

1. Discuss 3 theories of elastic failure. Outline how and why they have been developed

There are five generally used theories of elastic failure, I will discuss in this report three types and outline why and how they have been developed. I will discuss the following;

Von - Mises & Hencky theory, which relates to distortion energy in a material (Edge, 2020)

Rankine's theory – Maximum Principal(normal) stress theory

Guest- Tresca theory – Shear strain energy (IDC, 2020)

They have been developed because one kind of failure theory cannot apply to all materials. Engineers will need to prove their theory for their specific needs, and the different theories focus more on those stresses.

Von-Mises and Guest-Tresca theory agree much better with experimental observations (Annabattula, 2020).

Von-Mises

Von Mises theory of elastic failure is based on determining the distortion energy in a material. This theory states that a structural material is safe so long as the maximum value of distortion energy per unit volume in the material stays smaller than the distortion energy per unit volume required to cause yield in a tensile test of the same material (Edge, 2020) , or Yielding occurs when maximum distortion energy is equal to the distortion energy at yielding in a uniaxial tensile test. Distortion comes from the product shape changing, whilst the volume remains the same.

This can be determined by the expression;

$$u_d = \frac{1 + \nu}{3E} \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]$$

Where u_d is a general 3 dimensional case of distortional energy where it shows the principal stress (distortion) . This is simplified by finding out a simple tensile test (sim) at its time of failure is:

$$u_{d,sim} = \frac{1 + \nu}{3E} \sigma_y^2$$

This is connected to the Von Mises theory by stating the equation in [*the condition of failure*] is greater than the yield strength

$$\left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2} \right]^{1/2} \geq \sigma_y$$

Where the left most section of the equation is referred to as the Von Mises Stress it allows the theory to be simplified again with;

$$\sigma_v \geq \sigma_y$$

The above equation means that the Von Mises Stress is greater than the yield strength of the material in a simple tensile test, it will fail. This simple equation allows for the user to effectively see if the stress is too great for the material, and if additional design needs to be included to increase the load supporting capability. This also considers that there is no Factor of Safety applied. (Mathew, 2012)

Rankine's Theory

Is used only for brittle materials, and not recommended for ductile material. Failure of the structure occurs when the major principal tensile stress reaches a value beyond the elastic limit, and minor principal stress reaches a value beyond its elastic limit in simple compression. (Engineering, 2020)

$$\begin{aligned}\sigma_1 &= \sigma_y, \sigma_u \\ \sigma_3 &= -\sigma_y, -\sigma_u\end{aligned}$$

Where $\sigma_1 = \text{maximum stress}$, $\sigma_3 = \text{minimum stress}$ & $\sigma_y = \text{yield strength(ductile)}$, $\sigma_u = \text{ultimate strength (brittle)}$

Therefore if the maximum stress is equal to the yield strength or ultimate strength it will fail, or if the minimum stress is equal to the yield or ultimate strength it will also fail.

A limitation of this theory is that if σ_1 and σ_2 are the same, it means they are both σ_{MAX} – meaning when failure does occur, it could be on the X plane or the Y plane. Having two potential points of failure is difficult to precisely predict the point of failure.

The reason behind its lack of suitability for ductile materials is because they are weak in shear stress. Whilst brittle materials are weak in tension, they are safe in use of static loading conditions, such as uniaxial and biaxial. (Bhandari, 2013). This is a reason it has been developed, in aid of brittle materials.

Guest-Tresca

Comparable to Von Mises, the Guest-Tresca theory is highly favoured for its accuracy and reliability.

Yielding occurs when the maximum shear stress is equal to the shear stress at yielding in a uniaxial tensile test or $\tau_{\text{max}} = \tau_y$

It can also be displayed as $\sigma_1 - \sigma_3 = \sigma_y$ which means if the maximum minus the minimum stress equal the yield strength, failure is likely.

In 1864 Henri Tresca was one of the first to use a similar theory to relate to metallic material. He discovered that stress on metal were independent of average normal stresses whilst dependant on shear stress. Researching microscopic isotropic polycrystals in relation to the maximum shear stress

of materials written by saint-Venant – a theory recommended for brittle materials. (Britannica, 2020)

Both Guest-Tresca and Von Mises theories of failure can be represented on Mohr's circle.

