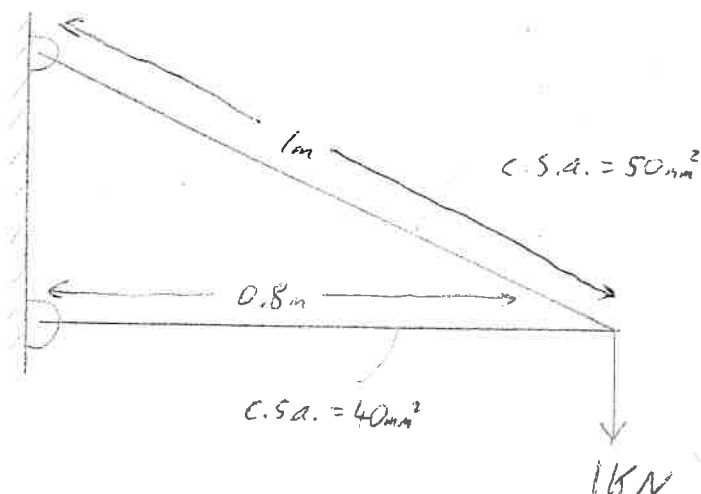
$$= \begin{bmatrix} -2.71 \times 10^{-3} \\ -8.479 \times 10^{-3} \end{bmatrix}$$

$$F = \frac{200 \times 10^9 \times 40 \times 10^{-6}}{1.7} \times -2.71 \times 10^{-3} = \underline{\underline{-18.07 \text{ kN}}}$$

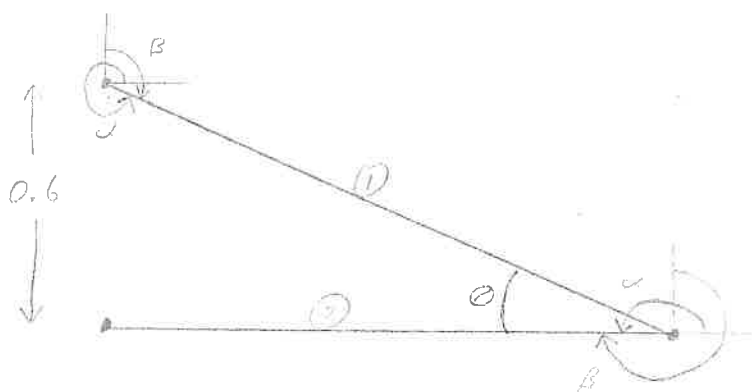
(5)

EXAMPLE: Simple 2D truss

(24)



$$E_{\text{steel}} = 200 \text{ GPa}$$



S^0_H
 C^0_H
 t^0_A

$$\sin \theta = \frac{0.6}{1} \therefore \theta = 36.87^\circ$$

For element 1: $\beta = 90 + 36.87 = 126.87$

$$\alpha = 360 - 36.87 = 323.13$$

$$m = -0.6$$

$$L = 0.8$$

$$[K^{(0)}] = \frac{AE}{L} \begin{bmatrix} L^2 & Lm & -L^2 & -Lm \\ Lm & m^2 & -Lm & -m^2 \\ -L^2 & -Lm & L^2 & Lm \\ -Lm & -m^2 & Lm & m^2 \end{bmatrix}$$

$$= \frac{50 \times 10^{-6} \times 2 \times 10^{11}}{1} \begin{bmatrix} 0.64 & -0.48 & -0.64 & 0.48 \\ -0.48 & 0.36 & 0.48 & -0.36 \\ -0.64 & 0.48 & 0.64 & -0.48 \\ 0.48 & -0.36 & -0.48 & 0.36 \end{bmatrix} \text{ N/m}$$

For element 2: $m = \frac{2}{0.8} = 0$
 $l = \frac{-0.8}{0.8} = -1$

$$[K^{(2)}] = \frac{40 \times 10^{-6} \times 2 \times 10^9}{0.8} \begin{bmatrix} u_2^0 & v_2^0 & u_3^0 & v_3^0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} u_1^0 \\ v_1^0 \\ u_3^0 \\ v_3^0 \end{matrix} \text{ N/m}$$

$$[K^0] = \begin{matrix} & u_1^0 & v_1^0 & u_2^0 & v_2^0 & u_3^0 & v_3^0 \\ \begin{matrix} u_1^0 \\ v_1^0 \\ u_2^0 \\ v_2^0 \\ u_3^0 \\ v_3^0 \end{matrix} & \begin{bmatrix} 0.64 & -0.48 & -0.64 & -0.48 & 0 & 0 \\ -0.48 & -0.36 & -0.48 & -0.36 & 0 & 0 \\ -0.36 & -0.48 & -0.64 & -0.48 & -1 & 0 \\ 0.48 & -0.36 & -0.48 & -0.36 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \times 10^{-3} \text{ N}$$

minimise P.E.

$$[K^0] \begin{Bmatrix} u_1^0 \\ v_1^0 \\ u_2^0 \\ v_2^0 \\ u_3^0 \\ v_3^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -1000 \\ 0 \\ 0 \end{Bmatrix}$$

$$[K^0] \{U^0\} = \{F\}$$

$$[K^0] \begin{Bmatrix} 0 \\ 0 \\ u_2^0 \\ v_2^0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} R_{1x}^0 \\ R_{1y}^0 \\ 0 \\ -1000 \\ R_{3x}^0 \\ R_{3y}^0 \end{Bmatrix}$$

$$u_2^0 = -0.133 \text{ mm}$$

$$v_2^0 = -0.456 \text{ mm}$$

To FIND STRESS/STRAIN we must first go to LOCAL COORDINATES:
element ①:

$$\begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{Bmatrix} \begin{Bmatrix} -0.133 \\ -0.456 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ v_2 \end{Bmatrix} = \begin{Bmatrix} -0.167 \\ -0.455 \end{Bmatrix}$$

$$\epsilon^{(1)} = \frac{-u_1 + u_2}{L} = 167 \times 10^{-6}$$

$$\sigma^{(1)} = E \epsilon^{(1)} = 33.4 \text{ MPa}$$