Mohr's circle (Figure 1) is used graphically to plot the predicted state of stress points in different orientations. (Shah, 2017) Used to evaluate principal stress, maximum shear stress, and normal stresses on a given plane. (Shukla, 2017)

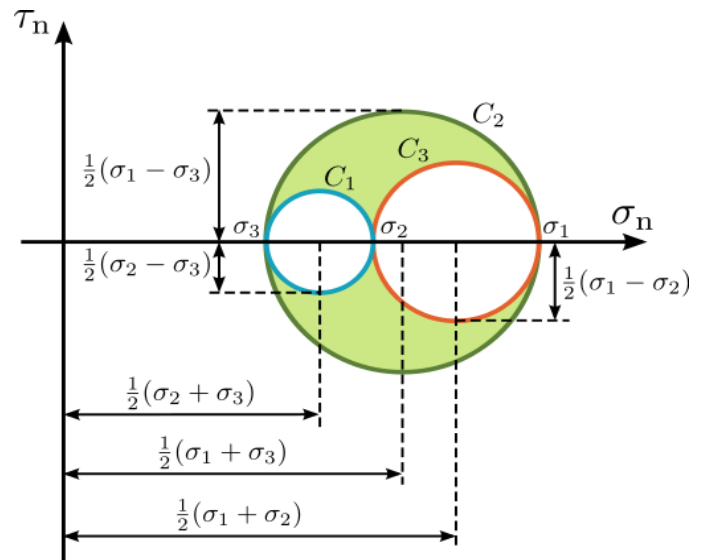


Figure 1 (Jain, 2013)

Figure 2 compares the von Mises and Tresca criterion in a 2D representation. Outside of the retrospective lines, the material will reach its maximum yielding point and is likely to fail. (Ho, 2010)

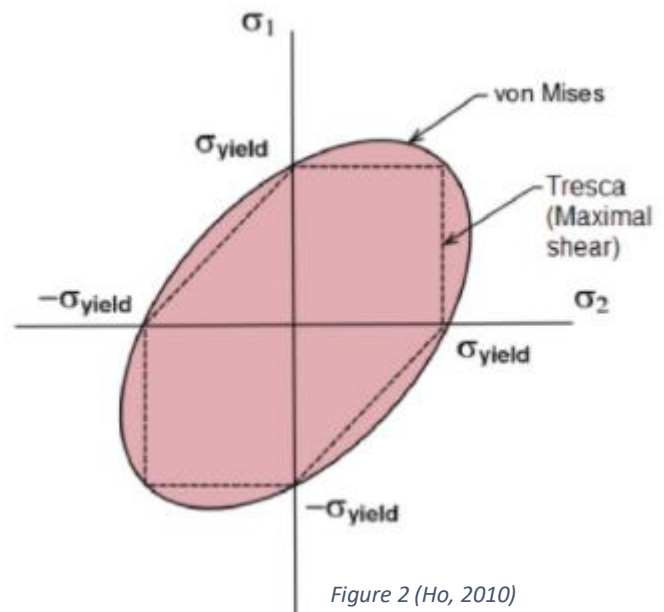


Figure 2 (Ho, 2010)

2. Choose a component made from a material of your choice. Discuss what theory of elastic failure should be used when looking at developing this component.

With the progressing cycling industry there are new technologies being developed to design and manufacture lighter and stronger bicycle frames. In the current industry, aluminium frames are either made from 6061 or 7005 (road.cc, 2020). For this study I will be choosing a bike frame manufactured out of 6061 with a T6 temper to increase the mechanical properties of the alloy.

The T6 has more than double the yield strength of T4, with T6 reporting 35 MPa and a tensile strength of 42MPa, and T4 reporting 16 MPa yield strength and 30Mpa tensile strength. (Toolbox, 2008)

The composition of aluminium alloy 6061 is seen in the *figure 3* below.

FIGURE 3. COMPOSITION OF ALUMINIUM ALLOY 6061

Element	Si	Fe	Cu	Mn	Mg
Weight %	0.514	0.23	0.161	0.071	0.96
Element	Ni	Zn	Ti	Cr	Al
Weight %	0.010	0.015	0.031	0.103	Bal.

Figure 3 (Basithrahman. A, 2016)

The mechanical properties of aluminium 6061 are found in *figure 4*.

Material properties of aluminum 6061		
Properties	Metric	Unit
Modulus Young (E)	68.9	GPa
Poison's ratio (ν)	0.30	-
Density	2.700	kg/m ³
Yield Stress	276	MPa

Figure 4 (George E. Totten, 2003)

Due to the ductility of the material, using the Von-mises theory of failure is recommended. This is due to the high reliability of tensile stress sample tests to real world experimental purposes.

However, there are other suitable theories that could be applied. The Tresca-Guest theory is also viable.

Rider-induced load failure is also a possibility, which is the loads that are associated with the weight the rider will put on the seat, pedal and handlebars of the bike when climbing a hill or moving on a flat surface. In turn, the forces will give greater readings on the bike frame where the top tube and seat tube meet, where the bottom bracket and the seat tube meet, and the head tube where both bottom bracket and top tube meet. These tubes can be seen in *figure 5*.

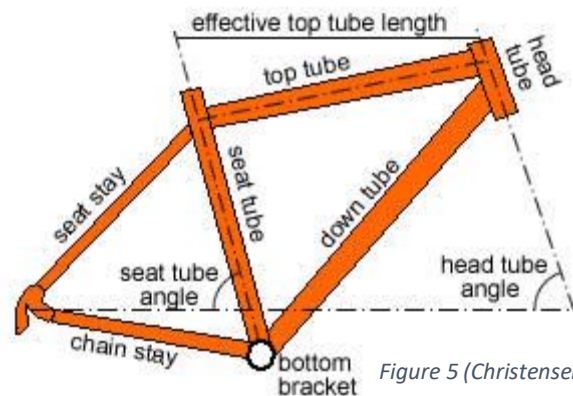


Figure 5 (Christensen, 2017)

As most bike frames are manufactured from metal, it is very rare that catastrophic failure would occur with simple loading. The cause would likely be a dynamic impact such as a crash. If a bike frame manufactured from aluminium alloy 6061 would fail from static loading, it would show signs of elastic failure in cracks, bulges, and bends. (Robson Forensic, 2015)

Safety is paramount for bike frames. If a bike frame undergoes elastic failure, its impact can be very dangerous and has the potential to significantly injure the rider or surrounding riders. Analysing where elastic failure is likely to occur and designing the frame to support these findings is significant to reduce the risk of injury occurring. (Kailas Khutal, 2019)

Due to the environment that the bike will be used in, it has an increased chance of suffering from a dynamic load failure than a static load failure. Said load could encounter the frame from a multitude of directions and angles. Road-induced loads are another possibility of failure. This is the stresses that a bike frame will receive when the wheels pass over different types of terrain and the irregularities that goes with it. (M. L. Hull, 1988)

To focus on only one type of stress such as Rankin's theory of maximum normal stress would isolate itself from realistic external forces. In addition to this, Rankine's theory of elastic failure is better suited for brittle materials, where the weakest stress is tension as opposed to shear stress being the weakest stress for ductile materials. (Sawhney, 2017) However, it could be applied to a bike frame made of carbon fibre, which is an increasingly popular choice for high end bike frames and competitive races. (EL-HAJJAR, 2012)

Using the von-Mises stress analysis will significantly lower the cost of testing if used effectively, due to the cost of destructive testing of a real bike frame, mimicking loads to apply to the tubes of a frame under Finite Element Analysis (FEA) heavily reduces the cost (Alonzo, 2015). Using von-Mises stress also clearly outlines that if the maximum stress is greater than the yield strength, it will fail (Chien-Cheng Lin, 2017). Allowing strengthening of the component when still in the design phase.

Figure 6 shows the vertical loading force of 700N, which is the equivalent to 71.3kg in body weight. The rider weight is doubled due to the G force of riding on an uneven surface that the bike is traveling over.

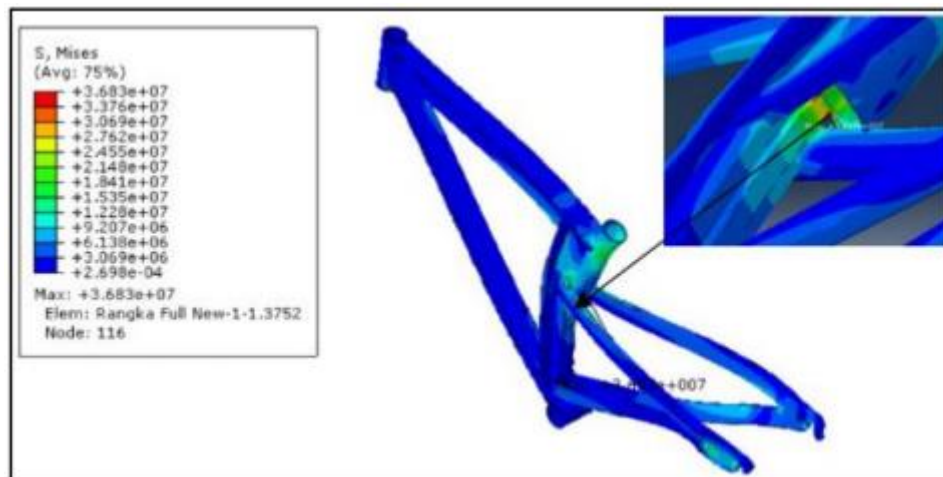


Figure 6 (Akhyar Akhyar, 2019)

The maximum von-Mises stress in the seat tube and seat tube area is 36.83 MPa. Compared with the yield strength in *figure 3*, of 276 MPa. The total vertical load of the rider would need to be seven point five times greater for the frame to experience failure. Through this visualising and clarity, using von-Mises theory of elastic failure is suitable for this component and material choice.

3. Choose a research paper from Science Direct that discusses the theory of elastic failure identified in task 2 and summarise this paper. Relate this to your discussion in Task 2.

The paper I am appraising “Von mises yield criterion and nonlinearly hardening variable thickness rotating annular disks with rigid inclusion” written by Ahmet N Eraslan in 2002.

The paper analyses the use of annular disks mounted on circular shafts, which are commonly used through engineering. Comparing these to similar studies recent to the papers release date, some of which were their own studies, however Ahmet Eraslan referenced papers that used Tresca’s criterion. (Eraslan, 2002)

The paper discusses in depth regarding the Von-Mises yield condition and how it works with Swift’s hardening law, that assumes nonlinear strain hardening, breaking down the fundamental equations that are required to work out the results. Through a detailed compact paragraph, all relevant forces and figured were applied to the appropriate equations. The findings stated that the Von-Mises criterion is 3% to 4% higher than results in Eraslan’s previous paper when using Tresca’s criterion. This statement was backed up with comparisons of professional line graphs, displaying the stress and displacement and its radii with two different disks, where nonlinear hardening disks are represented by the plots ($m=2$) of which the only difference was a slight spike on the 0.5 “dimensionless radius” at the stress and displacement Y axis.

The author illustrates the variation of plastic limit angular velocity against the effects of the hardening parameters, where the higher the parameter the higher the angular velocity. The data shows that when the linear material will have a higher angular velocity, whilst nonlinear material remains slightly lower.

The paper highlights that larger deformations occur in nonlinear disks, whilst the geometry of the variable thickness disks significantly affects the plastic limit angular velocities. The paper highlights the importance of reducing the disk edge thickness. To determine the increase in plastic limit angular velocity, both the thickness ratio, and the reduction in disk thickness are important parameters. The plastic limit angular velocity and the thickness ratio simultaneously increase.

Due to the layout of the paper, digesting the information is difficult and confusing. If the author had stated their findings as they did in "*4.1 Inelastic deformations*" then supported these further with a table that clearly outlined where each measurement came from, the paper would be more reader friendly.

The drawback of this paper is that its dated to 2002, although the von-Mises theory of elastic failure remains the same, there have been technological advances to maintain more accurate readings such as updates to software.

If the paper were to be tested in today's experimental environment, the author would need to increase the sample amount. Three disks have been tested in this paper, which is small scale. The author does not state the disk diameter, which would influence the overall tensile stress limit on the disks (Joong-Ki Hwang, 2019), however, the author is testing the numerical solution. I believe in addition to the "dimensionless" solution, the conclusion would be better made if it were trailed with a physical disk to test. Contrary to this statement, I understand that there are limitations to the resources that the author has available.

This experiment is only completed once using Von-Mises stress for the results, to gain more reliable information then the experiment needs to be repeated under the exact conditions. The paper does not state the material of the disks apart from the statement of linear and nonlinear materials, which is vital in understanding the expected results versus the actual results. This is due to the varying maximum yield strength of materials. In my previous task the material for the bicycle frame was aluminium 6061-T6, this level of detail needed to be used for reliable data.

However, the paper does not state that it is sponsored or that it has any affiliation with influential corporations or individuals, making this experiment unbiased. The author stated that the test was completed previously using the Tresca yield criterion, by undergoing two experiments using similar but different methods to gain results, the author has backed their statement that the critical angular velocities increase as the edge thickness reduces. The increase of 3% to 4% using the Von-Mises yield criterion show that the disks have a capacity to reach a higher stress, with thinner edges, however knowing the two sets of figures the safer figure is the one found using the Tresca yield criterion.

Given that a physical item was used to experiment Von-Mises criterion in my previous task, this could not be accomplished in this paper.

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